Breakup of One-Neutron Halo Nuclei within Eikonal Model

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The breakup of a one-neutron halo nucleus is studied in the eikonal approximation. Both the nuclear and Coulomb breakup processes are taken into account. The continuum states, which are usually approximated by plane waves, are constrained to satisfy the condition of orthogonality to the bound states of the halo nucleus. Taking full account of the recoil effect, the neutron-core relative energy distribution of the breakup cross section is calculated for \textsuperscript{11}Be and \textsuperscript{19}C projectiles incident on \textsuperscript{12}C and \textsuperscript{208}Pb targets and compared to experimental data. All the data regarding the energy distribution for a \textsuperscript{208}Pb target at beam energies of 68 and 520 MeV/nucleon are accurately reproduced, except for its peak at small relative energy. The experimental data regarding the relative energy distribution for the \textsuperscript{11}Be+\textsuperscript{12}C system are also accurately reproduced. It is shown that use of orthogonalized plane waves leads to reduced cross sections and is necessary to account for the experimental data. The effect of the no-recoil limit is quantified.

§1. Introduction

Halo nuclei exhibit some extreme properties, such as very large reaction and breakup cross sections.\textsuperscript{1–3)} The last neutron of one-neutron halo nuclei is very weakly bound, with a separation energy of typically 0.5 MeV or less, and is in a state of low angular momentum (\(l=0,1\)). Halo nuclei exhibit an enhanced electric dipole (E1) strength distribution at low excitation energies. The Coulomb dissociation of \textsuperscript{11}Be\textsuperscript{4)} is a good example of this case. Coulomb dissociation is a process in which a projectile is excited by absorbing virtual photons when it passes a high Z target, and subsequently decays into a breakup channel involving a few fragments. It is a useful spectroscopic tool for investigating halo structure, because the uncertainty with regard to the nuclear interaction between the projectile and the target is believed to play a minor role. However, extracting nuclear structure information, for example, spectroscopic factors, from the cross sections requires a reliable description of the reaction mechanism involving continuum states.

Nuclear and Coulomb breakup reactions of one-neutron halo nuclei have been studied both experimentally and theoretically. (See, e.g., Refs. 1) and 3) for an extensive list of references.) The techniques that have been developed for the calculation of the breakup of halo nuclei include a numerical solution of the three-dimensional time-dependent Schrödinger equation (TDM),\textsuperscript{5–7) a coupled-channels method with discretized continuum states (CDCC),\textsuperscript{8–10) and an eikonal approximation.\textsuperscript{11–14) In the TDM, the Coulomb and nuclear interactions between the projectile and the target are treated as external fields, and the relative motion between them is assumed to follow a classical trajectory. The CDCC can describe the breakup process quantum
mechanically, but the inclusion of continuum states will require a great deal of computational time at higher energies, as it involves a large number of partial waves. Both the TDM and CDCC calculations have to this time been applied mostly to the breakup of one-neutron halo nuclei. Very recently, a CDCC calculation\(^\text{15}\) was carried out for \(^6\text{He}+\(^{12}\text{C}\) scattering around 40 MeV/nucleon. On the other hand, the eikonal approximation has the merit of simplicity and gives virtually the same result as the TDM for energies higher than 60 MeV/nucleon.\(^\text{16}\) The eikonal approximation has flexibility with regard to treating both light and heavy targets on an equal footing as well as one-neutron and one-proton halo\(^\text{17}\) nuclei. In addition, an extension to two-neutron halo projectiles\(^\text{18,19}\) can be made relatively easily in the eikonal approximation.

The eikonal approximation has been used to calculate the nuclear breakup of one-nucleon halo nuclei,\(^\text{11,12}\) Both nuclear and Coulomb breakups are considered in Ref. 13) within the framework of the eikonal approximation. The authors of Ref. 13) have resolved the well-known divergence problem which arises from the Coulomb amplitude treated in the eikonal approximation and applied it to investigate the Coulomb and nuclear breakups of \(^{11}\text{Be}\) and \(^{19}\text{C}\). In their analysis, however, two approximations are used. First, a plane wave (PW) is used for the continuum states. Second the no-recoil limit is used, where the impact parameter of the projectile is assumed to coincide with that of the core.

