Many-Particle and Many-Hole States in Neutron-Rich Ne Isotopes Related to Broken $N = 20$ Shell Closure

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The low-lying level structures of $^{26}$Ne, $^{28}$Ne and $^{30}$Ne which are related to the breaking of the $N=20$ shell closure have been studied in the framework of the deformed-basis anti-symmetrized molecular dynamics plus generator coordinate method using the Gogny D1S force. The properties of the many-particle and many-hole states are studied as well as that of the ground band. We predict that the negative-parity states, in which neutrons are promoted into the $pf$-orbit from the $sd$ orbit, have a small excitation energy in the cases of $^{28}$Ne and $^{30}$Ne. We regard this to be a typical phenomena accompanying the breaking of the $N=20$ shell closure. It is also found that the neutron $4p4h$ structure of $^{30}$Ne appears at low excitation energy, which contains $\alpha + ^{16}$O correlations.

§1. Introduction

In recent years, many experimental and theoretical efforts to investigate the properties of nuclei away from the stability valley have shown the variety of the nuclear binding systems that is beyond our standard understanding of the nuclear properties established in the study of stable nuclei. The discovery of the neutron-halo phenomenon$^1)$ proved that the basic concept of the density saturation does not hold near the neutron drip-line. Another important concept of the shell structure and the magic number is also under reconsideration because of their rearrangement in neutron-rich nuclei which is deduced from the breaking of the neutron shell closure in neutron-rich $N=20$ isotones.

The breaking of the $N=20$ shell closure in neutron-rich nuclei was first pointed out from the observation of the anomalous ground state spin of $^{31}$Na$^2$) associated with the prolate deformation. Theoretically, it was suggested that the large deformation which is caused by the promotion of the neutrons from the $sd$-shell to the $f$-shell possibly overcomes the $N=20$ shell effect in the neutron-rich $N=20$ isotones such as $^{31}$Na and $^{32}$Mg.$^3$) Shell model studies$^4)^{-}6)$ also support neutron promotion into the $pf$-shell, and a striking result was found for $^{32}$Mg,$^7$) namely that the neutron $2p2h$ configuration (two holes in the $sd$ shell and two particles in the $pf$ shell) dominates the ground state of $^{32}$Mg, and a large $B(E2; 0^+_1 \rightarrow 2^+_1)$ due to the large deformation was predicted. Since the first observation of the $B(E2; 0^+_1 \rightarrow 2^+_1)$ value in $^{32}$Mg,$^8$) which confirmed the shell model prediction, many experimental and theoretical studies have been devoted to $Z \sim 10$ and $N \sim 20$ nuclei.$^9)^{-}24)$ Presently, a systematic large deformation and breaking of the $N=20$ shell closure are theoretically expected in the ground states of $^{28-32}$Ne,$^{29-33}$Na and $^{28-34}$Mg and have been
exponentially investigated in $^{32,34}\text{Mg}$. Recently, a new $NN$ effective interaction, which has a stronger $(\sigma \cdot \sigma)(\tau \cdot \tau)$ term, was suggested to explain the drastic change of the location of the neutron drip-line from oxygen isotopes to fluorine isotopes based on a Monte Carlo shell model study.\textsuperscript{15} They argued that the extension of the neutron drip-line in F isotopes is caused by the mixing of the neutron $0p0h$, $2p2h$ and $4p4h$ configurations, which is driven not by the deformation but by the stronger $(\sigma \cdot \sigma)(\tau \cdot \tau)$ interaction.

However, the experimental and theoretical information of low-lying states of neutron-rich isotopes is not yet sufficient. In particular, negative-parity states of even-even nuclei are still not confirmed in experiments,\textsuperscript{18,24} and few theoretical studies have been made.\textsuperscript{6} The neutron $2p2h$ dominance in the ground states of neutron-rich $N=20$ isotones implies that one or three neutrons can be easily promoted into the $pf$-shell with a small excitation energy, which leads to the existence of low-lying negative-parity states. Indeed, a shell model study\textsuperscript{6} has shown the possible existence of low-lying neutron $1h\omega$ and $3h\omega$ states in this mass region. Furthermore, the parity of the ground state of even-odd nuclei $^{33}\text{Mg}$ is identified to be the positive\textsuperscript{25,26} that means the neutron $1h\omega$ structure of the ground state. Therefore it is important to study negative-parity states to understand the structure of neutron-rich $N \sim 20$ nuclei. In the same sense, the excited positive-parity bands that have the neutron $0p0h$ and $4p4h$ structures dominantly are also important. In particular, in the case of the $4p4h$ structure, due to the presence of the four neutrons in the $pf$-shell, the protons that are almost frozen in the ground state might be activated. Actually, we show that ($\alpha+^{16}\text{O}+\text{valence-neutrons type}$) cluster-like correlations can exist in the neutron $4p4h$ structure of $^{30}\text{Ne}$, which appears in a rather low energy region.

