

A robust optimization approach for real-time multiple source drinking water blending problem

Wei Peng, Rene V. Mayorga and Syed Imran

ABSTRACT

Although many optimization methods can be applied to real-time multiple source drinking water blending problems, the field still lacks an approach to rapidly produce a robust optimal solution by simultaneously optimizing multiple objectives. This paper develops a fuzzy multiple response surface methodology (FMRS) to achieve this objective. In the FMRS, experimental data are fitted to mean response surface models while the residuals (the error between the predicted response of the mean model and the measured data of the real system) are fitted to standard deviation models. Fuzzy linear programming using the min-operator approach is applied to optimize the multiple objectives. Six scenarios are designed based on data from a real-time multiple source drinking water blending operation. The results show the FMRS is a robust, computationally efficient optimization approach. The FMRS could be extended to other real-time multi-objective non-linear optimization problems.

Key words | fuzzy optimization, multi-objective, real-time operation, robust optimum, water quality

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INTRODUCTION

Water utilities commonly practice the drinking water blending of multiple sources to meet seasonal demand, reduce their reliance on stressed sources, or to comply with drinking water guidelines (Imran *et al.* 2006). However, introducing water from a new source may alter the original balanced environment in water networks. As a consequence, pipe corrosion, loss of disinfectant residual, and microbiological growth will deteriorate water quality in distribution systems. Optimizing the blending ratio of water from different sources may minimize this water deterioration; an appropriate optimization method is needed to achieve this.

Multiple source drinking water blending is a real-time multi-objective non-linear optimization problem. The previous approaches with regard to this field are intelligent optimization, model predictive control and optimization, and non-linear optimization that contributed to improving product quality, optimizing production rate, reducing production costs, and minimizing pollution. However, intelligent optimization has a high requirement of prior operation knowledge that highly depends on expert

experience, which may not be available (Huang *et al.* 2008). Traditional model predictive control and optimization is capable of dealing with simple non-linear systems or is used in applications with slow dynamics (Wang & Boyd 2010). Conventional non-linear optimization methodologies, such as gradient-based algorithms and genetic algorithms (Mehrez *et al.* 1992; Ostfeld & Shamir 1996; Yang *et al.* 2000; Tu *et al.* 2005), cannot to rapidly produce a robust optimum objective. For example, gradient-based algorithms may only find local optimums and have a large development cost; genetic algorithms are time-consuming when used to solve large-scale non-linear problems.

Imran *et al.* (2006) described a method that optimized drinking water blending of multiple sources for metal corrosion abatement and monochloramine residual control in distribution systems, which based on the non-linear empirical water quality models. However, this optimization method may need to be improved in three respects. First, the optimization may not be robust because the assumption of homogeneous variance for the response of empirical

model may not be valid. Second, this method is time consuming and not easy to apply because of the high-order non-linear models involved. Third, Imran's study used the weighted optimization method that may not be conducive to handling the multi-objective functions.

This paper proposes a novel means, namely fuzzy multiple response surface methodology (FMRS), to solve the multiple-source drinking water blending problem. This FMRS can improve Imran's method in the three aforementioned respects. Basically, the FMRS integrates a dual response surface methodology (DRSM) with fuzzy linear programming (FLP). The DRSM is derived from response surface methodology (RSM), which is a statistical technique used in empirical study. The RSM approximates the true response surface, estimates the parameters, and works well in solving real response surface problems (Myers et al. 2004; Anderson-Cook et al. 2009). As a consequence, it searches for an optimal set of input variables to optimize the response by using a set of designed experiments. A RSM model is easy to apply and can be quickly solved by any commercial solver because it contains a quadratic model rather than a high-order non-linear model. The DRSM builds two quadratic empirical models, one for the mean and one for the standard deviation, and then optimizes one of the responses subjected to an appropriate constraint given by the other (Lin & Tu 1995; Kim & Lin 1998). As the second empirical model considers the standard deviation rather than assumes a homogeneous variance to the mean empirical model, a DRSM model can produce robust results.

Use of a weighted optimization method to solve a multi-objective problem assumes that the decision-maker knows the characteristics of all individual objective functions. However, this assumption usually is not realistic. The FLP using the min-operator approach is a distinctive technology to efficiently solve a multi-objective problem without the above assumption. In a fuzzy environment, the objective functions and constraints can be characterized into a series of fuzzy sets by their membership functions. This fuzzy-based tool can then define an optimal solution according to the intersection of the fuzzy sets with the highest degree of membership. This decision making simultaneously satisfies the objective functions and constraints without any difference among them, and synchronous optimization is ensured (Zimmermann 1978).

In this paper, a FMRS is developed to produce a robust solution in response to the uncertainty of water quality in the real-time multiple source drinking water blending problem. It includes three major steps. First, each original non-linear empirical model will be converted into two quadratic models: one for the mean and one for the standard deviation. Second, the FMRS will be built up that integrates the DRSM and FLP. Finally, a decision support system will be produced to guide decisions on drinking water blending in a water distribution system in accordance to six designed scenarios. The FMRS could be extended to other real-time multi-objective non-linear optimization problems.

METHODOLOGY

DRSM

Box & Wilson (1951) first used the RSM to study the relationship between a response and a set of input variables. Vining & Myers (1990) first fitted second-order polynomial models for a mean and standard deviation separately.