In this paper we investigate the soundness of the above two approximations by calculating the Coulomb and nuclear breakup amplitudes more accurately. We find that these approximations should be used only under certain conditions. We first modify the PW so as to describe the continuum states for the breakup more realistically. We then carry out complete calculations within the eikonal approximation. Finally, we compare our results with those obtained using the PW or no-recoil limit. The effects of these approximations are illustrated by several examples, including the dipole strength functions of \(^{11}\text{Be}\) and \(^{19}\text{C}\) and the relative energy distributions of the fragments coming from the projectiles \(^{11}\text{Be}\) and \(^{19}\text{C}\) incident on \(^{12}\text{C}\) and \(^{208}\text{Pb}\) targets. The calculation given here is still simple and fairly accurate, and it is expected to shed light on the problem of extracting a spectroscopic factor.

This paper is organized as follows. Section 2 presents a theoretical formulation for the elastic breakup amplitude as well as a treatment of continuum states. In §3 we investigate the dipole strength function using modified continuum states. Our results are presented in §4 in which we apply our theory to the one-neutron breakup cross sections of \(^{11}\text{Be}\) and \(^{19}\text{C}\). Also, the effect of the no-recoil limit on the cross sections is studied. Finally, a summary is given in §5.

\section*{§2. Coulomb and nuclear breakup}

\subsection*{2.1. Elastic breakup amplitude}

We assume that a projectile has a one-neutron halo structure. Its wave function
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is determined by solving the three-body Schrödinger equation

\[
\begin{aligned}
&\left\{ \frac{P^2}{2M_{PT}} + H_{nC} + V_{nT}^N \left( R + \frac{A_C}{A_P} r \right) + V_{CT}^N \left( R - \frac{1}{A_P} r \right) \\
&\quad + \frac{Z_C Z_T e^2}{|R - \frac{1}{A_P} r|} \right\} \Psi(r, \zeta, R) = E \Psi(r, \zeta, R),
\end{aligned}
\]

(2.1)

where \( R \) and \( r \) are the projectile-target and the neutron-core relative coordinates, respectively, \( \zeta \) is the spin coordinate of the halo-neutron, and \( P \) and \( M_{PT} \) are the momentum operator and the reduced mass of the projectile-target relative motion, respectively. Further, \( A_P \) and \( A_C (= A_P - 1) \) are the mass numbers of the halo nucleus (projectile) and the core nucleus, and \( Z_C \) and \( Z_T \) are the atomic numbers of the core and target nucleus, respectively. The core-target and neutron-target interactions, \( V_{CT}^N \) and \( V_{nT}^N \), are appropriate optical potentials. Finally, \( H_{nC} \) is the internal Hamiltonian for the neutron-core relative motion.

Ignoring the \( H_{nC} \) term, Eq. (2.1) is solved in the eikonal approximation, which is valid for intermediate- and high-energy reactions. In this approximation, the wave function of the halo nucleus after the reaction is, in the projectile-rest frame, given by

\[
\psi(r, \zeta, b) = S(s, b) \psi_{nljm}^{gs}(r, \zeta),
\]

(2.2)

where \( \psi_{nljm}^{gs}(r, \zeta) \) is the ground-state wave function of the halo nucleus,

\[
\psi_{nljm}^{gs}(r, \zeta) = \sum_{m_l m_s} |l m_l \frac{1}{2} m_s | jm \rangle u_{nljm_l}^{gs}(r) \chi_{\frac{1}{2} m_s}(\zeta),
\]

(2.3)

with \( u_{nljm_l}^{gs}(r) = u_{nlj}^{gs}(r) Y_{lm_l}(\hat{r}) \), and the S-matrix operator \( S \) is given by

\[
S(s, b) = S_{nT}^N \left( b + \frac{A_C}{A_P} s \right) S_{CT}^N \left( b - \frac{1}{A_P} s \right) S_{CT}^C \left( b - \frac{1}{A_P} s \right).
\]