The purpose of this article is to provide theoretical information on the properties of the negative-parity states and the excited bands of the positive-parity of the even-even Ne isotopes in order to understand the structure of the neutron-rich $sd$-shell nuclei around the broken $N=20$ shell closure. The low-lying level structures of $^{26}\text{Ne}$, $^{28}\text{Ne}$ and $^{30}\text{Ne}$ are studied by using the deformed-basis AMD+GCM framework (deformed-basis antisymmetrized molecular dynamics plus generator coordinate method). The deformed-basis AMD framework has now been confirmed to accurately describe the mean-field structure like the Hartree-Fock method, in addition to its ability to accurately describe the cluster structure. Since in AMD, the energy variation is made after the parity projection, the AMD can describe negative parity states which almost no Hartree-Fock calculations can treat. The use of AMD also enables us to study the possible existence of the $\alpha+^{16}\text{O}+\text{valence-neutrons type}$ structure in neutron-rich Ne isotopes. Because the low-lying states of $^{20}\text{Ne}$ have the $\alpha+^{16}\text{O}$ cluster structure, the cluster correlation may survive even in the neutron-rich isotopes, as in the case of neutron-rich Be isotopes which inherit the cluster structure of $^{8}\text{Be}$. It will be shown that the excitation energy of the negative-parity states becomes lower around $N=20$ because of the breaking of the $N=20$ shell closure and the $\alpha+^{16}\text{O}$ cluster correlations appears in the positive-parity $4p4h$ states.

The contents of this article are as follows. In the next section, the theoretical framework of the deformed-basis AMD is briefly explained. In §3, the obtained
energy curves and level schemes are discussed. The $E2$ transition probabilities are also studied. The density distribution and the possible existence of the cluster core in the ground and excited states of $^{26}$Ne, $^{28}$Ne and $^{30}$Ne are examined. In the last section, we summarize this work.

§2. Theoretical framework

In this section, the framework of the deformed-basis AMD+GCM is explained briefly. For a more detailed explanation of the framework of the deformed-basis AMD, readers are referred to references.\textsuperscript{27,28} The intrinsic wave function of the system of mass $A$ is given by a Slater determinant of the single-particle wave packets $\phi_i(r)$:

$$\Phi_{\text{int}} = \frac{1}{\sqrt{A!}} \det\{\phi_1, \phi_2, \ldots, \phi_A\},$$

$$\phi_i(r) = \phi_i(r) \chi_i \xi_i.$$  \hspace{1cm} (2.1)

Here, the single-particle wave packet $\phi_i$ consists of the spatial $\phi_i$, spin $\chi_i$ and isospin $\xi_i$ parts. The deformed-basis AMD employs a triaxially deformed Gaussian centered at $Z_i$ as the spatial part of the single-particle wave packet:

$$\phi_i(r) \propto \exp \left\{ - \sum_{\sigma=x,y,z} \nu_\sigma (r_\sigma - Z_\sigma) \right\},$$

$$\chi_i = \alpha_i \chi^\uparrow + \beta_i \chi^\downarrow, \quad |\alpha_i|^2 + |\beta_i|^2 = 1,$$

$$\xi_i = \text{proton or neutron}.$$  \hspace{1cm} (2.2)

Here, the complex number parameter $Z_i$, which represents the center of the Gaussian in the phase space takes an independent value for each nucleon. The width parameters $\nu_x, \nu_y$ and $\nu_z$ are real number parameters and take independent values for the $x$, $y$ and $z$ directions, but they are common for all nucleons. The spin part $\chi_i$ is parametrized by $\alpha_i$ and $\beta_i$, and the isospin part $\xi_i$ is fixed to up (proton) or down (neutron). $Z_i, \nu_x, \nu_y, \nu_z$ and $\alpha_i, \beta_i$ are the variational parameters and are optimized by the method of frictional cooling. The advantage of the triaxially deformable single-particle wave packet is that it makes it possible to describe the cluster-like structure and deformed mean-field structure within a single framework, which was discussed in Ref. 28).

As the variational wave function, we employ the parity projected wave function in the same way as in many other AMD studies:

$$\Phi^\pm = P^\pm \Phi_{\text{int}} = \left( \frac{1 \pm P_x}{2} \right) \Phi_{\text{int}}.$$  \hspace{1cm} (2.4)

Here, $P_x$ is the parity operator and $\Phi_{\text{int}}$ is the intrinsic wave function given in Eq. (2.1). Parity projection makes it possible to determine the different structure of the intrinsic state for the different parity states.

The Hamiltonian used in this study is

$$\hat{H} = \hat{T} + \hat{V}_n + \hat{V}_c - \hat{T}_g,$$  \hspace{1cm} (2.5)
where $\hat{T}$ and $\hat{T}_g$ are the total kinetic energy and the energy of the center-of-mass motion, respectively. We have used the Gogny force with the D1S parameter set as the effective nuclear force $\hat{V}_n$. The Coulomb force $\hat{V}_c$ is approximated by the sum of seven Gaussians. The energy variation is made under a constraint on the nuclear quadrupole deformation by adding to $\hat{H}$ the constraint potential $V_{\text{cnst}} = v_{\text{cnst}}(\langle \beta \rangle^2 - \beta_0^2)^2$ with a large positive value of $v_{\text{cnst}}$. At the end of the variational calculation, the expectation value of $V_{\text{cnst}}$ should be zero in principle, and in the actual calculation, we confirm that it is less than 0.1 keV. The optimized wave function is denoted by $\Phi^\pm(\beta_0)$. Here it should be noted that this constraint does not apply to the deformation parameter $\gamma$, which means that $\Phi^\pm(\beta_0)$ with positive $\beta_0$ can be not only prolate but also oblate.