$$C_{\mu} = h_0 + \mathbf{S}^T \mathbf{h} + \mathbf{S}^T \mathbf{H} \mathbf{S} \quad (1)$$

$$C_{\sigma} = g_0 + \mathbf{S}^T \mathbf{g} + \mathbf{S}^T \mathbf{G} \mathbf{S} \quad (2)$$

where

$$g_0 = \alpha_0, h_0 = \beta_0, \mathbf{g} = (\alpha_1, \alpha_2, \dots, \alpha_k)^T, \mathbf{h} = (\beta_1, \beta_2, \dots, \beta_k)^T,$$

and

$$\mathbf{H} = \frac{1}{2} \begin{bmatrix} 2\beta_{11} & \beta_{12} & \dots & \beta_{1k} \\ \beta_{21} & 2\beta_{22} & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k1} & \beta_{k2} & \dots & 2\beta_{kk} \end{bmatrix} \quad (3)$$

$$\mathbf{G} = \frac{1}{2} \begin{bmatrix} 2\alpha_{11} & \alpha_{12} & \dots & \alpha_{1k} \\ \alpha_{21} & 2\alpha_{22} & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{k1} & \alpha_{k2} & \dots & 2\alpha_{kk} \end{bmatrix} \quad (4)$$

where $k = 1, 2, \dots, k$; h_0 and g_0 are the appropriate scalars of the estimates for the constant terms; \mathbf{h} and \mathbf{g} , are the

appropriate vectors of the estimates for the coefficients of the linear terms; \mathbf{H} and \mathbf{G} are the appropriate matrixes of the estimates for the coefficients of the interaction terms; C_μ and C_σ are the responses of the mean and standard deviation, respectively; \mathbf{S} and \mathbf{S}^T are $(k \times 1)$ vectors of the input variables and their transpose, respectively.

FLP using the min-operator approach

Starting from a FLP model:

$$\begin{aligned} \text{Min } f &= \sum_{j=1}^n c_j x_j, \\ \text{s.t. } \sum_{j=1}^n a_{ij} x_j &\tilde{<} b_i, \\ x_j &\geq 0. \end{aligned} \quad (5)$$

where $i = 1, 2, \dots, m; j = 1, 2, \dots, n; c_j$ is the coefficient with respect to x_j in the objective function; x_j express positive decision variables; a_{ij} represent the coefficient with respect to x_j in the i th constraint; b_i the value in the right-hand side of the i th constraint; the symbol ' $\tilde{<}$ ' represents fuzzy inequality, which means the decision-maker permits some violations of the constraints.

If the goal is fuzzy, a decision-maker can establish an aspiration level ' f' ' to achieve the desired objective function value, and thus Model (5) can be converted to:

$$\begin{aligned} \text{Find } x_j &\geq 0, \\ \text{such that } \sum_{j=1}^n c_j x_j &\tilde{<} f', \\ \sum_{j=1}^n a_{ij} x_j &\tilde{<} b_i. \end{aligned} \quad (6)$$

where no distinction is made between fuzzy objectives and fuzzy constraints.

According to Bellman & Zadeh's (1970) concept of maximizing decision, the solution to Model (6) will be x^* , and the membership function (satisfaction degree) of the optimal decision is:

$$\mu_D(x^*) = \max_x \mu_D(x) = \max_x \min_i [\mu_G(x), \mu_C(x)] \quad (7)$$

where $\mu_G(x)$ and $\mu_C(x)$ are the membership functions of the goal and the constraints, $\mu_D(x)$ may be called 'the satisfaction degree of a decision'. In order to obtain x^* , let $\lambda = \mu_D(x) = \min_i [\mu_G(x), \mu_C(x)]$, Equation (7) is then transformed to an equivalent linear programming model as follows:

$$\begin{aligned} \text{Max } \lambda \\ \text{s.t. } \lambda &\leq \frac{d_i + \Delta d_i - \sum_{j=1}^n E_{ij} x_j}{\Delta d_i}, \quad i = 1, 2, \dots, m + 1 \\ x &\geq 0, \quad 0 \leq \lambda \leq 1. \end{aligned} \quad (8)$$

where

$$\sum_{j=1}^n E_{ij} x_j$$

is the i th row of

$$\left[\sum_{j=1}^n c_j x_j, \sum_{j=1}^n a_{1j} x_j, \dots, \sum_{j=1}^n a_{ij} x_j, \dots, \sum_{j=1}^n a_{mj} x_j \right]^T;$$

d_i is the i th row of $[f', b_1, \dots, b_i, \dots, b_m]^T$; Δd_i is the tolerance of the right-hand side, $\Delta d_i \geq 0$.

As this decision process simultaneously satisfying objective functions and constraints without a difference between the former and latter, a synchronous optimization is achieved.

FMRSM

In this study, we will generate two types of response surface models: one for mean and one for standard deviation. The mean models are directly fitted by the measured data; the standard deviation models are fitted by the residuals. The residual here is defined as the error between the predicted output (response) of the mean model and the measured data of a real system.

Assume that there are n types of contaminants in a water distribution system. The concentration of a contaminant is considered as one response that is influenced by the concentrations of water quality parameters (input variables). Water quality parameters include alkalinity, calcium, silica, sodium, pH, and conductivity. The relationship between each contaminant and its corresponding water quality parameters is fitted into a mean response surface model. The residual values of this mean model are used to fit a standard deviation response surface model.

A general dual response surface optimization model is shown as follows:

$$\text{Min } (C)_n \quad (9a)$$

$$\text{Min } (C_r)_n \quad (9b)$$

subject to:

$$(C)_n = (h_0 + \mathbf{S}^T \mathbf{h} + \mathbf{S}^T \mathbf{H} \mathbf{S})_n \leq (L)_n \quad (9c)$$

$$(C_r)_n = (g_0 + \mathbf{S}^T \mathbf{g} + \mathbf{S}^T \mathbf{G} \mathbf{S})_n \leq (L_r)_n \quad (9d)$$

$$S_k = \sum_{i=1}^I x_i U_{ki} \quad (9e)$$

$$0 \leq x_i \leq 1 \quad (9f)$$

$$\sum_{i=1}^I x_i = 1 \quad (9g)$$

where C is the predicted contaminant from the mean model, C_r is the predicted residual from the standard deviation model, n is the number of water quality contaminants, L and L_r are the limitations corresponding to C and C_r , respectively; S_k is the k th input variable, k is an index for the input variables (water quality parameters); x_i is the percentage of water source i , I is an index for the water sources; U_{ki} is the coefficient of the k th input variable for water source i .