(2.4)

Here, the impact parameter \( b \) for the projectile-target reaction is perpendicular to the beam (z) direction, and \( s \) is the projection of \( r \) onto the xy-plane. Note that \( b + \frac{A_C}{A_P} s \) and \( b - \frac{1}{A_P} s \) denote the neutron-target and core-target impact parameters, respectively. The nuclear S-matrix is related to the corresponding optical potential as

\[
S^N(b) = \exp \left\{ -i \frac{\hbar}{v} \int_{-\infty}^{\infty} dz V^N(b + z \hat{z}) \right\},
\]

(2.5)

where \( v \) is the velocity of the relative motion, and \( \hat{z} \) is the unit vector in the z direction. In the calculation of the S-matrix the spin dependence of the optical potential is ignored, and therefore the S-matrix operator is just a function.

The Coulomb potential in Eq. (2.1) is that which is chiefly responsible for projectile-target Rutherford scattering. It is convenient for the description of the breakup process to subtract the part corresponding to the elastic scattering from the beginning. The Coulomb part that is effective for the breakup is thus given by

\[
V_{CT}^C \left( R - \frac{1}{A_P} r \right) = \frac{Z_C Z_T e^2}{|R - \frac{1}{A_P} r|} - \frac{Z_C Z_T e^2}{R}.
\]

(2.6)
The Coulomb S-matrix in Eq. (2.4) is then given by

$$S_{CT}^C(b - \frac{1}{A_p}s) = e^{2i\eta \log|b - \frac{1}{A_p}s| - 2i\eta \log b} = e^{i\eta \log \left(1 - \frac{2b.s}{A_p b} + \frac{s^2}{A_p b^2}\right)}, \quad (2.7)$$

where $\eta = \frac{Z_C Z_T e^2}{\hbar v}$ is the Sommerfeld parameter and $\hat{b} = \frac{b}{b}$ is a unit vector.

The breakup component of the halo nucleus wave function is obtained by subtracting the ground-state component from Eq. (2.2). In the elastic breakup, the core nucleus and the halo neutron are elastically scattered from the target, but they are diverted differently. Let $ψ_{km_s}(r, ζ)$ denote a continuum state with core-neutron asymptotic momentum $k$ and spin direction $m_s$. Then, the amplitude for the elastic breakup into this state is given by the two-dimensional Fourier transform

$$A_n(q; km_s, nljm) = \frac{iK}{2\pi} \int db e^{-iq.b} A_n(b; km_s, nljm), \quad (2.8)$$

with

$$A_n(b; km_s, nljm) = \langle ψ_{km_s}(r, ζ)|S(s, b) - 1|ψ_{nljm}b(r, ζ)\rangle, \quad (2.9)$$

where $q$ is the momentum transferred to the projectile. The differential elastic breakup cross section is obtained by averaging over the $z$ component of the angular momentum of the initial state and summing over the spin direction of the continuum state:

$$\frac{dσ_n^el}{dqdk} = \frac{1}{K^2} \frac{1}{2j + 1} \sum_{m_s} \sum_{m=-j}^j |A_n(q; km_s, nljm)|^2. \quad (2.10)$$

Integrating over $q$ leads to

$$\frac{dσ_n^el}{dk} = \frac{1}{2j + 1} \sum_{m_s} \sum_{m=-j}^j \int db |A_n(b; km_s, nljm)|^2. \quad (2.11)$$

Then, using the relation $dk = \frac{\mu}{\hbar^2} k dE_k dΩ_k (E_k = \frac{\hbar^2 k^2}{2\mu})$, where $\mu$ is the core-neutron reduced mass, we obtain the basic formula to calculate the (relative) energy distribution of the elastic breakup cross section:

$$\frac{dσ_n^el}{dE_k} = \frac{\mu k}{\hbar^2} \frac{1}{2j + 1} \sum_{m_s} \sum_{m=-j}^j \int dΩ_k db |A_n(b; km_s, nljm)|^2$$

$$≡ \int db \frac{dP_n^el(b)}{dE_k}. \quad (2.12)$$

Here, $P_n^el(b)$ is the elastic breakup probability.

