the peak acceleration of the containment vessel foundation was approximately equal to the peak free-field value for a soil with a shear wave velocity of 2000 fps [5, 6].[7] However, the spectrum calculated with the containment vessel foundation acceleration was found to be reduced at the fixed-base natural frequencies of the containment vessel where as for the Alexander building the spectrum was increased. Thus, in the first case, seismic inertia loads acting on the structure are reduced and in the latter case loads slightly increased for stiff soils.

Significant interaction effects caused by the vertical and rocking modes have also been observed by Isenberg [7] and Chiapetta [8] using finite-element methods. These trends are consistent with observations made by the discussers.

The authors wish to thank Professor Rainer for the careful review of their work. However, the authors do not feel that the conclusions are erroneous. A great deal of effort was spent comparing results of the presented analysis with finite-element results [7, 8]. Excellent agreement was obtained with the finite-element work of Isenberg, [7, pp. 132–137]. The authors realize that the discusser did not have the opportunity to study referenced document.

In the analysis of the subject paper, the acceleration term, $\ddot{u}$, denotes the structure acceleration at the ground. Given this ground motion, the force acting on each mass of the structure, m_i, can be calculated by the formula [9]

$$F_i = -m_i \sum_a \bar{X}_{ia} P_a \omega_a \int_0^t \ddot{u}(\tau) \sin \omega_a (t - \tau) d\tau \qquad (1)$$

The modal participation factor is defined by

$$P_a = \frac{\sum_i m_i \bar{X}_{ia}}{\sum_i m_i \bar{X}_{ia}{}^2} \qquad (2)$$

where $\bar{X}_{ia}$ and ω_a are the mode shapes and circular frequencies obtained with the structure fixed at the base. When normal mode theory is applied to a linear N-mass structure, the fixed-base natural frequencies appear in the resulting equation. It is for this reason that only the fixed-base frequencies are of significance in this analysis. The use of these fixed-base frequencies is well known in naval shock [10].

When study of this phenomenon was initiated, a true resonance of the structure was expected. Many input frequencies were employed attempting to obtain resonance in this study as well as previous work [11]. None were found. However, it should be pointed out that shock loads increase at the resonant frequency even though a true resonance was not obtained, Fig. 6.

In reference [6], a preliminary study of structural damping was made. Spectrum responses of both the free-field input $\ddot{u}_p(t)$ and $\ddot{u}(t)$ were calculated assuming the same damping applied to each motion. Interaction effects were still found to be significant. The work of the discussers Lee and Wesley would also apply to this point.

The effective mass presented in equation (2) of the subject paper is a result of the linear N-mass approximation. This equation is obtained by applying normal mode theory of the N-mass system and is not a concept. In the discussion by Tajimi referenced by the discusser, the same equation (equation (30)) is used in the analysis.

The authors feel that the greatest weakness of this analysis is that the foundation is assumed to be two dimensional. The effect of the third dimension on this transient analysis is not known. Because of the good agreement with finite-element results, the approximation of the constant stress boundary condition is felt not to be significant.

References

5 Scavuzzo, R. J., Bailey, J. L., and Raftopoulos, D. D., "Lateral Structure-Foundation Interaction of Nuclear Power Plants During Earthquake Loading," USAEC Contract No. AT-(40-1)3822, Technical Report No. 2, The Research Foundation, The University of Toledo, Aug. 1969, p. 44.

6 Scavuzzo, R. J., Raftopoulos, D. D., and Bailey, J. L., "Lateral Structure-Foundation Interaction of Nuclear Power Plants With Large Base Masses," USAEC Contract No. AT-(40-1)3822, Technical Report No. 3, The Research Foundation, University of Toledo, Sept. 1969, p. 59.

7 Isenberg, J., "Interaction Between Soil and Nuclear Reactor Foundations During Earthquakes," USAEC Contract No. AT-(40-1)3822, ORO-3822-5, Ababian-Jacobsen Associate, Los Angeles, Calif., June 1970.

8 Chiapetta, R., "Effect of Soil-Structure Interaction on the Response of Reactor Structures to Seismic Ground Motion," USAEC Contract No. AT-(40-1) 3822, ORO-3882-4, IIT Research Institute, Chicago, Ill., Apr. 1970.

9 O'Hara, G. J., and Cuniff, P. F., "Elements of Normal Mode Theory," NRL Report 6002, Nov. 1963.

10 O'Hara, G. J., "Background for Mechanical Shock Design of Ship Systems," NRL Report 6267, March 12, 1965.

11 Scavuzzo, R. J., "Foundation-Structure Interaction in the Analysis of Wave Motions," *Bulletin of the Seismological Society of America*, Vol 57, 1967, pp. 735–746.