The objectives of Equations (9a) and (9b) are to minimize contaminants (responses). These contaminants are constrained by Equations (9c) and (9d). Applying the above fuzzy linear programming of Equation (8) to the general response surface optimization model of Equation (9), the membership functions of the goal and constraint can be defined as:

$$\lambda = \mu_D(C) = \min[\mu_G(C), \mu_C(C)]$$

$$= \begin{cases} 1 & \text{if } C \leq L, \\ \frac{L + \Delta L - C}{\Delta L} & \text{if } L \leq C \leq L + \Delta L, \\ 0 & \text{if } C \geq L + \Delta L. \end{cases} \quad (10)$$

$$\lambda = \mu_D(C_r) = \min[\mu_G(C_r), \mu_C(C_r)]$$

$$= \begin{cases} 1 & \text{if } C_r \leq L_r, \\ \frac{L_r + \Delta L_r - C_r}{\Delta L_r} & \text{if } L_r \leq C_r \leq L_r + \Delta L_r, \\ 0 & \text{if } C_r \geq L_r + \Delta L_r. \end{cases} \quad (11)$$

where ΔL and ΔL_r are the tolerances of contaminant and contaminant variance, respectively. It notes that the assumption of linear membership function is sufficient for this study. Any non-linear membership function assumption would make the computation more complicated. A general FMRS model can be generated as:

$$\text{Max } \lambda \quad (12a)$$

subject to:

$$\mu(C)_n \geq \lambda \quad (12b)$$

$$\mu(C_r)_n \geq \lambda \quad (12c)$$

$$(C)_n = (h_0 + \mathbf{S}^T \mathbf{h} + \mathbf{S}^T \mathbf{H} \mathbf{S})_n \quad (12d)$$

$$(C_r)_n = (g_0 + \mathbf{S}^T \mathbf{g} + \mathbf{S}^T \mathbf{G} \mathbf{S})_n \quad (12e)$$

$$0 \leq \lambda \leq 1 \quad (12f)$$

$$\text{Equations (9e), (9f), (9g), (10), and (11).} \quad (12g)$$

MODELING

A fixed source has a stable water quality, referred to as a 'balanced environment', in the distribution system. In other word, this balanced environment has stable concentrations of water quality parameters (alkalinity, calcium, silica, sodium, pH, conductivity, etc.). Any change to a fixed historical source will break the original balance and cause deterioration of water quality through corrosion, loss of disinfectant residual, and microbiological growth in a distribution system. Introducing water from a new source may alter the concentration of water quality

parameters in water distribution systems. According to a previous study (Chang & Jung 2004), the biomass accumulation is influenced to a greater extent by the nature of the supporting material (such as unlined ductile iron) than by water quality parameters. Meanwhile, Imran *et al.* (2006) confirmed that loss of disinfectant residual is affected to a greater extent by the delivery distance and retention time. Their researches showed that metal corrosions are more sensitive to the changes of water quality parameters than other types of contaminants. Therefore, this paper focuses on the relationships between types of metal corrosion and their corresponding water quality parameters in the distribution system. Metal corrosion here includes iron corrosion, lead corrosion, and copper corrosion, and three sets of metal corrosion models are fitted by the real experimental data.

Quadratic polynomial (DRSM) model

Tampa Bay Water operates a water distribution system to deliver drinking water for three cities and counties in Florida, USA. In order to reduce groundwater (GW) withdrawal as is required by the local government, a pilot distribution system was constructed to simulate the full-scale operational water distribution system of Tampa Bay Water (Imran *et al.* 2006). According to the two-year recorded data of the pilot distribution system, the statistical non-linear copper, lead, and color (iron) corrosion models were developed (Xiao 2004; Imran *et al.* 2005, 2006; Taylor *et al.* 2005). These non-linear corrosion models can explicitly describing the relationships between the metal releases and water quality parameters. However, they implied the variances of the corrosions of these non-linear models are relatively small and stable, but when the variances are not small, these non-linear corrosion models could be misleading. Moreover, the non-linear equations will be time consuming during the optimization process.

We convert these non-linear models into two types of quadratic polynomial models: corrosion (mean) and its standard deviation models. The metal corrosions are designed as responses and their related water quality parameters are designed as input variables in the corrosion models. And then the residuals, where residual is defined as the error between the predicted corrosion of mean model and the measured data of a real system, are used for fitting the

standard deviation models. Because the variances of the corrosions are considered through these standard deviation models, the optimization is to be robust.

Notably, the water quality parameter of pH is non-conservative. It is defined as a negative decimal logarithm of the molar concentration of the hydrogen ion activity in a solution. Because the pH value of blends cannot be obtained from a mass balance equation, we chose the molar concentration of the activity of hydrogen ions A_{H^+} as a substitution ($A_{H^+} = 10^{-pH}$). The corrosions and their standard deviation models are generated as follows.

Iron-release model

$$C_{Fe} = (10)^{-3}(-9,763 + 355T - 2Alk + 17Cl - 43Na - 6.5SO_4 + 1,259DO + 699HRT - 0.1T \bullet Alk - 0.4T \bullet Cl + 1T \bullet Na + 0.1T \bullet SO_4 - 21T \bullet DO + 3.9T \bullet HRT + 0.1Alk \bullet Na - 0.3Alk \bullet DO - 0.2Alk \bullet HRT + 0.1Cl \bullet Na - Cl \bullet DO + 0.9Cl \bullet HRT + 1.9Na \bullet DO + 0.2Na \bullet HRT + 0.5SO_4 \bullet DO + 0.2SO_4 \bullet HRT + 8.2DO \bullet HRT - 4.2T^2 + 0.03Alk^2 - 0.04Cl^2 + 0.1Na^2 - 47.7DO^2 - 118.4HRT^2) \quad (13a)$$

$$R^2 = 0.817$$

$$C_r^{Fe} = (10)^{-4}(-65,091 - 123T + 23Alk - 73Cl + 196Na - 39SO_4 + 2,601DO + 37,656HRT - 0.3T \bullet Alk + 1.7T \bullet Cl - 4T \bullet Na + 0.7T \bullet SO_4 - 20T \bullet DO + 11T \bullet HRT - 0.1Alk \bullet Na - 1.4Alk \bullet DO + 0.1Alk \bullet HRT - 0.1Cl \bullet Na + 5.2Cl \bullet DO + 0.9Cl \bullet HRT - 13.8Na \bullet DO - 0.1Na \bullet HRT + 2.7SO_4 \bullet DO + 0.2SO_4 \bullet HRT + 0.3DO \bullet HRT + 6.9T^2 - 0.01Alk^2 - 0.04Cl^2 + 0.28Na^2 - 0.01SO_4^2 - 112.7DO^2 - 5,431HRT^2) \quad (13b)$$