From the optimized wave function, we project out the eigenstate of the total angular momentum $J$:

$$\Phi^{J\pm}_{MK}(\beta_0) = P^{J\pm}_{MK} \Phi^\pm(\beta_0) = P^{J\pm}_{MK} \Phi_{\text{int}}(\beta_0).$$  \hspace{1cm} (2.6)$$

Here, $P^{J\pm}_{MK}$ is the total angular momentum projector. The integrals over the three Euler angles included in the $P^{J\pm}_{MK}$ were evaluated by numerical integration.

Furthermore, we superpose the wave functions $\Phi^{J\pm}_{MK}$ which have the same parity and angular momentum but have different value of the deformation parameter $\beta_0$ and $K$. Thus, the final wave function of the system becomes

$$\Phi_n^{J\pm} = c_n \Phi^{J\pm}_{MK}(\beta_0) + c'_n \Phi^{J\pm}_{MK'}(\beta'_0) + \cdots,$$  \hspace{1cm} (2.7)$$

where quantum numbers other than the total angular momentum and parity are represented by $n$. The coefficients $c_n$, $c'_n$, $\cdots$ are determined by the Hill-Wheeler equation,

$$\delta \left( \langle \Phi_n^{J\pm} | \hat{H} | \Phi_n^{J\pm} \rangle - \epsilon_n \langle \Phi_n^{J\pm} | \Phi_n^{J\pm} \rangle \right) = 0. \hspace{1cm} (2.8)$$

§3. Low-lying level structure of neutron-rich Ne isotopes

We have studied low-lying level schemes of the neutron-rich Ne isotopes, $^{26}\text{Ne}$, $^{28}\text{Ne}$ and $^{30}\text{Ne}$ to study the change of the shell structure toward $N=20$ and the mechanism of the nuclear excitation in these isotopes. For the positive-parity levels, the possible existence of the $\alpha+^{16}\text{O}$ core is also examined.

Before discussing the obtained results, we explain the analysis of the obtained wave function and the notation used in this study to clarify our discussion. In this study, the single-particle structure of the obtained wave function is analyzed by constructing the Hartree-Fock single-particle Hamiltonian from the obtained AMD wave function \cite{29} as follows.

When the optimized wave function $\Phi^\pm = P^{\pm} \frac{1}{\sqrt{A!}} \det \{ \varphi_1, \varphi_2, \ldots, \varphi_A \}$ is given, we calculate the orthonormalized basis $\phi_\alpha$, which is a linear combination of the single-particle wave packets $\varphi_i$:

$$\phi_\alpha = \frac{1}{\sqrt{\mu_\alpha}} \sum_{i=1}^{A} c_{i\alpha} \varphi_i.$$  \hspace{1cm} (3.1)$$
Here, \( \mu_\alpha \) and \( c_{i\alpha} \) are the eigenvalue and eigenvector of the overlap matrix \( B_{ij} \equiv \langle \varphi_i | \varphi_j \rangle \):

\[
\sum_{j=1}^{A} B_{ij} c_{j\alpha} = \mu_\alpha c_{i\alpha}.
\] (3.2)

It is clear that \( \phi_\alpha \) is orthonormalized from this relation. Using this basis set of \( \phi_\alpha \), we calculate the Hartree-Fock single-particle Hamiltonian \( h_{\alpha\beta} \), which is defined as

\[
h_{\alpha\beta} \equiv \langle \phi_\alpha | \hat{t} | \phi_\beta \rangle + \sum_{\gamma=1}^{A} \langle \phi_\alpha \phi_\gamma | \hat{v}_{\text{Gogny}} + \hat{v}_{\text{Coulomb}} | \phi_\beta \phi_\gamma - \phi_\gamma \phi_\beta \rangle + \frac{1}{2} \sum_{\gamma=1}^{A} \langle \phi_\gamma \phi_\delta | \phi_\alpha^* \phi_\beta \partial \hat{v}_{\text{Gogny}} / \partial \rho | \phi_\delta \phi_\rho - \phi_\rho \phi_\delta \rangle,
\] (3.3)

where \( \hat{t} \), \( \hat{v}_{\text{Gogny}} \) and \( \hat{v}_{\text{Coulomb}} \) denote the kinetic operator, the Gogny force, and the Coulomb potential. \( \partial \hat{v}_{\text{Gogny}} / \partial \rho \) denotes the derivative of the density dependent term of the Gogny force.

