Strength of the $\Sigma$ Single-Particle Potential in Nuclei from Semiclassical Distorted Wave Model Analysis of the ($\pi^-, K^+$) Inclusive Spectrum

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The semiclassical distorted wave model is developed to describe ($\pi^-, K^+$) inclusive spectra related to $\Sigma^-$ formation measured at KEK with $p_\pi = 1.2$ GeV/c. The shape and magnitude of the spectrum obtained with a $^{28}\text{Si}$ target are satisfactorily reproduced using a repulsive $\Sigma$-nucleus potential whose strength is of the order of 30–50 MeV. This strength is not as large as the value of more than 100 MeV obtained using the estimation presented in the report of the experiment.

The study of the $\Sigma$-$N$ interaction is essential to understanding the physics of octet baryons. However, in contrast to the other strange particle, $\Lambda$, the $\Sigma$ single-particle (s.p.) potential in nuclear media, which reflects basic properties of the $\Sigma$-$N$ interaction, has not been established, because experimental information is scarce. Even its sign has not been established.

$\Sigma$ formation spectra in ($\pi, K$) and ($K, \pi$) reactions with nuclei are not believed to have narrow peaks, because of the strong $\Sigma N \rightarrow \Lambda N$ coupling. In spite of this, however, the early ($\pi, K$) experimental spectra$^1$ were interpreted as indicating an attractive $\Sigma$ s.p. particle potential of approximately 10 MeV$^2,3)$. The experimental discovery of $^4\Sigma$He$^4,5)$ has shown that the $\Sigma$-$N$ interaction in the $T = 1/2$ channel is sufficiently attractive to support the bound state, as discussed by Harada.$^6$ It has been recognized, however, that due to the strong isospin dependence, it is unlikely that $\Sigma$ bound states will be observed in heavier nuclei. This conjecture is supported by experimental results.$^7$ In a different context, Batty, Friedman and Gal$^8$ reexamined the $\Sigma^-$ atomic data and concluded that the $\Sigma$ potential might be repulsive in a nucleus. The analysis of the ($\pi, K$) spectra from BNL$^9$ by Dąbrowski$^{10}$ also suggests that the $\Sigma$ potential is repulsive of the order of 20 MeV.

Theoretical studies have also been inconclusive for the $\Sigma$-$N$ interaction. In a standard OBEP model for hyperon-nucleon interactions, there are uncertainties in the coupling constants, although $SU_3$ relations are assumed. In the 1970s, the Nijmegen group constructed hard-core hyperon-nucleon potentials, models D and F.$^{11}$ The calculation of Yamamoto and Bando$^{12}$ showed that model D yields $-16.3$ MeV for the $\Sigma$ potential in nuclear matter ($k_F = 1.35$ fm$^{-1}$) and model F repulsive...
5.3 MeV. The later soft-core version\textsuperscript{13) was shown\textsuperscript{14) to predict a smaller attraction than model D.

In recent years, a non-relativistic $SU_6$ quark model has been developed by the Kyoto-Niigata group\textsuperscript{15)-(17) to obtain a unified description of octet baryon-baryon interactions. $G$ matrix calculations in the lowest order Brueckner theory\textsuperscript{18) with this potential yield the result that the $\Sigma$ s.p. potential in symmetric nuclear matter is repulsive of the order of 20 MeV due to a strong repulsion in the $T = \frac{3}{2}$ channel, which originates from quark Pauli effects.

Recently, $(\pi^-, K^+)$ spectra corresponding to $\Sigma$ formation were measured employing various nuclei targets at KEK\textsuperscript{19) with better precision, using 1.2 GeV/c $\pi^-$. In a very noteworthy experimental study\textsuperscript{19) with $^{28}\text{Si}$ it was found that the $\Sigma$ potential deduced from their DWIA analysis is strongly repulsive, as large as 100 MeV.

The determination of the $\Sigma-N$ interaction should be of fundamental importance in the study of such problems as those of neutron star matter and heavy ion collisions, because the baryonic component of such hadronic matter, especially the hyperon admixture, is governed by the basic baryon-baryon interactions. Considering the importance of understanding the $\Sigma-N$ interaction for our description of the whole octet baryon-baryon interactions, it is desirable to carry out an independent analysis of the KEK experiments. In this paper, we develop a semiclassical method for the DWIA approach and apply it to $(\pi^{\pm}, K^\pm)$ inclusive reaction. The semiclassical distorted wave (SCDW) model was originally considered for describing intermediate energy nucleon inelastic reactions with target nuclei.\textsuperscript{20) Applications to various $(p, p')$ and $(p, n)$ inclusive spectra\textsuperscript{21),22) have demonstrated that the method works well.