$$R^2 = 0.99$$

where C_{Fe} is the mean response of iron corrosion in mg/L, C_r^{Fe} is the standard deviation response of iron corrosion; T is the temperature in °C; Alk is the concentration of alkalinity in mg/L as calcium carbonate ($CaCO_3$); Na, SO_4 , and Cl are the concentrations of sodium, sulfates and chlorides in mg/L, respectively; DO is the dissolved oxygen content in

mg/L; R^2 is the correlation coefficient; and HRT is the hydraulic retention time in days. The original non-linear iron-release model was developed by Imran *et al.* (2006).

Copper-release model

$$C_{Cu} = (10)^{-3}(-252.3 + 35T + 2.4Alk - 12.3A_{H^+} + 1.4SO_4 - 15SiO_2 + 0.1T \bullet Alk + 1.2T \bullet A_{H^+} - 0.5T \bullet SiO_2 + 0.04Alk \bullet A_{H^+} + 0.01Alk \bullet SO_4 - 0.14Alk \bullet SiO_2 + 0.04A_{H^+} \bullet SO_4 + 0.34A_{H^+} \bullet SiO_2 - 0.08SO_4 \bullet SiO_2 - 0.77T^2 - 0.68A_{H^+}^2 + 1.34SiO_2^2) \quad (14a)$$

$$R^2 = 0.997$$

$$C_r^{Cu} = (10)^{-4}(-1,132 + 124T + 4.8Alk - 77A_{H^+} + 3SO_4 - 6SiO_2 - 0.2T \bullet Alk + 2.6T \bullet A_{H^+} - 0.1T \bullet SO_4 + 0.3T \bullet SiO_2 + 0.09Alk \bullet A_{H^+} - 0.03Alk \bullet SiO_2 + 0.14A_{H^+} \bullet SO_4 + 1.27A_{H^+} \bullet SiO_2 - 0.17SO_4 \bullet SiO_2 - 2.92T^2 - 0.01Alk^2 - 0.81A_{H^+}^2 - 0.07SiO_2^2) \quad (14b)$$

$$R^2 = 0.999$$

where C_{Cu} is the mean response of copper corrosion in mg/L, C_r^{Cu} is the standard deviation response of copper corrosion; T is the temperature in °C; Alk is the concentration of alkalinity in mg/L as calcium carbonate ($CaCO_3$); SO_4 and SiO_2 are the concentrations of sulfates and silica in mg/L, respectively; R^2 is the correlation coefficient; A_{H^+} is the molar concentration of the activity of hydrogen ions in 10^9 mol/L (in order to keep the same order of magnitude as the pH value). The original non-linear copper-release model was generated by Xiao (2004).

Lead-release model

$$C_{Pb} = (10)^{-3}(6,239 - 159T + 49Alk - 1,527A_{H^+} - 25,7Cl + 66SO_4 - 0.2T \bullet Alk + 35.3T \bullet A_{H^+} + 8.3T \bullet Cl - 2T \bullet SO_4 + 3.76Alk \bullet A_{H^+} + 1.25Alk \bullet Cl - 0.29Alk \bullet SO_4 + 11.6A_{H^+} \bullet Cl + 1.8A_{H^+} \bullet SO_4 - 1.4Cl \bullet SO_4 - 1.2T^2 - 0.26Alk^2 - 18.6A_{H^+}^2 + 2.18Cl^2 + 0.12SO_4^2) \quad (15a)$$

$$R^2 = 0.993$$

$$C_r^{Pb} = (10)^{-4}(-49,578 + 4,041T + 188Alk - 2,854A_{H^+} + 471Cl + 50SO_4 - 8.7T \bullet Alk + 62.3T \bullet A_{H^+} - 5.8T \bullet Cl - 8T \bullet SO_4 + 11Alk \bullet A_{H^+} - 1.5Alk \bullet Cl + 0.004Alk \bullet SO_4 - 5.4A_{H^+} \bullet Cl + 10.3A_{H^+} \bullet SO_4 + 0.4Cl \bullet SO_4 - 65.5T^2 - 0.22Alk^2 - 13.8A_{H^+}^2 - 1.1Cl^2 + 0.05SO_4^2) \quad (15b)$$

$$R^2 = 0.998$$

where C_{Pb} is the mean response of lead corrosion in mg/L, C_r^{Pb} is the standard deviation response of lead corrosion; T is the temperature in °C; Alk is the concentration of alkalinity in mg/L as calcium carbonate ($CaCO_3$); SO_4 and SiO_2 are the concentrations of sulfates and silica in mg/L, respectively; R^2 is the correlation coefficient; A_{H^+} is the molar concentration of the activity of hydrogen ions in 10^9 mol/L (in order to keep the same order of magnitude as the pH value). The original non-linear lead-release model was fitted by Taylor *et al.* (2005).

Membership function design

Membership function of corrosion model

Simplify Equation (10) as follows:

$$\mu(C) = \begin{cases} 1 & \text{if } C \leq L, \\ \frac{L + \Delta L - C}{\Delta L} & \text{if } L \leq C \leq L + \Delta L, \\ 0 & \text{if } C \geq L + \Delta L \end{cases} \quad (16)$$

where ΔL is assumed as the tolerance of metal corrosion. Figure 1(a) shows the linear membership function as described by Equation (16). It demonstrates the fact that the decision-maker does not accept a metal corrosion over $L + \Delta L$; The degree of satisfaction is equal to 1 when a metal corrosion is lower than L ; and the membership function value would decrease monotonically from 1 at $C = L$, to 0 at $C = L + \Delta L$. Thus, the assumption of linear membership function is sufficient for this study. Any non-linear membership function assumption would make the computation more complicated.