Through the diagonalization of \( h_{\alpha\beta} \), we obtain the single-particle energy \( \epsilon_s \) and the single-particle wave function \( \tilde{\phi}_s \):

\[
\sum_{\beta=1}^{A} h_{\alpha\beta} f_{\beta s} = \epsilon_s f_{\alpha s},
\] (3.4)

\[
\tilde{\phi}_s = \sum_{\alpha=1}^{A} f_{\alpha s} \phi_\alpha.
\] (3.5)

We note that the single-particle energy \( \epsilon_s \) and wave function \( \tilde{\phi}_s \) are obtained for occupied states but not for unoccupied states from this method. Furthermore, since the actual variational calculation is made after the parity projection (the superposition of the two Slater determinants), it does not allow a naive interpretation of \( \phi^\pm (\beta_0) \) provided by the single-particle picture. However, the single-particle structure obtained with this method is useful to understand the particle-hole structure of the obtained wave function. From the parity \( \pi_s^{\pm} = \langle \tilde{\phi}_s | P^{\pm} | \tilde{\phi}_s \rangle \) and the angular momentum \( \langle \tilde{\phi}_s | \hat{j}_z | \tilde{\phi}_s \rangle \) in the intrinsic frame, we have determined the particle-hole structure. From this analysis, we find that the obtained wave functions have various types of single-particle structure. For example, in the case of the positive-parity states of \( ^{30}\text{Ne} \), neutron 0p0h, 2p2h and 4p4h structures with respect to \( N=20 \) appear in the low-lying states. When protons are excited, proton 2p0h, 3p1h and 4p2h structures with respect to \( Z=8 \) are combined with the neutron \( ph \) structures. For convenience, in the following, ‘mpnh structure’ refers to the neutron \( ph \) structure. When we need to distinguish it from that of protons, it will be written explicitly.

3.1. Energy curves

The energy curves as functions of the matter quadrupole deformation parameter \( \beta \) obtained before and after the angular momentum projection are plotted in Fig.
Fig. 1. Obtained energy curves of the positive-parity states of $^{26}$Ne (a), $^{28}$Ne (b) and $^{30}$Ne (c). In each panel, the energies of the parity-projected intrinsic state (dashed curve) and the parity and angular momentum-projected states are plotted as functions of the matter quadrupole deformation parameter $\beta$.

Fig. 2. The neutron single-particle energies of $^{26}$Ne (a), $^{28}$Ne (b) and $^{30}$Ne (c) obtained from the intrinsic wave functions on the positive-parity energy curves. The positive (negative) parity single-particle orbits are plotted by the solid (dashed) curves. Each single-particle orbit is occupied by two neutrons.

1 for positive-parity states. The angular-momentum-projected energy curve of the positive-parity state of $^{30}$Ne [Fig. 1 (c)] has an energy minimum around $\beta \sim 0.4$ in each spin state. The energy gain of this deformed state is due to the restoration of the rotational symmetry and it amounts to about 5 MeV in the case of the $J^\pi=0^+$ state. This deformed minimum state has a neutron $2p2h$ structure. The neutron single-particle energies of $^{30}$Ne are plotted as functions of the deformation in Fig. 2 (c). From $\beta=0$ to $\beta=0.2$, all neutrons are almost frozen in the $N=20$ shell closure. This inactivity of neutrons explains the absence of the $6^+$ and $8^+$ states in this region, since only two protons in the $sd$-shell contribute to the total spin of the system. From $\beta=0.2$ to $\beta=0.5$, two neutrons are promoted into the $pf$-shell ($2p2h$), and the system is most deeply bound. The largely-deformed state ($\beta > 0.5$) has a $4p4h$ structure in which four neutrons are promoted into the $pf$-shell. This state has not been investigated in detail in other model studies. One reason for this may be that the stability of the mean-field structure of this state is doubtful. Indeed,
in our result, this state does not appear as a local minimum on the energy curve, but, rather, as a shoulder around $\beta=0.5$. However, when we superpose the wave functions on the energy curve, this state contributes to the $K^\pi=0^+_3$ band and is stabilized through the orthogonalisation with respect to the lower two bands. The parity projection before variation also helps to lower the energy of the $4p4h$ structure. The energy of this state is lowered by about 2 MeV by the parity projection, while the $0p0h$ and $2p2h$ structures are not so strongly affected (by less than 1 MeV). Note that the $4p4h$ state has a parity asymmetric intrinsic density distribution. This asymmetry is the reason why the energy of the $4p4h$ structure is lowered by the parity projection. Furthermore, the proton density distribution implies the possible existence of an $\alpha+^{16}\text{O}$ cluster structure. Indeed, the overlap between the proton wave function of this state and that of $^{20}\text{Ne}$, which has an $\alpha+^{16}\text{O}$ cluster structure, is quite large, amounting to about 80%. Therefore, the interpretation of this state offered by the $\alpha+^{16}\text{O}$-valance-neutron models will be appropriate. In the present calculation, we did not find another state which has the apparent $\alpha+^{16}\text{O}$-valance-neutron-like density distribution. The overlap of the proton wave function with that of the $^{20}\text{Ne}$ wave function is largest in this state, and it decreases in $^{28}\text{Ne}$ and $^{26}\text{Ne}$ as the neutron number decreases. The existence of the $0p0h$, $2p2h$ and $4p4h$ structures within a small excitation energy (lying within about 5 MeV measured from the lowest $2p2h$ structure) implies the softness of the neutrons in the $0d_{3/2}$ orbit of this nucleus. In other words, it does not cost much energy to promote neutrons from the shell which originates in the spherical $0d_{3/2}$ shell into the higher shell which originates in the spherical $pf$-shell. It can be attributed to the large deformation caused by the neutron excitation, the restoration of the rotational symmetry and the reduced $N=20$ energy gap owes to the excess of neutrons.