The double differential cross section for the $(\pi, K)$ hyperon ($Y$) production inclusive reaction is expressed as

$$
\frac{d^2\sigma}{dWd\Omega} = \frac{\omega_i\omega_f}{(2\pi)^2} \int \int dr dr' \sum_{p,h} \frac{1}{4\omega_i\omega_f} \chi_f^{(-)}(r)v_{f,p,i,h}\chi_i^{(+)}(r') \times v_{f,p,i,h}^{*} \phi_{p}^{*}(r)\phi_{h}(r')\phi_{p}(r')\delta(W - \epsilon_p + \epsilon_h)\theta(\epsilon_F - \epsilon_h),
$$

where $\chi_i^{(+)}$ and $\chi_f^{(-)}$ represent the incident pion and final kaon wave functions with energies $\omega_i$ and $\omega_f$, respectively, and $W = \omega_i - \omega_f$ is the energy transfer. Further, $p$ and $h$ denote the unobserved outgoing hyperon ($A$ or $\Sigma$) and nucleon hole states, and the Fermi energy of the target nucleus is represented by $\epsilon_F$. The transition strength of the elementary process $\pi + N \rightarrow K + Y$ is represented by $v_{f,p,i,h}$, which depends on the energy and angle of the scattering particles, although this dependence is not expressed explicitly. Denoting the c.m. and relative coordinates of $r$ and $r'$ by $R = \frac{r + r'}{2}$ and $s = r' - r$, respectively, we introduce the following semiclassical approximation:

$$
\chi_f^{(-)} \left( R \pm \frac{1}{2}s \right) \simeq e^{\pm i\frac{s}{2}k_f(R)}\chi_f^{(-)}(R),
$$

$$
\chi_i^{(+)} \left( R \pm \frac{1}{2}s \right) \simeq e^{\pm i\frac{s}{2}k_i(R)}\chi_i^{(+)}(R).
$$

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The hyperon wave functions \( \phi_p(r) \) and \( \phi_p(r') \) are treated in the same way. In these expressions, \( k(R) \) is the local classical momentum at the position \( R \), which is defined as follows. First, the quantum mechanical expectation value of the momentum is given by
\[
k_i(R) = \frac{\Re \{ \chi^{(\pm)*}(R)(-i)\nabla \chi^{(\pm)}(R) \}}{|\chi^{(\pm)}(R)|^2},
\]
where \( \Re \) represents the operation of taking the real part, and then the magnitude is renormalized using the energy-momentum relation \( \frac{\hbar^2}{2m} k^2(R) + U_R(R) = E \). Here, \( U_R(R) \) is the real part of the optical potential which describes the distorted wave function \( \chi \) of each particle with energy \( E \). Along with the above approximation, we employ the Thomas-Fermi approximation for the summation of hole states. Explicitly, the Bloch density
\[
C(r, r'; \beta) \equiv \sum_i \phi_i(r) \phi_i^*(r') e^{-\beta \epsilon_i},
\]
where the summation with respect to the single particle states \( \phi_i \) of the potential \( U \) with energy \( \epsilon_i \) goes over the entire spectrum, is replaced by that in the Thomas-Fermi approximation, \( C_{TF}(r, r'; \beta) = \frac{2}{(2\pi)^3} \int dK e^{-\beta(U_r(R) + \frac{\hbar^2}{2m} K^2)} \delta(K \cdot s). \) The function \( \theta(\epsilon_F - \epsilon_h) \) in Eq. (1) actually restricts the integration over \( K \) below the local Fermi momentum \( k_F(R) \), defined in terms of the nucleon density \( \rho_r(R) \) (with \( r \) specifying the proton in the present case) as \( k_F(R) = [3\pi^2 \rho_r(R)]^{1/3} \).