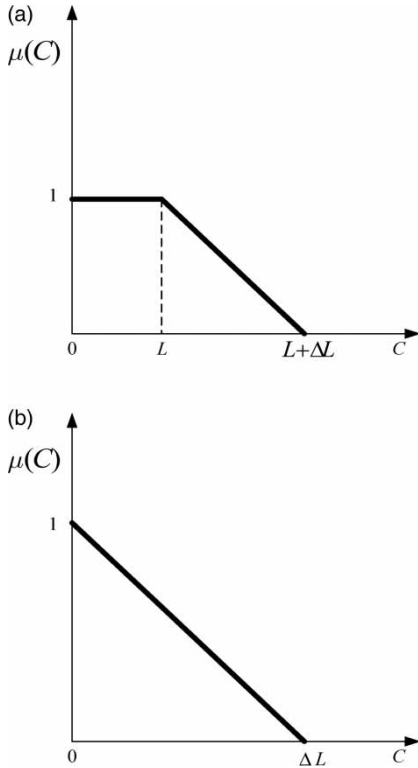


Figure 1 | (a) Membership function of corrosion model. (b) Membership function of corrosion model with $L = 0$.

The Lead and Copper Rule action level of US EPA for copper stipulates 90% of samples have a copper concentration less than 1.3 mg/L and a lead concentration less than 15 $\mu\text{g/L}$. The National Secondary Drinking Water Regulations suggests the maximum contaminant level for iron is 0.3 mg/L. The regulations can be used to determine either the limitation of L ($L_{\text{Cu}} = 1.3 \text{ mg/L}$; $L_{\text{Pb}} = 15 \text{ }\mu\text{g/L}$; $L_{\text{Fe}} = 1.3 \text{ mg/L}$) or the tolerant of ΔL ($\Delta L_{\text{Cu}} = 1.3 \text{ mg/L}$; $\Delta L_{\text{Pb}} = 15 \text{ }\mu\text{g/L}$; $\Delta L_{\text{Fe}} = 1.3 \text{ mg/L}$). If the former regulation is chosen, the values of ΔL still need to be determined. However, we did not find any reference regarding to the fuzzy regulation. If the latter is chosen, we can define $L = 0$. Then the membership function value can be reasonable, which is described as decreasing monotonically from 1 at $C = 0$, to 0 at $C = \Delta L$ as shown in Figure 1(b). The three membership functions, with corrosion levels less than the regulations can be expressed as:

$$\mu(C_{\text{Fe}}) = \frac{0.3 - C_{\text{Fe}}}{0.3} \quad (17)$$

$$\mu(C_{\text{Cu}}) = \frac{1.3 - C_{\text{Cu}}}{1.3} \quad (18)$$

$$\mu(C_{\text{Pb}}) = \frac{15 - C_{\text{Pb}}}{15} \quad (19)$$

Membership function of standard deviation model

The residual is the error between the predicted metal release of corrosion model and the measured data of a real system. We found that three types of residual values, obtained from the comparison of corrosion prediction and measurement, are normally distributed. They are shown in Figure 2. If a normal distribution is expressed as $f(x)$, a non-linear membership function could be standardized as follows:

$$\mu(C_r) = \sqrt{2\pi\sigma^2} f(x) = e^{-\frac{(x - \mu)^2}{2\sigma^2}} \quad (20)$$

However, the non-linear membership functions will cause the optimization to be time consuming. Thus, we linearize the non-linear membership functions (shown in Figure 3) as follows:

$$\mu(C_r) = \begin{cases} \frac{L_r - |C_r|}{L_r} & \text{if } -L_r \leq C_r \leq L_r, \\ 0 & \text{if } C_r > L_r \text{ or } C_r < -L_r. \end{cases} \quad (21)$$

To determine the value of L_r , we should take into consideration the corrosion limitations ($L_{\text{Cu}} = 1.3 \text{ mg/L}$; $L_{\text{Pb}} = 15 \text{ }\mu\text{g/L}$; $L_{\text{Fe}} = 1.3 \text{ mg/L}$). Assuming the predicted residuals do not exceed 20% of the corrosion regulations, the membership functions can be expressed as:

$$\mu(C_r^{\text{Fe}}) = \frac{0.06 - |C_r^{\text{Fe}}|}{0.06} \quad (22)$$

$$\mu(C_r^{\text{Cu}}) = \frac{0.26 - |C_r^{\text{Cu}}|}{0.26} \quad (23)$$

$$\mu(C_r^{\text{Pb}}) = \frac{3 - |C_r^{\text{Pb}}|}{3} \quad (24)$$

Thus, a specific FMRS can be generated for the multiple source drinking water blending problems.