As the $S$-matrix is assumed to be spin-independent, the factorization of the orbital and spin parts, $ψ_{km_s}(r, ζ) = φ_k(r)χ_{zm_s}(ζ)$, allows us to integrate over the spin coordinate. This results in the replacement

$$\frac{1}{2j + 1} \sum_{m_s} \sum_{m=-j}^j |A_n(b; km_s, nljm)|^2 \rightarrow \frac{1}{2l + 1} \sum_{m_l=-l}^l |a_n(b; k, nljm_l)|^2, \quad (2.13)$$
where
\[ a_{-n}(b; k, nljm_l) = \langle \phi_k(r)|S(s, b) - 1|u_{nljm_l}^{gs}(r) \rangle. \] (2.14)

The above factorization is in general not rigorous, but it may be permissible if the spin-orbit potential in \( H_{nC} \) is not so strong that the orbital functions it produces for spin-orbit partners with \( j = l + \frac{1}{2} \) and \( j = l - \frac{1}{2} \) are similar.

2.2. Coulomb breakup

The elastic breakup amplitude (2.14) can be separated into nuclear and Coulomb parts as

\[ a_{-n}(b; k, nljm_l) = a_{-n}^N(b; k, nljm_l) + a_{-n}^C(b; k, nljm_l), \] (2.15)

with

\[ a_{-n}^N(b; k, nljm_l) = \langle \phi_k(r)|\left\{ S_{nT}^N(b + \frac{A_C}{A_p}s)S_{CT}^N(b - \frac{1}{A_p}s) - 1\right\}|u_{nljm_l}^{gs}(r) \rangle, \] (2.16)

\[ a_{-n}^C(b; k, nljm_l) = \langle \phi_k(r)|S_{nT}^N(b + \frac{A_C}{A_p}s)S_{CT}^N(b - \frac{1}{A_p}s) \times \left\{ S_{CT}^C(b - \frac{1}{A_p}s) - 1\right\}|u_{nljm_l}^{gs}(r) \rangle. \] (2.17)

This separation is made in such a way that the nuclear part of the elastic breakup remains unchanged even if the Coulomb force between the core and the target is switched off. We note that the Coulomb part contains the nuclear \( S \)-matrices, \( S_{nT}^N \) and \( S_{CT}^N \), which basically determine the impact parameter dependence of the Coulomb breakup process. The magnitude of the nuclear \( S \)-matrix is related to the imaginary part of the optical potential [see Eq. (2.5)]:

\[ |S_{CT}^N(b)| = \exp \left\{ \frac{1}{\hbar v} \int_{-\infty}^{\infty} dz \text{Im}V_N(b + z) \right\}. \] (2.18)

As the imaginary part of the optical potential is negative, \( |S_{CT}^N(b)| \) becomes small for small values of \( b \). However, \( S_{CT}^N(b) \) approaches unity for values of \( b \) that are close to or greater than the potential range. For this reason, the amplitude for the Coulomb breakup \( a_{-n}^C \) becomes significant for \( b \) larger than a cutoff impact parameter \( b_{\text{min}} \), which is \( b_{\text{min}} \sim 1.4 (A_T^{1/3} + A_P^{1/3}) \) (fm).

Coulomb breakup dominates nuclear breakup in the case of a heavy target. The calculation of the amplitude \( a_{-n}^C \), however, poses a problem, because the quantity \( S_{CT}^C \) in Eq. (2.7) cannot be used in the above formula for all values of \( b \). As is well known, the eikonal approximation ignores energy conservation in the reaction process. This leads to a logarithmic divergence in the Coulomb breakup cross section. To avoid this divergence, a cutoff impact parameter was used in Ref. 19) for the Coulomb breakup, but its treatment there is unsatisfactory. As shown in Ref. 13), this problem can be better remedied by replacing such a term of the eikonal Coulomb phase that leads to the divergence by that derived from a perturbation theory for the Coulomb excitation in which energy conservation is properly taken into account.
Approximating $S_{CT}^C(b - \frac{1}{Ap}s)$ of Eq. (2.7) by $S_{CT}^C(b - \frac{1}{Ap}s) \approx e^{-iQ \hat{b} \cdot s}$, which is valid because $\frac{s}{Apb_{\min}}$ is small, we use the following $E_k$-dependent expression

$$S_{CT}^C(b - \frac{1}{Ap}s) \to e^{-iQ \hat{b} \cdot s} + iQ\hat{b} \cdot s - iQ\xi\{K_1(\xi)\hat{b} \cdot s + iK_0(\xi)z\}. \quad (2.19)$$