The energy curve of the positive-parity state of $^{28}\text{Ne}$ [Fig. 1 (b)] has an oblately deformed minimum at $\beta=0.25$, which has $0p2h$ structure. We also find that the prolately deformed $2p4h$ structure has a minimum at $\beta=0.5$ whose energy is about 1 MeV above the oblately deformed $0p2h$ state. This small excitation energy of the $2p4h$ structure also implies the softness of the neutron’s $0d_{3/2}$ orbit again. Since two different structures coexist within the small excitation energy, when we superpose these wave functions, a strong mixing between these configurations results. This is discussed in the next subsection. The change of the shape from oblate to prolate is the reason why the single particle energies as functions of $\beta$ are discontinuous around $\beta=0.35$ in $^{28}\text{Ne}$. The energy of the neutron $0d_{3/2}$ orbit of $^{28}\text{Ne}$ becomes lower as

![Fig. 3. Proton and neutron intrinsic density distributions of the neutron $4p4h$ structure of $^{30}\text{Ne}$. For the sake of comparison, those of the $2p2h$ structure are also shown.](image-url)
deformation becomes larger in the oblately deformed region \((0 \leq \beta \leq 0.45)\), while that of \(^{30}\text{Ne}\) (lower \(0d_{3/2}\) of \(^{30}\text{Ne}\)), which is prolately deformed, becomes higher in the region, \(0.1 \leq \beta \leq 0.45\). This behavior of the lower \(0d_{3/2}\) orbits is qualitatively the same as that described by the Nilsson model.\(^{30}\) i.e., \([N, n_z, m_l, \Omega] = [2,1,1,3/2]\) orbit for the oblate state and \([N, n_z, m_l, \Omega] = [2,0,0,1/2]\) orbit for the prolate state. Because the neutron number \(N = 18\) prefers oblate deformation in the small deformed region, owing to the deformed shell effect, the \(0p2h\) structure of \(^{28}\text{Ne}\) has an oblate shape. Contrastingly, the neutron \(2p4h\) state has a prolate shape, because the proton number \(N = 10\) prefers a prolate deformation, though the neutron’s shell effects are comparable for the prolate and oblate deformations. In this nucleus, the \(4p6h\) structure does not appear as an energy minimum or a shoulder on the energy curve. When we calculate the more deformed state, it appears at about 20 MeV above the \(0p2h\) state, and it does not seem to be a stable structure. We believe that the absence of the stable \(4p6h\) structure implies the hardness of the \(N = 16\) deformed shell. Here, by the deformed \(N = 16\) shell, we mean a shell of \(sd\) orbits without the orbits coming from the spherical \(d_{3/2}\) orbit.

The energy curve of the positive-parity state of \(^{26}\text{Ne}\) [Fig. 1 (a)] has an energy minimum at \(\beta = 0.25\), which has the \(0p4h\) structure. It also has the \(2p6h\) shoulder in which two neutrons are promoted into the \(pf\)-shell, but its excitation energy is rather high (about 7 MeV in the case of the \(0^+\) state). We note that the \(2p6h\) structure of this nucleus is a superdeformed state. The deformed orbit configuration of this state is the same as that of the closed shell state with the magic number of the superdeformation \(N = 16\). However, the single-particle energy diagram of Fig. 2 (a) does not show clearly the deformed magic number \(N = 16\).

Here, we make some additional comments on the single particle energy diagrams of Fig. 2. Except the oblate region in \(^{28}\text{Ne}\), the single particle energy diagrams of the three isotopes are very similar. From this similarity we can understand why the deformation of the \(2\hbar\omega\)-jump structure becomes larger for lighter Ne isotopes reaching the superdeformation in \(^{26}\text{Ne}\). It is because the uppermost occupied orbit which crosses first the intruder \(pf\)-orbit changes to lower \(sd\)-orbit as going to lighter isotope, which results in larger value of \(\beta\) of the crossing point.