$$\text{Max } \lambda \quad (25a)$$

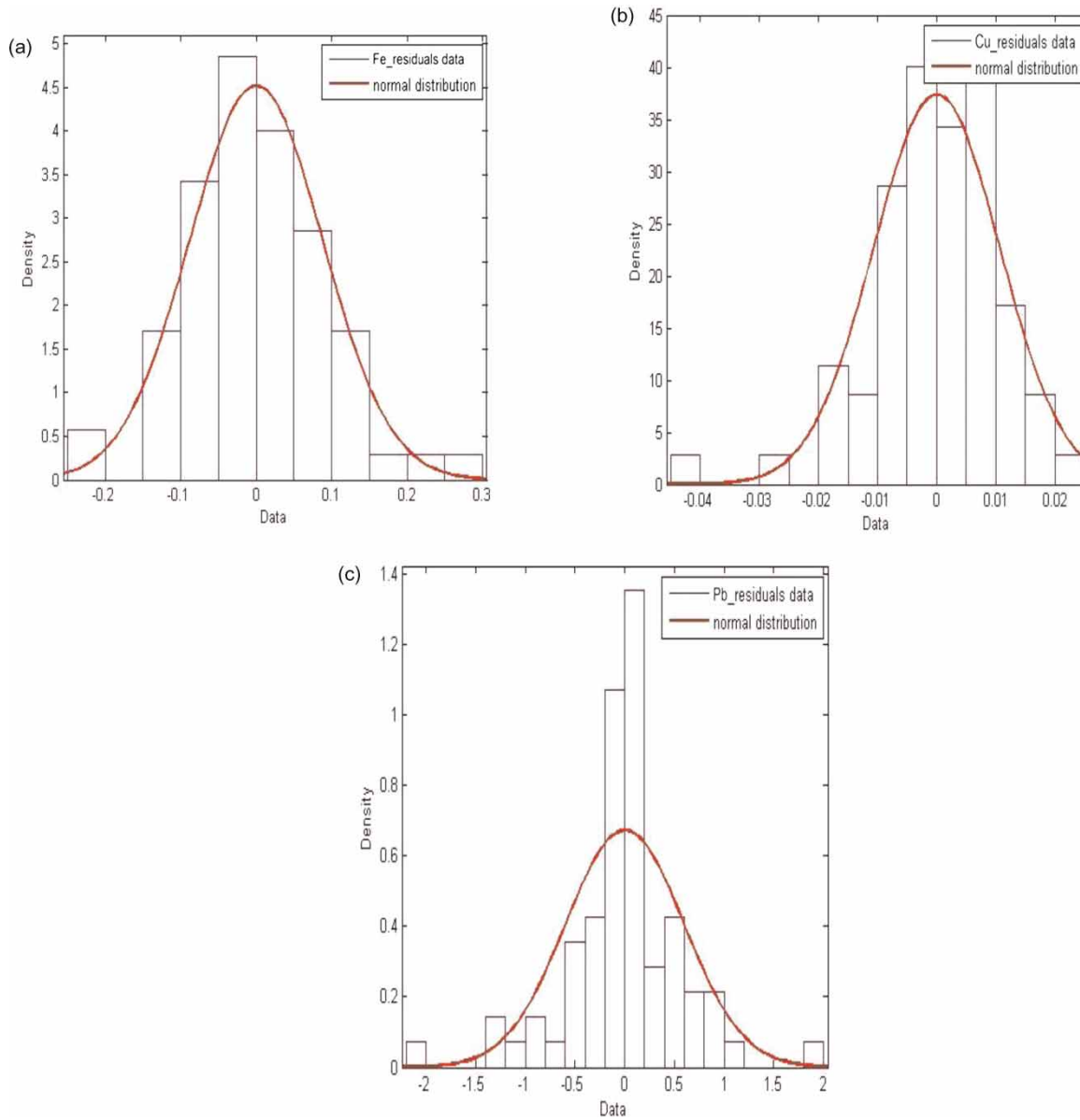


Figure 2 | (a) The normal distribution for the residual of iron with $\mu_{Fe} = 0$, $\sigma_{Fe} = 0.0883$. (b) The normal distribution for the residual of copper with $\mu_{Cu} = 0$, $\sigma_{Cu} = 0.0107$. (c) The normal distribution for the residual of lead with $\mu_{Pb} = 0$, $\sigma_{Pb} = 0.5961$.

subject to:

$$(13a), (14a), (15a) \tag{25d}$$

$$\frac{0.3 - C_{Fe}}{0.3} \geq \lambda, \frac{1.3 - C_{Cu}}{1.3} \geq \lambda, \frac{15 - C_{Pb}}{15} \geq \lambda \tag{25b} \quad (13b), (14b), (15b) \tag{25e}$$

$$0 \leq \lambda \leq 1 \tag{25f}$$

$$\frac{0.06 - |C_r^{Fe}|}{0.06} \geq \lambda, \frac{0.26 - |C_r^{Cu}|}{0.26} \geq \lambda, \frac{3 - |C_r^{Pb}|}{3} \geq \lambda \tag{25c} \quad (9e), (9f), \text{ and } (9g) \tag{25g}$$

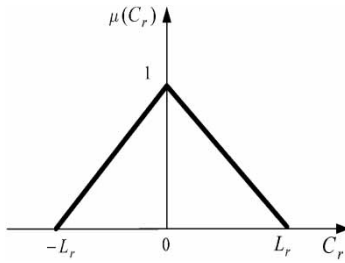


Figure 3 | Membership function of residual model.

Scenario design

Assume three plants produce GW, surface-water (SW), and desalinated water (DW) for a distribution system. Their water quality information is described in Table 1. In the blend, define the ratio of GW as x_1 , the ratio of SW as x_2 , and the ratio of DW as x_3 . Several scenarios are defined as follows:

Scenario 1

Assume three plants are fully operational with the following three situations.

1. No limitation of production capacity or delivery capacity.
2. It is reported that some connections may only receive DW in the range 0–81% and SW in the range of 0–90%

Table 1 | Water quality of the source waters

Parameter	GW	SW	DW
Alkalinity (mg/L) as CaCO ₃	225	50	50
Calcium (mg/L) as CaCO ₃	200	50	50
Dissolved oxygen (mg/L)	8	8	8
pH	7.9	8.2	8.3
Silica (mg/L)	14	7	1
Sodium (mg/L)	10	15	30
Chloride (mg/L)	15	10	50
Sulfates (mg/L)	10	180	30
Temperature (°C)	25	25	25

Source: Imran et al. (2006).

GW: groundwater; SW: surface water; DW: desalinated water.