Introducing the above approximations, we obtain the following expression for the differential cross section, whose detailed derivation is given in a separate paper:
\[
\frac{d^2\sigma}{dWd\Omega} = \frac{\omega_I \omega_f p_f}{(2\pi)^2 p_i} \int dR \int_{K < k_F(R)} dK \sum_p \frac{2}{4\omega_i \omega_f} |\chi_f^{-1}(R)|^2 |\chi_i^{(+))(R)}|^2 |\phi_p(R)|^2
\times |\nu_{f,p,i,h}(K, k_i)|^2 \delta(K + k_i(R) - k_f(R) - k_p(R))
\times \delta \left( W - \epsilon_p + \frac{\hbar^2}{2m} K^2 + U_r(R) \right).
\]
This expression has the simple interpretation that the reaction in which \( \pi + N \) yields \( K + Y \) takes place at the position \( R \) and satisfies conservation of local semiclassical momentum. These momenta, \( k_i(R), k_f(R) \) and \( k_p(R) \), are calculated with Eq. (4), using \( \pi, K \) and \( Y \) distorted wave functions in an optical model description. It should be stressed that we have avoided the naive introduction of the averaged differential cross section of the elementary process over the proton momentum distribution \( \rho(k) \),
\[
\frac{d\sigma(\pi^- p \rightarrow K^+ \Sigma^-)}{d\Omega} = \frac{\int \rho(k) \frac{d\sigma}{d\Omega}(\Omega_k) \delta(k - P) dk}{\int \rho(k) \delta(k - P) dk}
\]
with \( P = k_K + k_Y - k_\pi \), which is used in the analysis of Ref. 19).
Fig. 1. \((\pi^+, K^+)\) \(\Lambda\) formation inclusive spectra with a \(^{28}\text{Si}\) target at \(\theta_K = 6^\circ \pm 2^\circ\) for pions with \(p_\pi = 1.2\) GeV/c. These results were obtained with various choices of \(U^0_\Lambda\) in a Woods-Saxon potential form. The KEK data\(^27\) are also displayed.

Fig. 2. \((\pi^-, K^+)\) \(\Sigma\) formation inclusive spectra with a \(^{28}\text{Si}\) target at \(\theta_K = 6^\circ \pm 2^\circ\) for pions with \(p_\pi = 1.2\) GeV/c. These results were obtained with various choices of \(U^0_\Sigma\) in a Woods-Saxon potential form. The KEK data\(^19\) are also displayed.

The transition strength \(v_{f,p,i,h}\) is related to the elementary cross section by

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \frac{E_N E_Y}{s} \frac{k_K}{k_\pi} |v|^2, \tag{9}
\]

where \(s\) is the invariant mass squared. We are able to account for the angular dependence of the \(\pi + N \rightarrow K + Y\) elementary process.

The local Fermi momentum \(k_F(R)\) for \(^{28}\text{Si}\) is prepared with the nucleon density distribution obtained using the density-dependent Hartree-Fock method of Campi-Sprung\(^25\). The distorted waves for the incident pion and outgoing kaon are described by the simple absorptive potential \(U(r) = -i\frac{k_\pi^2}{2\pi} b_0 \rho(r)\), with \(\rho(r)\) being the nucleon density distribution. The parameter \(b_0\) is related to the spin-isospin averaged total cross section of the elementary process by \(b_0 \sim 1/k_\pi \langle \sigma_{tot} \rangle\). Referring to the PDG data\(^26\), we use \(b_0 = 0.58\) fm\(^3\) for the incident 1.2 GeV/c pion and \(b_0 = \frac{1}{2p_K}(2.1(\log p_K - 2) + 0.84)\) fm\(^3\) \((p_K\) in fm\(^{-1}\)) for the outgoing kaon.

A treatment of the unobserved hyperon is in order. Actually, the hyperon optical potential should be complex, because there are inelastic processes. The effects of these inelastic channels can be treated using the Green function method. Here we adopt a simplified prescription, employing a real local potential of the standard Woods-Saxon form, \(U_Y(r) = U^0_Y/\{1+\exp((r-r_0)/a)\}\), and we convolute the result of the calculated spectrum with a Lorentz-type distribution function. The half width is taken to be of 5 MeV for the \(\Lambda\) and of 20 MeV for the \(\Sigma\), based on the imaginary part of the \(\Lambda\) and \(\Sigma\) s.p. potentials\(^18\) in nuclear matter calculated with the quark model potential FSS. As the first application, we use the standard geometry parameters, \(r_0 = 1.2 \times (A - 1)^{1/3}\) fm and \(a = 0.6\) fm. The Coulomb interaction is incorporated.

We first apply our model to the \((\pi^+, K^+)\) \(\Lambda\) formation inclusive spectrum obtained with a \(^{28}\text{Si}\) target measured at KEK\(^27\). The strength and angular dependence
of the elementary process are parameterized according to the available experimental
data.\textsuperscript{28,29)} Figure 1 displays the calculated spectra with various strengths of the
\( \Lambda \) s.p. potential, \( V_A^0 = -50, -30, -10 \) and 10 MeV, respectively, to understand
the potential dependence of the calculated spectra, though the \( \Lambda \) s.p. potential has
been established as \( V_A^0 \sim -30 \) MeV from various \( \Lambda \) hypernuclear data. Bearing in
mind various ambiguities in the elementary amplitudes that would be modified in
a nuclear medium and additional two-step contributions, our model is found to be
capable of describing the inclusive spectra.