Next, we discuss the energy curves of the negative-parity-states (Fig. 4). The energy curves of the positive-parity states exhibit softness to excite neutrons in the shell, which originates in the spherical \(0d_{3/2}\) shell, and hardness to excite neutrons below \(N = 16\). We find that this feature is inherited in the negative-parity states. Specifically, the \(1p1h\) and \(3p3h\) structures of \(^{30}\text{Ne}\) and the \(1p3h\) structure of \(^{28}\text{Ne}\) have small excitation energies, while the \(3p5h\) structure of \(^{26}\text{Ne}\) and the \(1p5h\) structures of \(^{26}\text{Ne}\) have larger excitation energies. In the negative-parity states of \(^{30}\text{Ne}\), there are two low-lying groups of minimums. The curves that are minimized around \(\beta = 0.45\) are the \(K\pi = 1^-\) rotational band members that have the \(3p3h\) structure. Around \(\beta = 0.3\), there is \(J\pi = 2^-, 3^-, 4^-\) and \(5^-\) states which have the \(1p1h\) structure. The order of these states is changed by the GCM calculation. This will be discussed in the next subsection. The interesting point is that both the \(1p1h\) and \(3p3h\) structures have small excitation energies, 3.6 MeV and 4 MeV as measured from the positive-parity
Fig. 4. Obtained energy curves for the negative parity states of $^{26}$Ne (a), $^{28}$Ne (b) and $^{30}$Ne (c). In each panel, the energies of the parity projected intrinsic state (dashed curve) and the parity and angular momentum projected states are plotted as functions of the matter quadrupole deformation parameter $\beta$.

Fig. 5. The proton and neutron single particle energies of $^{26}$Ne (a) (b), $^{28}$Ne (c) (d) and $^{30}$Ne (e) (f) obtained from the intrinsic wave functions on the negative-parity energy curves. The positive (negative) party single particle orbits are plotted by the solid (dashed) curves. The single particle orbits given by the thin (bold) curves are occupied by two (one) protons or neutrons.
2p2h state. When we compare the energy curves of the positive- and negative-parity states of \( ^{30}\text{Ne} \), we find that the neutron excitation and nuclear deformation are correlated. As the deformation becomes larger, neutrons in the shell that originates in the spherical \( 0d_{3/2} \) shell are promoted into the \( pf \)-shell, with \( m=0, 1, 2, 3 \) and 4, in the order \( mpmh \). The energy loss due to the neutron promotion is not large, and it is always comparable with the energy gain due to the deformation of the intrinsic state and the restoration of the rotational symmetry. Thus the neutron \( 0\text{-}4p \) structure appears in the low-lying state. It is interesting that the \( J^\pi=1^- (K^\pi=1^-) \) state, which has a neutron \( 3p3h \) structure is even lower than the states that have a neutron \( 1p1h \) structure. In the shell model calculation, the \( 1p1h \) structure is lower than the \( 3p3h \) structure.\(^{6} \) Though the energy loss due to the neutron promotion in the \( 3p3h \) structure is larger than that for the \( 1p1h \) structure, a larger deformation and smaller angular momentum provide a larger energy gain due to the deformation of the intrinsic state and the restoration of the rotational symmetry. We also find an interesting structure around \( \beta=0.6 \), which appears as a \( K^\pi=2^- \) band. Because the deformation is too large, it seems that the neutron \( 3p3h \) structure is not able to form a stable mean-field. But the neutron \( 5p5h \) structure costs a large amount of energy because of the hardness of the \( N=16 \) shell. Therefore, a proton in the \( 0p \)-shell is drafted to make a negative parity state. Therefore, in this structure, the system has a proton \( 3p1h \) and neutron \( 4p4h \) structure in which the \( ^{16}\text{O} \) core is excited. In other words, this state can be understood as the state in which the neutron \( 4p4h \) state of the positive-parity state is excited by the proton excitation.

The energy curve of the negative-parity state of \( ^{28}\text{Ne} \) has two minima. The lower one, which is minimized at around \( \beta=0.3 \), has a \( 1p3h \) structure. It produces \( J=3^- \) and \( 5^- \) states, and both have small excitation energies. By contrast, the \( 3p5h \) state, which produces a \( K^\pi=1^- \) band around \( \beta=0.55 \), has a much larger excitation energy due to the hardness of the deformed \( N=16 \) shell. Because of the hardness of the deformed \( N=16 \) shell closure, the negative-parity states of \( ^{26}\text{Ne} \) do not appear in the low-energy region. \( ^{26}\text{Ne} \) has \( 1p5h \) structure around \( \beta=0.5 \), but its energy is about 6MeV higher than the positive-parity \( 0p4h \) state. The proton excitation is also found in this nucleus. In the largely deformed region (\( \beta>0.6 \)), the system chooses the proton \( 3p1h \) and neutron \( 2p6h \) structures instead of the neutron \( 3p7h \) structure. This choice is due to the stability of the superdeformed configuration of the neutron.

### 3.2. Low-lying level scheme

After the angular momentum projection, we superposed the wave functions obtained by the variation and diagonalized (the GCM calculation). The obtained level schemes of \( ^{26}\text{Ne}, ^{28}\text{Ne} \) and \( ^{30}\text{Ne} \) are shown Fig. 6. First, we discuss \( ^{30}\text{Ne} \). In the positive-parity state, we have obtained three bands. The ground, second and third bands have dominant contribution from the