(Imran et al. 2006). The constraints are modified as:

$$0 \leq x_1 \leq 1, \quad 0 \leq x_2 \leq 0.9, \quad 0 \leq x_3 \leq 0.5 \quad (26)$$

3. A water plant's production capacity may not satisfy the total water demand in many cities. Imran et al. (2006), for example, described how the production capacities of SW and DW plants could only satisfy approximately 41 and 16% of the total water demands. Moreover, the GW withdrawals were reduced to 76% of the original amount. The constraints are modified as:

$$0 \leq x_1 \leq 0.76, \quad 0 \leq x_2 \leq 0.41, \quad 0 \leq x_3 \leq 0.16 \quad (27)$$

Scenario 2: One source is off line

1. GW is off line. The constraint is modified as:

$$x_1 = 0, \quad 0 \leq x_2 \leq 1, \quad 0 \leq x_3 \leq 1, \quad x_2 + x_3 = 1. \quad (28)$$

2. SW is off line.

$$x_2 = 0, \quad 0 \leq x_1 \leq 1, \quad 0 \leq x_3 \leq 1, \quad x_1 + x_3 = 1. \quad (29)$$

3. DW is off line.

$$x_3 = 0, \quad 0 \leq x_2 \leq 1, \quad 0 \leq x_1 \leq 1, \quad x_2 + x_1 = 1. \quad (30)$$

Scenario 3: Varying chloride concentration

Chloride has an important effect on the releases of lead and iron. To analyze this, two different chloride concentrations are designated for the DW: 100 and 300 mg/L. A large increase in chloride concentration in the DW may result in the optimal ratio of the DW dropping to a very low value (down to 0) in the blend. We constrain x_3 to a value equal to or greater than 16%.

$$0.16 \leq x_3 \leq 1 \quad (31)$$

Scenario 4: Varying pH value

The pH value is also an important influence on the three metal corrosions. We consider the following pH values: (1) pH = 7.2 in DW, (2) pH = 8.5 in GW.

Scenario 5: Varying alkalinity concentration

We consider the following alkalinity concentrations: (1) Alk = 50 mg/L in GW, (2) Alk = 300 mg/L in GW.

Scenario 6: Varying sulfate concentration

Two sulfate concentrations are considered: (1) $SO_4 = 30$ mg/L in SW, (2) $SO_4 = 360$ mg/L in SW.

RESULTS AND DISCUSSION

The water quality parameters described by Table 1 are substituted into Equation (25). Commercial software is used to

solve this specific FMRS model. Based on the above scenarios, the results are generated and shown in Table 2. The run time is only several milliseconds, which shows that the FMRS is a rapidly methodology used for solving the real-time multiple-source drinking water blending problems.

In order to demonstrate the advantage of FMRS, a fuzzy response surface optimization (FRSO) model is generated for comparison. This FRSO model has the same structure as the FMRS model except that its constraints do not have standard deviation models. Based on the same water quality parameter inputs and scenarios as FMRS, the results of FRSO are generated and shown in Table 3.

Table 2 shows that the FMRS cannot find feasible solutions under the scenarios of 2-1, 2-3, 3-1, 3-2, and 4-1. Table 3 shows The FRSO cannot find feasible solutions under the scenarios of 2-1 and 3-2. Compared with the FRSO, obviously, the FMRS cannot find feasible solutions under the scenarios of 2-3, 3-1, and 4-1. This is because the predicted variances of metal releases are not small

Table 2 | Results of the FMRS in various scenarios

	Scenario 1 Capacity			Scenario 2 Operation			Scenario 3 Chloride		Scenario 4 pH		Scenario 5 Alkalinity		Scenario 6 Sulfates	
	1	2	3	1	2	3	1	2	1	2	1	2	1	2
λ	0.54	0.52	0.41	–	0.46	–	–	–	–	0.54	0.76	0.45	0.74	0.49
x_1	0.33	0.36	0.65	–	0.43	–	–	–	–	0.4	0.71	0.21	0.02	0.31
x_2	0.07	0.14	0.19	–	0	–	–	–	–	0.06	0	0.09	0.98	0.03
x_3	0.60	0.5	0.16	–	0.57	–	–	–	–	0.54	0.29	0.71	0	0.66
C_{Pb}	6.99	5.73	2.85	–	8.03	–	–	–	–	6.96	0.72	8.21	0.52	7.66
C_{Cu}	0.61	0.62	0.76	–	0.66	–	–	–	–	0.6	0.32	0.71	0.33	0.6
C_{Fe}	0.14	0.14	0.15	–	0	–	–	–	–	0.14	0	0.16	0.08	0.15
Alk	107	113	165	–	126	–	–	–	–	120	50	123	53	114
SO_4	34.3	43.9	44.7	–	21.3	–	–	–	–	30.3	15.8	39.5	29.7	32.2
Cl	35.8	31.8	19.7	–	34.8	–	–	–	–	33.7	25.2	39.1	10.1	38.2
pH	8.12	8.1	7.99	–	8.29	–	–	–	–	8.46	7.98	8.17	8.19	8.13
C_r^{Fe}	0	0.03	0	–	0.01	–	–	–	–	0.1	0	0.16	0.55	0
C_r^{Cu}	0.02	0.01	0.05	–	0.03	–	–	–	–	0.12	0.58	0.02	0.26	0.02
C_r^{Pb}	0.17	0.19	0.37	–	0.23	–	–	–	–	0.23	0.36	0.22	0.67	0.17

1. x_1 is the blending ratio of groundwater, x_2 is the blending ratio of surface water, and x_3 is the blending ratio of desalinated water.

2. C_{Pb} is in $\mu\text{g/L}$; others are in mg/L .

3. The magnitude of C_r^{Fe} , C_r^{Cu} and C_r^{Pb} is 10^{-4} .

Table 3 | Results of the FRSO in various scenarios

	Scenario 1 Capacity			Scenario 2 Operation			Scenario 3 Chloride		Scenario 4 pH		Scenario 5 Alkalinity		Scenario 6 Sulfates	
	1	2	3	1	2	3	1	2	1	2	1	2	1	2
λ	0.54	0.52	0.46	–	0.46	0.49	0.47	–	0.53	0.54	0.76	0.45	0.74	0.52
x_1	0.33	0.36	0.54	–	0.43	0.48	0.5	–	0.3	0.4	0.71	0.21	0.02	0.34
x_2	0.07	0.14	0.30	–	0	0.52	0.22	–	0.6	0.06	0	0.09	0.98	0.04
x_3	0.60	0.5	0.16	–	0.57	0	0.29	–	0.1	0.54	0.29	0.71	0	0.62

x_1 is the blending ratio of groundwater, x_2 is the blending ratio of surface water, and x_3 is the blending ratio of desalinated water.

under these three scenarios. In other words, without the standard deviation models, the FRSO could be misleading. Hence, the FMRS is a robust methodology.