[7] Numbers in brackets designate References at end of Closure.

Normal Impact of an Infinite Elastic Beam by a Semi-Infinite Elastic Rod[1]

A. J. Durelli.[2] The following comments refer to the experiment conducted by the author. The experiment was designed to obtain results in terms of bending moments. For this reason strain gages were affixed at opposite points on the top and bottom surface of the beam and the difference in their outputs was recorded. It is the discusser's opinion that this way of designing the experiment may have hidden some important features of the phenomenon.

The author assumes in his thoretical analysis that the dynamic response of a beam to normal impact consists entirely of bending waves. It would be interesting to verify this assumption experimentally by observing the top and bottom strain gage signals separately and noting whether the required antisymmetry between the output signals exists. If longitudinal waves are also present in the beam then the author's assumption of an infinite beam in his theoretical solution may no longer be valid since reflections of the faster longitudinal waves could occur before bending waves reach the ends of the beam. This possibility could be checked if the author would specify the length of the beam used in his experiment. The existence of higher symmetric and antisymmetric modes of bar waves might also be anticipated. All of these possibilities underline the importance of conducting an experiment from which the state of stress can be determined independently of assumptions about the types of waves propagating in the beam.

The author does not seem to be acquainted with a paper of the discusser in which results are given from a beam impacted by a striker [1].[3] Because of differences in several of the parameters a comparison with the author's results is not immediate, but it seems that the photoelastic approach would have given the author much more information, Figs. 1 and 2 of this Discussion. The strains observed photoelastically did not display everywhere and at all times antisymmetry assumed by the author and effects of the end supports on the stresses distribution were observed before stresses associated with a bending type of deformation of the

[1] By S. Ranganath published in the June, 1971, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 38, TRANS. ASME, Vol. 93, Series E, pp. 455–460.

[2] Professor, Department of Civil and Mechanical Engineering, The Catholic University, Washington, D. C.

[3] Numbers in brackets designate References at end of Discussion.

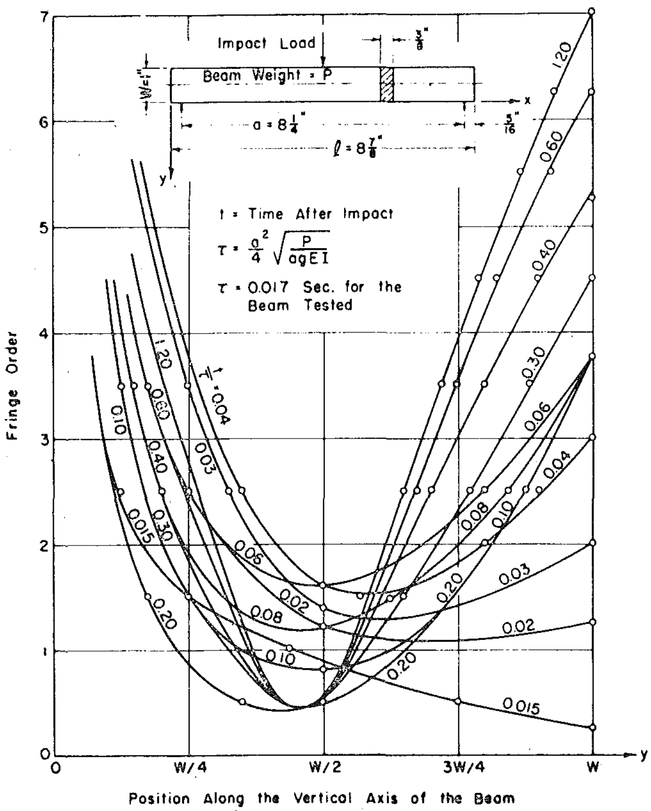

Fig. 1 Photoelastic fringe order along center line of a simply supported beam under central impact as a function of time

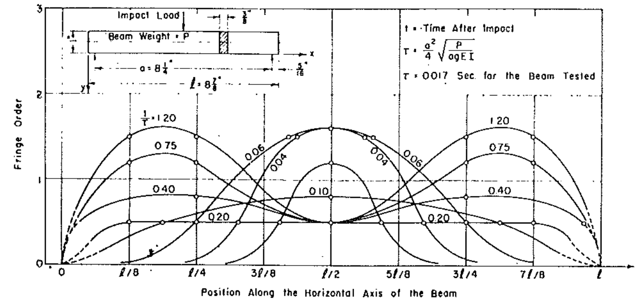

Fig. 2 Photoelastic fringe order along horizontal axis of a simply supported beam under central impact as a function of time