Figure 2 compares calculated \( \Sigma^- \) formation (\( \pi^-, K^+ \)) inclusive spectra with the
KEK experimental data. In the present calculation, we assume an isotropic angular
dependence. The energy dependence of \( |v|^2 \) is taken from the parameterization by
Tsushima et al.\textsuperscript{24)} Their overall strength was normalized by a factor of 0.82 to match
the experimental data taken at KEK.\textsuperscript{19)} Several curves correspond to the assumed
\( \Sigma \) potential with \( U_\Sigma^0 = -10, 10, 30, 50 \) and 90 MeV. No overall renormalization
factor is introduced. It is seen that the shape and absolute value are satisfactorily
reproduced by a repulsive strength of 30 MeV. Expecting contributions from multi-
step processes, the actual repulsive strength may be large as \( \sim 50 \) MeV, though it
is premature to draw a final conclusion before taking into account various effects
discussed below. This order of the repulsive magnitude of the \( \Sigma \) s.p. potential is
in line with the estimation of Dąbrowski\textsuperscript{10)} for the BNL data that the \( \Sigma^- \)-nucleus
potential for \( ^9 \text{Be} \) is approximately 20 MeV. It is noted in Ref. 19) that the peak
position at an energy as high as 150 MeV is difficult to reproduce if the repulsion of
the \( \Sigma \)-nucleus potential is not so strong. Present calculations suggest, however, that
it is not necessary for the \( \Sigma \) s.p. potential to be strongly repulsive. An attractive
potential fails because it predicts much larger cross sections. On the other hand, a
stronger repulsive \( \Sigma \) s.p. potential tends to underestimate the cross section. The
reason that the result obtained here differs from that of Ref. 19) might be related
to the fact that we did not use the factorization approximation represented by the
average cross section, Eq. (8). This point deserves further investigation.

For the \( SU_6 \) quark model, it is interesting to observe that this order of magnitude
of a few tens of MeV is consistent with the prediction yielded by the calculation in
nuclear matter.\textsuperscript{18)} The quark model description of the \( \Sigma N \) interaction predicts
a definite strong repulsive nature in the isospin \( T = \frac{3}{2} \) channel due to the quark
antisymmetrization effect. Thus, it is hypothesized that the \( \Sigma^- \)-nucleus potential
becomes more repulsive in the case of a neutron excess. In this respect, analysis of
the (\( \pi^-, K^+ \)) data with heavier nuclei targets would be interesting for the purpose
of investigating whether such a quantitative isospin dependence actually exists.

There are various simplified treatments in the present calculations. The smearing
caused by the Lorentz-type convolution should be replaced by the Green’s function
method. A quantitative estimation of the contribution from multi-step processes
is needed. A more sophisticated description of the elementary process has to be
employed. Contributions from more than two-step processes tend to increase the
cross section. On the other hand, the possible modification of the elementary process
in nuclear media would reduce the cross section. These problems are future subjects
to be investigated. The present SCDW framework serves as a quantitatively reliable
model to study the possible change in properties of intermediate baryonic states.

In summary, we have developed a semiclassical distorted wave model for \((\pi, K)\) inclusive spectra corresponding to \(\Lambda\) and \(\Sigma\) formation. The expression of the double differential cross section consists of the incoming pion distorted wave function, the outgoing kaon distorted wave function and the undetected hyperon distorted wave function at each collision point, where conservation of the classical local momentum holds. The bound nucleon in the target nucleus is described by a local Fermi gas model. The present framework can be easily applied to describe other inclusive spectra, such as \((K, \pi)\), \((\pi, \eta)\), \((\gamma, K)\), and so on. It is also straightforward to consider multistep contributions, as was done for \((p, p')\) and \((p, n)\) inclusive spectra.\(^{21, 22}\) The application of this model to the \((\pi^-, K^+)\) inclusive spectrum obtained with \(^{28}\)Si targets at KEK\(^{19}\) has shown that the spectrum is satisfactorily reproduced by a repulsive \(\Sigma\)-nucleus potential of the order of 30–50 MeV. This magnitude, in turn, constrains the \(\Sigma-N\) potential model and thereby improve our understanding of the interactions between complete octet baryons.

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27) P. K. Saha, private communication.