The results of two models (the FMRS and FRSO) under scenarios of 1-3 and 6-2 are different. The values of λ (satisfaction degree) of the FMRS are smaller than that of the FRSO. This is because the FMRS includes the standard deviation model, which makes the feasible solution region shrink. Consequently, the optimal solution λ is reduced.

Unlike a weighted optimization method, the FMRS does not need an assumption that the decision-maker knew the characteristics of all individual objective functions. It characterizes the three metal release models, three standard deviation models, and relevant constraints into a series of fuzzy sets by their membership functions. It then defines an optimal solution λ according to the intersection of the fuzzy sets with the highest degree of membership under a designed scenario. This decision-making simultaneously satisfies the objective functions and constraints without any difference among them. Thus, the FMRS can perform a synchronous optimization.

However, we found that some results may not be reasonable for real life. For example, the optimal blending ratio of GW:SW:DW = 0.33:0.07:0.60 derived from the scenario of 1.1 (no limitation of delivery capacity or production capacity) is not a reasonable result. This is due to DW being very costly and few cities can afford such a large proportion of DW in water distribution systems. There are two ways to fix this unreasonable result: restrict the ratio of DW in the FMRS model such as the scenario of 1.3 (the ratio of DW is less than 0.16), or modify the chemical component of source waters prior to distribution. Obviously, the latter is

more realistic, and water utilities are more willing to accept it. Our future work will focus on the latter.

CONCLUSIONS

This paper has successfully developed a FMRS approach by combining DRSM and FLP. The developed FMRS improves Imran's optimization method on dealing with the multi-objective non-linear drinking water blending problem in a water distribution system. In the FMRS, the experimental data were fitted into mean response surface models, and the residuals, the error between the predicted response and measured data, were fitted into standard deviation models. A fuzzy optimization method was used to deal with the multiple objectives. The proposed FMRS was applied to a real case, and based on six designed scenarios. The results show: (1) the FMRS is a robust methodology; (2) the FMRS is a rapidly methodology; (3) the FMRS can perform a synchronous optimization. The FMRS could be extended to other real-time multi-objective non-linear optimization problems in the areas of water treatments and water distribution systems.

REFERENCES

- Anderson-Cook, C. M., Borror, C. M. & Montgomery, D. C. 2009 Response surface design evaluation and comparison. *J. Stat. Plann. Infer.* **139**, 629–641.
- Bellman, R. E. & Zadeh, L. A. 1970 Decision making in a fuzzy environment. *Manage. Sci.* **17**, 141–164.
- Box, G. E. P. & Wilson, K. B. 1951 On the experimental attainment of optimum conditions. *J. Roy. Stat. Soc. B.* **13**, 1–45.

- Chang, Y. C. & Jung, K. 2004 Effect of distribution system materials and water quality on heterotrophic plate counts and biofilm proliferation. *J. Microbiol. Biotechnol.* **14**, 1114–1119.
- Huang, Y. F., Wang, G. Q., Huang, G. H., Xiao, H. N. & Chakma, A. 2008 IPCS: an integrated process control system for enhanced in-situ bioremediation. *Environ. Pollut.* **151**, 460–469.
- Imran, S., Dietz, J. D., Mutoti, G., Taylor, J. S., Randall, A. A. & Cooper, C. D. 2005 Red water release in drinking water distribution systems. *J. Am. Waterworks Ass.* **97**, 93–100.
- Imran, S., Dietz, J. D., Mutoti, G., Xiao, W., Taylor, J. S. & Desai, V. 2006 Optimizing source water blends for corrosion and residual control in distribution systems. *J. Am. Waterworks Ass.* **98**, 107–115.
- Kim, K.-J. & Lin, D. 1998 Dual response surface optimization: a fuzzy modeling approach. *J. Quality Technol.* **30**, 1–10.
- Lin, D. & Tu, W. 1995 Dual response surface optimization. *J. Quality Technol.* **27**, 34–39.
- Mehrez, C., Percia, C. & Oron, G. 1992 Optimal operation of a multisource and multi-quality regional water system. *Water Resour. Res.* **128**, 1199–1206.
- Myers, R. H., Montgomery, D. C., Vining, G. G., Borror, C. M. & Kowalski, S. M. 2004 Response surface methodology: a retrospective and literature survey. *J. Quality Technol.* **36**, 53–77.
- Ostfeld, A. & Shamir, U. 1996 Design of optimal reliable multi-quality water supply systems. *J. Water Res. Pl. ASCE* **122**, 322–333.
- Taylor, J. S., Dietz, J. D., Randall, A. A., Hong, S. K., Norris, C. D., Mulford, L. A., Arevalo, J. M., Imran, S., Le Puil, M., Liu, S., Mutoti, I., Tang, J., Xiao, W., Cullen, C., Heaviside, R., Mehta, A., Patel, M., Vasquez, F. & Webb, D. 2005 Effects of blending on distribution system water quality. AwwaRF, Report No. 91065F, Denver.
- Tu, M. Y., Tsai, F. T.-C. & Yeh, W. W.-G. 2005 Optimization of water distribution and water quality by hybrid genetic algorithm. *J. Water Res. Pl. ASCE* **131**, 431–440.
- Vining, G. G. & Myers, R. H. 1990 Combining taguchi and response surface philosophies: a dual response approach. *J. Quality Technol.* **22**, 38–45.
- Wang, Y. & Boyd, S. 2010 Fast model predictive control using online optimization. *IEEE T. Contr. Syst. T.* **18**, 267–278.
- Xiao, W. 2004 Effect of source water blend on copper release in a pipe distribution system: thermodynamic and empirical models. Doctoral Dissertation, University of Central Florida, Orlando.
- Yang, S. L., Sun, Y.-H. & Yeh, W. W.-G. 2000 Optimization of regional water distribution system with blend requirements. *J. Water Res. Pl. ASCE* **26**, 229–235.
- Zimmermann, H.-J. 1978 Fuzzy programming and linear programming with several objective functions. *Fuzzy Set. Syst.* **1**, 45–55.

First received 3 July 2010; accepted in revised form 16 January 2012