| Table I. \( E2 \) transition probabilities. The experimental data is taken from Ref. 20). |
|---------------------------------|---------|--------|
| \( ^{30}\text{Ne}; 0^+_1 \rightarrow 2^+_{1} \) | 283 | \( ^{30}\text{Ne}; 0^+_1 \rightarrow 2^+_{1} \) |
| \( 0^+_2 \rightarrow 2^+_{2} \) | 88 | 208 |
| \( 0^+_3 \rightarrow 2^+_{3} \) | 393 | 112 |
| \( 0^+_1 \rightarrow 2^+_{1} \) | 203 | 109 |
| \( 0^+_2 \rightarrow 2^+_{2} \) | 239 | \( ^{28}\text{Ne}; 0^+_1 \rightarrow 2^+_{1} \) |
| \( 0^+_2 \rightarrow 2^+_{2} \) | 412 | 269±136 |
| \( ^{28}\text{Ne}; 0^+_1 \rightarrow 2^+_{1} \) | | 228±41 |
2p2h, 0p0h and 4p4h structure, respectively. The mixing between these three neutron configurations is most strong in the 0\(^+\) state and becomes smaller as the angular momentum increases. Similarly to the results of other theoretical studies, we find that the 2\(^+\) state has a small excitation energy (0.88 MeV) and a large B(E\(_{2};\)0\(^+\) \rightarrow 2\(^+\)) value (Table I) due to the dominance of the large deformed 2p2h configuration in the ground band. The mixings between different neutron configurations are weaker in the 2\(^+\) and 4\(^+\) states than in the ground state. Therefore, the mixing between the neutron configurations lowers the ground state energy by a greater amount than the higher spin states and causes the spectrum of the ground band to deviate slightly from the rotational one, giving \(E(2^+)/E(0^+)\)=3.08. The second band shows the vibrational spectra. The third band has a 4p4h dominant structure, and this band
exhibits the greatly deformed rotational spectrum and has large intra-band $E2$ transition probabilities (Table I). Even if we increase the number of basis states in the GCM calculation, the energy and wave function of the third band, as well as those of the ground and second bands, are not affected. Therefore, we conclude that this band is stable. The small excitation energy of this band is important, because it implies the softness of the neutrons above the deformed $N=16$ shell. In the same sense, the low-lying negative-parity states are also important, though their spectra are complicated due to a rather strong mixing between the $1p1h$ and $3p3h$ structures. As discussed in the previous section, due to the large energy gain, which is due to the larger deformation and the smaller angular momentum, the $1^-$ state becomes the lowest negative-parity state. The $3^{-}$ state is lowered by the strong mixing between the $1p1h$ states and the $3p3h$ structure, and in this state, the $1p1h$ structure is dominant, while the $3^{-}$ state becomes higher and has a $3p3h$ dominant structure. The $5^{-}$ state is also lowered by the mixing between two configurations, and in this state, the neutron $1p1h$ structure is dominant. There is not strong mixing between the $2^{-}$ and $2^+_1$ states and between the $4^{-}$ and $4^+_2$ states, and therefore the $2^{-}_2$ and $4^+_2$ states, which have $1p1h$ dominant structure, have higher excitation energies than the $5^{-}_1$ and $3^{-}_1$ states, which also have $1p1h$ structure. The $K^\pi=2^-$ band, in which a proton in the $p$-shell is excited into the $sd$-shell combined with the neutron $4p4h$ structure, does not mix with other states and is left in the higher energy region. The obtained low-lying states of $^{30}$Ne are governed by the neutron’s particle-hole configurations. The softness of neutrons in the $0d_{5/2}$ orbit with respect to promotion into the $pf$-shells produces the low-lying $0p0h$ and $2p2h$ rotational bands in $^{30}$Ne. This trend also exists in the case of negative-parity states. In particular, the lowest negative-parity band, the $K^\pi=1^-$ band, which has neutron $3p3h$ structure dominantly, is deformed as strongly as the ground band. It can be regarded as the $1h\omega$ excitation from the ground band in the same deformed mean-field as the ground band. The small excitation energy $K^\pi=0^+_3$ band, which has neutron $4p4h$ structure dominantly can be also regarded as exhibiting the softness of the $0d_{3/2}$ orbit. However, as stated above, this band has a ‘$\alpha+^{16}\text{O}+\text{valance neutron}$ nature, as in the case of the neutron-rich Be isotopes.$^{31,32}$ This implies the coexistence of a deformed mean-field structure and a molecular orbital structure and their interplay, which has not been clearly seen in the case of the Be isotopes.

The ground band of $^{28}$Ne has a strong mixing between the oblatelly deformed $0p2h$ structure and the prolately deformed $2p4h$ structure. This mixing is strongest in the case of the $0^+$ states and it lowers the ground state’s energy by about 1.9 MeV. This mixing becomes smaller rapidly as the angular momentum becomes larger. As a result, the energy gap between the $0^+_1$ state and the $2^+_1$ state becomes larger, and the $0^+_1$, $2^+_1$ and $4^+_1$ states of the ground band possess vibrational character: $E_x(4^+_1)/E_x(2^+_1)=2.17$. The strong mixing also causes the $E2$ transition probability between the ground state and the $2^+_1$ state to be smaller than that of $^{30}$Ne (Table I), because the amount of the largely deformed component becomes smaller than the ground band of $^{30}$Ne. The excitation energy of $0^+_2$ becomes higher through the mixing. And this is the reason of the small energy gap between $0^+_2$ and $2^+_2$ states. In
the case of $^{30}$Ne, the low-lying $1p1h$ and $3p3h$ states are mixed strongly. However, in this nucleus, the mixing between $1p3h$ and $3p5h$ structures is no longer strong, because of their large energy difference. As a result, the $3^{-}_1$ and $5^{-}_1$ states which have $1p3h$ structure appear in the lower energy region together with the positive-parity bands, while the $K^{π}=1^{-}$ band, which has $3p5h$ structure, is left in the higher energy region. The softness of neutrons in $0d_{3/2}$ induces strong mixing between the neutron $0p2h$ and $2p4h$ configurations, but the hardness of the deformed neutron $N=16$ shell causes the mixing between the $1p3h$ and $3p5h$ states to be weak.