Here, $Q = \frac{2\eta}{Apb}$ is the classical Coulomb momentum transfer to the neutron due to the core recoil, and $\xi = \frac{(E_k - E_{gs})b}{h\nu}$, where $E_{gs}$ is the ground-state energy relative to the neutron+core threshold of the projectile, is the adiabaticity parameter which represents the ratio of the reaction time to the nuclear interaction time. The quantity $K_n(x)$ is the $n$-th order modified Bessel function. It should be noted that the leading term of the operator, $S_{CT}^C(b - \frac{1}{Ap}s) - 1$, is given by the last term on the right-hand side of Eq. (2.19), which is linear in the coordinate $r$ and is responsible for the electric dipole excitation into the continuum states. The rest of the terms in $S_{CT}^C - 1$ contribute to the higher-order Coulomb breakup process.

Corresponding to the decomposition of Eq. (2.15), we can define the nuclear and Coulomb elastic breakup cross sections. The differential nuclear breakup cross section $\frac{d\sigma_N}{dk}$ is defined by Eqs. (2.11) and (2.13) with $a_{-n}$ replaced by $a_{N-n}$, and the differential Coulomb breakup cross section $\frac{d\sigma_C}{dk}$ is similarly defined by $a_{C-n}$. The one-neutron elastic breakup cross section consists of the sum of the nuclear and Coulomb breakup cross sections and their interference term.

### 2.3. Continuum states

To calculate the breakup amplitude, we have to specify the continuum state $\psi_{kms}(r, \zeta)$ or its orbital part, $\phi_k(r)$. Both $\psi_{kms}$ and the ground-state wave function $\psi_{nلجm}^{gs}$ are eigenfunctions of $H_{NC}$. Note that there may exist other bound-state eigenfunctions $\psi_{nلجm}^{bs}$ of $H_{NC}$. These should satisfy the orthogonality and closure relations

$$\langle \psi_{kms} | \psi_{nلجm}^{bs} \rangle = 0, \quad (2.20)$$

$$\int dk \sum_{m_s} |\psi_{kms}(r)|^2 = 1 - \sum_{nلجm \in bs} |\psi_{nلجm}^{bs} \rangle \langle \psi_{nلجm}^{bs}|, \quad (2.21)$$

where $\psi_{nلجm}^{bs}$ represents all the bound-state eigenfunctions, including the ground state. As it is computationally costly to use the exact solution $\psi_{kms}$ in the calculation of the breakup amplitude, we want to simplify this solution by using PW to as great an extent as possible. In fact, most calculations in the eikonal approximation carried out to this time have used the PW. In so doing, however, we must consider the fact that the closure relation plays an important role, especially in elastic breakup. This implies that the PW, $e^{ikr}$, must be modified to satisfy the closure relation or the orthogonality constraint. The minimal requirement is expected to be
preferred weakly on

Assuming that the radial parts of the bound-state eigenfunctions of a spin-orbit

wave (OPW) hereafter.

Therefore, we refer to the PW given in (2.22) or (2.23) as an orthogonalized plane

where the Fourier transform of a function \( \psi(r) \) is given by

Assuming that the radial parts of the bound-state eigenfunctions of a spin-orbit

partner depend weakly on \( j = l \pm \frac{1}{2} \) for a given \( l \) (that is, \( v_{nlj(=l+\frac{1}{2})m_i}^{bs} \approx u_{nlj(=l-\frac{1}{2})m_i}^{bs} \)),

we can use the approximation

Then, it is easy to show that the modified PW (2.23) satisfies the condition of

orthogonality to the bound-state eigenfunctions, i.e. \( \langle \phi_k | u_{nljm_i}^{bs} \rangle = 0 \), as well as the closure relation

Therefore, we refer to the PW given in (2.22) or (2.23) as an orthogonalized plane

wave (OPW) hereafter.