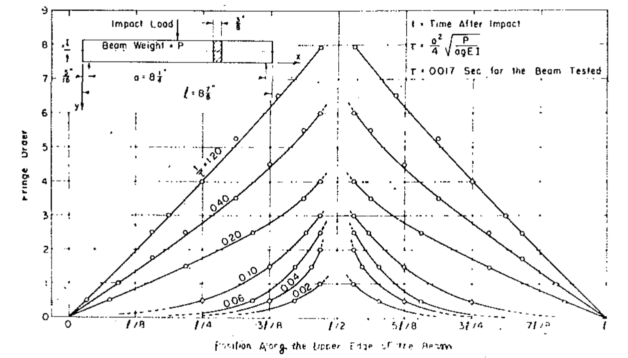

Fig. 3 Photoelastic fringe order along upper edge of a simply supported beam under central impact as a function of time

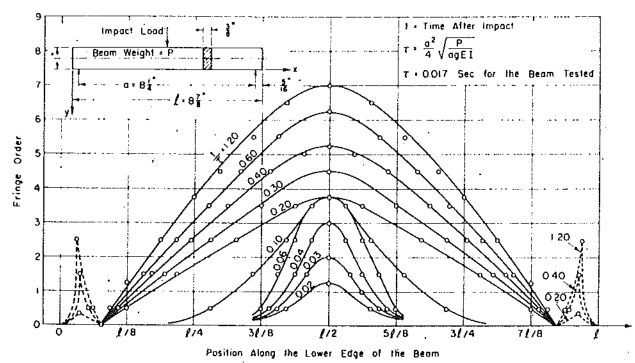

Fig. 4 Photoelastic fringe order along lower edge of a simply supported beam under central impact as a function of time

beam reached the end supports, Figs. 3 and 4 of this Discussion. It should be pointed out that techniques in dynamic photoelasticity have improved appreciably since the time the photoelastic tests on the beam were conducted, and good results can be obtained using hard materials as epoxy and microflash illumination [2, 3]. The interpretation of fringe orders in terms of stresses can also be a simple matter and the verification of theoretical assumptions concerning the types of waves propagating in a beam by observations of the instantaneous, full-field strain distribution can be readily accomplished [4].

References

1 Durelli, A. J., and Riley, W. F., "Experiments for the Determination of Transient Stress and Strain Distributions in Two-Dimensional Problems," Journal of Applied Mechanics, Vol. 24, Trans. ASME, Vol. 79, Mar. 1957, pp. 69–76.

2 Riley, W. F., and Durelli, A. J., "Stress Distribution in the Boundary of a Circular Hole in a Large Plate During Passage of a Stress Pulse of Short Duration," *Journal of Mechanical Engineering Science*, Vol. 3, No. 1, 1961, pp. 62–68.

3 Durelli, A. J., Clark, J. A., and Kochev, A., "Experimental Analysis of High-Frequency Stress Waves in a Ring," *Journal of Strain Analysis*, Vol. 4, No. 4, 1969, pp. 297–305.

4 Clark, J. A., and Durelli, A. J., "Optical Stress Analysis of Flexural Waves in a Bar," Journal of Applied Mechanics, Vol. 37, No. 2, Trans. ASME, Vol. 92, June 1970, Series E, pp. 331–338.

Author's Closure

The author thanks Dr. Durelli for his interest in the work The discusser remarks that the assumption of an infinite beam in the theoretical solution may not be valid because of reflections at the end of the beam. The steel beam used in the experiment was 36 in. long. The duration of the stress pulse was 100 μs and no reflections from the end could have reached the stations before this time. Thus the assumption of an infinite beam in the theoretical analysis was valid for the periods where the comparison of theory and experiment was made. Although in the results reported, the difference in the output of the top and bottom gages was taken to obtain the bending response, independent measurements of the output of the two gages were also made separately and there was no significant difference in the responses. In fact, the sum of the outputs of the two gages was also observed and it was found to be negligibly small. Thus the velocities of impact were so small that the longitudinal strain was insignificant. The effect of longitudinal strain cannot, however, be ignored for very large deflections. The author is presently working on the analysis of the elastic impact problem for large deflections including the effects of curvature.

As for the comment about the advantages of the photoelastic technique over strain-gage methods, the author makes no claim about the superiority of strain gages for obtaining the dynamic response. Strain gages were used in the experiments reported here because they provided a convenient, dependable means for examining the correctness of the main predictions of the Timoshenko beam theory.