In the ground band of the $^{26}$Ne, the mixing between the different neutron configurations is much weaker than $^{30}$Ne and $^{28}$Ne. The ground band of the $^{26}$Ne exhibits a vibrational character: $E_x(4^+_1)/E_x(2^+_1)=2.22$. Due to the hardness of the $N=16$ shell, neutron $ph$ states appear at more than 5 MeV above the ground state. In the positive parity, the superdeformed states that have $2p6h$ structure appear at about 6 MeV above the ground state. It seems that the magic number of the superdeformation, $N=16$, which has been studied in most detail for $^{32}$S, is valid even in the neutron-rich region. Indeed, in this superdeformed state, it is possible to excite a proton from the $p$-shell to the $sd$-shell while not affecting the stability of this state. This proton excited states appears as the $K^{π}=2^{-}$ state in the negative parity. The lowest negative-parity state is the $3^{-}_1$ state, but its excitation energy is higher than that of $^{30}$Ne and $^{28}$Ne. Because of the hardness of the deformed neutron $N=16$ shell, this nucleus does not have any low-lying particle-hole excited states. Instead, it has a superdeformed rotational band at about 7 MeV above the ground band, whose neutron configuration is almost the same as that of the predicted superdeformed state of $^{32}$S. This superdeformed neutron configuration is stable, as it allows proton excitation within the same superdeformed mean-field, which appears as the $K^{π}=2^{-}$ band at about 13 MeV above the ground state.

Finally, we compare our results with the experimental data and other theoretical results. The excitation energy of the $2^+_1$ state of $^{26}$Ne shows good agreement with experiments. However, in our result, the observed $0^+_2$ state is not obtained. We note that the experimentally observed $0^+_2$ state at 3.69 MeV does not correspond to the $0^+_2$ state obtained in the present calculation. In the HFB+GCM calculation, the prolately deformed local minimum that may have a neutron $0p4h$ structure appears and it corresponds to the experimentally observed $0^+_2$ state. Because in the variational calculation, we have imposed a constraint on the quadrupole deformation parameter $β$ but not on $γ$, we do not obtain a prolately deformed minimum that has the neutron $0p4h$ structure.
on the energy curve. The excitation energies of the $2^+_1$ and $4^+_1$ states of $^{28}$Ne and $^{30}$Ne are smaller than those obtained by the HFB+GCM calculation,\cite{17} while they are larger than those obtained by the Monte Carlo shell model study\cite{10} [Fig. 6 (d)]. The difference between our results and the HFB+GCM results can be attributed to the difference of the spatial symmetries of the wave functions, the difference of the generator coordinates used in the calculations and the lack of the paring effects in our calculation. However, for $B(E2;0^+ \rightarrow 2^+_1)$ we do not find such large difference between our results and HFB+GCM results. In $^{32}$Mg, the $3^-$ state at 2.32 MeV is proposed experimentally,\cite{18,24} though the assignment of the spin and parity of this state is still not determined. This state could correspond to the $1^-$ or $3^-$ state of $^{30}$Ne obtained in the present calculation.

§4. Summary

We have studied low-lying level schemes of $^{26}$Ne, $^{28}$Ne and $^{30}$Ne using the deformed-basis AMD+GCM method. The obtained energy curves and the level schemes of the positive parity states have demonstrated the softness of the neutrons in the deformed orbits coming from the $0d_{3/2}$ orbit with respect to promotion to the $pf$-shell and the hardness of neutrons below the $N=16$ deformed shell. This softness leads to the coexistence of the neutron $0p0h$, $2p2h$ and $4p4h$ states in $^{30}$Ne and the $0p2h$ and $2p4h$ states in $^{28}$Ne and to the breaking of the $N=20$ shell closure in $^{30}$Ne. We also found that this feature is inherited by the negative-parity states. These results yield the prediction that the neutron $1p1h$ and $3p3h$ states of $^{30}$Ne and the $1p3h$ state of $^{28}$Ne appear in the small excitation energy. We consider that the small excitation energies of negative parity states are typical phenomenon accompanying the breaking of the $N=20$ shell, but this has not been previously recognized, because almost no theoretical studies including the Hartree-Fock approach have been made for negative parity states. The proton excited states in the negative parity states of $^{30}$Ne and $^{26}$Ne, which are results of the hardness of the $N=16$ deformed shell, have also been obtained. The $\alpha+^{16}$O cluster correlations have been found to exist in the $4p4h$ structure of $^{30}$Ne.

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