Here we point out two important consequences of using the OPW. The first can

be seen from the halo-neutron removal cross section \( \sigma_{-n}^{N} \) due to the nuclear elastic breakup,

The cross section \( \frac{d\sigma_{-n}^{N}}{dk} \) is defined by Eqs. (2.11) and (2.13), with \( a_{-n} \) replaced by

\( a_{-n}^{N} \). The closure relation (2.26) allows us to obtain the nuclear elastic breakup probability,

\[
P_{-n}^{N}(b) = \frac{1}{2l+1} \sum_{m_i=-l}^{l} \left\{ |u_{nljm_i}^{bs}(r)|^2 |S_{n}^{N}(s,b)| - 1 |u_{nljm_i}^{gs}(r)|^2 \right\} - \sum_{n',j',m'_i \in bs} \frac{2j' + 1}{2(2l' + 1)} |u_{n'j'j'm'_i}^{bs}(r)|^2 |S_{n}^{N}(s,b)| - 1 |u_{nljm_i}^{gs}(r)|^2 \right\},
\]

\[
\sigma_{-n}^{N} = \int dk \frac{d\sigma_{-n}^{N}}{dk} = \int db P_{-n}^{N}(b).
\]
with $S^N(S, b) = S^N_{iT}(b + \frac{Ac}{AP}s)S^N_{CT}(b - \frac{1}{AP}s)$. The first term on the right-hand side of Eq. (2.28) is the norm of the scattered wave. It represents the probability obtained when the bound-states are not subtracted in Eq. (2.26), while the second term is the correction due to the orthogonality to those states. We see in §4.2 that the subtraction of the gs→gs term in particular among the second terms produces a significant reduction in the breakup cross section of $^{11}\text{Be}$. The second important consequence concerns the Coulomb breakup cross section. As noted above, the amplitude $a^C_n$ in Eq. (2.17) is dominated by the $E1$ operator for the neutron-core relative motion. Suppose that there exist bound states that have large dipole transition matrix elements with the ground state. Coulomb breakup to such states does not actually occur, because they are bound. If the OPW is used for the continuum state, the transition to such bound states is automatically excluded. However, if one uses the PW, such forbidden transitions are included, and as a result the Coulomb breakup cross section may be overestimated. We show in §4.2 that this is the case particularly for $^{11}\text{Be}$.

A resonance in a continuum cannot be represented by a single OPW. Therefore its contribution to the breakup cross section cannot be accounted for properly in the present treatment of the continuum states. Rather, one has to construct the resonance wave function explicitly to replace the OPW in order to evaluate its effect.

Before closing this subsection, we note that the matrix element of $S^CT(b - \frac{1}{AP}s)$ corresponding to the transition to the PW is expressed in terms of the Fourier transform as follows:

$$
\langle e^{i k \cdot r} | S^CT(b - \frac{1}{AP}s) - 1 | u^{gs}_{nljm}(r) \rangle \\
\rightarrow \tilde{u}^{gs}_{nljm}(k_\perp + Q\hat{b}, k_z) - \tilde{u}^{gs}_{nljm}(k) - Q\hat{b} \cdot \frac{\partial}{\partial k_\perp} \tilde{u}^{gs}_{nljm}(k_\perp, k_z) \\
+ Q\xi \left\{ K_1(\xi) \hat{b} \cdot \frac{\partial}{\partial k_\perp} + i K_0(\xi) \frac{\partial}{\partial k_z} \right\} \tilde{u}^{gs}_{nljm}(k_\perp, k_z),
$$

(2.29)

where $k = (k_\perp, k_z)$.

2.4. Inelastic breakup

Here, we consider an inelastic breakup to be such a process that the halo-neutron excites the target. The inelastic breakup probability is obtained as a part of the total absorption probability, which is given by the decrease of the norm of the wave function (2.2) from unity:

$$
P^{abs}(b) = 1 - \frac{1}{2l + 1} \sum_{m_l = -l}^{l} \langle u^{gs}_{nljm}(r) | S^N_{iT}(b + \frac{Ac}{AP}s)S^N_{CT}(b - \frac{1}{AP}s)| u^{gs}_{nljm}(r) \rangle^2
$$

$$
= \frac{1}{2l + 1} \sum_{m_l = -l}^{l} \langle u^{gs}_{nljm}(r) | 1 - |S^N_{iT}(b + \frac{Ac}{AP}s)S^N_{CT}(b - \frac{1}{AP}s)|^2 \rangle \times |u^{gs}_{nljm}(r)\rangle.
$$

(2.30)
Using the decomposition $1 - |SN_{nT}SN_{CT}|^2 = |SN_{CT}|^2(1 - |SN_{nT}|^2) + (1 - |SN_{CT}|^2)$, we surmise that the term $|SN_{CT}|^2(1 - |SN_{nT}|^2)$ is responsible for the inelastic breakup process. That is, the inelastic breakup probability is equal to the difference between two absorption probabilities, the total absorption due to both the neutron-target and core-target interactions, and the absorption due to the core-target interaction. The inelastic breakup probability is then given by

$$P_{\text{inel}}^{\text{n}}(b) = \frac{1}{2l + 1} \sum_{m_l = -l}^{l} |u_{nljm_l}^n(r)|^2 |S_{CT}^n(b - \frac{1}{Ap}s)|^2$$

$$\times \left\{ 1 - |S_{nT}^n(b + \frac{A_{CT}}{Ap}s)|^2 \right\} |u_{nljm_l}^n(r)|^2.$$  \hfill (2.31)

What can be measured experimentally in the case of the inelastic breakup of a one-neutron halo nucleus is the momentum distribution of the fragment. The momentum distribution of the fragment corresponding to this breakup process has been studied by many people including the present authors.\textsuperscript{12}

§3. Dipole strength function

As we use the OPW for the continuum state, it is desirable to check its usefulness by comparing the results it yields to results obtained employing exact continuum states that are solutions of the Schrödinger equation with $H_{nc}$. For this purpose, we calculate the $E1$ strength using the OPW and compare it to the result appearing in the literature. Such a comparison is made in §4.2. Here we present some formulae for the dipole strength functions calculated with the PW and OPW.

By decomposing the PW into partial waves, we obtain a continuum state with energy $E_k$ and angular momentum $l m_l$,

$$\phi_{E_klm_l}\chi_{\frac{1}{2}m_m} = \sqrt{\frac{2\mu_k}{\pi\hbar^2}} j_l(kr)Y_{lm_l}(\hat{r})\chi_{\frac{1}{2}m_m},$$  \hfill (3.1)

which is normalized as $\langle \phi_{E_klm_l} | \phi_{E_kl'm_l'} \rangle = \delta(E_k - E_{k'})\delta_{l,l'}\delta_{m_l,m_l'}$. The dipole strength function for this continuum state is given by

$$\frac{dB(E1 : \text{PW})}{dE_k} = \frac{1}{2j + 1} \sum_{m'l'm_lm_m} |\langle \phi_{E_kl'm_l}\chi_{\frac{1}{2}m_m} | M_{1\mu} | \psi_{nljm_l}^n \rangle|^2$$

$$= \frac{3}{4\pi} \left( \frac{Z_{Ce}}{Ap} \right)^2 \frac{2\mu_k}{\pi\hbar^2} \sum_{l'} \langle 010 \ | \ l'0 \rangle^2 D(l')^2,$$  \hfill (3.2)

where $M_{1\mu} = -\frac{Z_{Ce}}{Ap}ereY_{1\mu}(\hat{r})$ is the $E1$ operator and

$$D(l') = \int_{0}^{\infty} j_{l'}(kr)u_{nlj}^s(r)r^2dr$$  \hfill (3.3)

is the radial part of the dipole matrix element between the ground state and the continuum state.
In order to compare to a calculation using the exact continuum states, we derive the dipole strength calculated with the OPW. This is done by replacing the PW in Eq. (2.22) with that of Eq. (3.1), which yields

\[
\frac{dB(E1: \text{OPW})}{dE_k} = \frac{3}{4\pi} \left( \frac{Z_{Ce}}{A_P} \right)^2 \frac{2\mu k}{\pi\hbar^2} \left[ \sum_{l'} \langle l010 | l'0 \rangle^2 D(l')^2 - \frac{2l + 1}{6} \sum_{n'l'j'} \langle 2j' + 1 \rangle \langle l010 | l'0 \rangle^2 U(j'lj; \frac{1}{2}1)^2 \times \{ 2D(l')O(n'l'j')D'(n'l'j') - O(n'l'j')^2 D'(n'l'j')^2 \} \right],
\]

where \(U(j_1j_2j_3;j_1j_2j_3;j_1j_2j_3;JM | j_1j_2(j_12), j_3;JM)\) is a Racah coefficient in unitary form, \(O(n'l'j')\) is the radial part of the overlap between the bound state and the PW,

\[
O(n'l'j') = \int_0^\infty j_{l'}(kr)u^{bs}_{n'l'j'}(r)r^2dr,
\]

and \(D'(n'l'j')\) is the radial part of the dipole matrix element between the bound state and the ground state,

\[
D'(n'l'j') = \int_0^\infty u^{bs}_{n'l'j'}u^{gs}_{nlj}(r)r^3dr.
\]

\section{Results and discussions}

\subsection{Input data}

The ground state (\(\frac{1}{2}^+\)) and first excited state (\(\frac{1}{2}^-\)) of \(^{11}\)Be are assumed to have the pure single-particle configurations of \(1s_{1/2}\) and \(0p_{1/2}\) orbits coupled to the \(^{10}\)Be core, respectively. The corresponding wave functions are generated from the neutron-\(^{10}\)Be Woods-Saxon potential used in Ref. 7). The potential includes central and spin-orbit parts. The depth of the central term is chosen so as to reproduce the ground-state binding energy of 0.503 MeV, while that of the spin-orbit term is adjusted in order to locate the first excited state (\(\frac{3}{2}^-\)) in \(^{11}\)Be at the experimental excitation energy of 0.32 MeV. The \(0p_{3/2}\) and \(0s_{1/2}\) single-particle orbits are occupied by neutrons.

The ground-state wave function of \(^{19}\)C is obtained similarly by assuming a configuration in which the \(1s_{1/2}\) neutron is coupled to the \(0^+\) \(^{18}\)C core with a binding energy of 0.53 MeV. The depth of the central potential is chosen as \(-43.23\) MeV, and that of the spin-orbit part is taken to be the same as in the \(^{11}\)Be case. The radius and diffuseness parameters are set equal to 3.2 fm and 0.6 fm, respectively. The \(0d_{5/2}\) orbit becomes lower than the \(1s_{1/2}\) orbit. It is partly filled in our model, and therefore it is not considered a bound occupied orbit with respect to which the continuum state is orthogonalized.

Because there usually is no appropriate core-target optical potential, we calculate
the core-target $S$-matrix $S_{CT}^N$ in the optical limit of Glauber theory,\textsuperscript{12}) which gives

\[
S_{CT}^N(b) = \exp \left\{ - \int \int drdr' \rho_C(r)\rho_T(r')\Gamma(b + s - s') \right\},
\]

where the densities of the core and target are taken from Ref. 20) for $^{12}$C and $^{208}$Pb, from Ref. 12) for $^{18}$C, and from Ref. 21) for $^{10}$Be. The profile function $\Gamma$ is specified by $\sigma_{NN}$ (the total cross section of the nucleon-nucleon collision), $\alpha$ and $\beta$. These parameters are determined so as to fit empirical nucleon-nucleon scattering amplitudes: $\sigma_{NN}=82.5 (3.62)$ mb, $\alpha=0.97 (0.04)$ and $\beta=0.32 (0.125)$ fm$^2$ at 68 (520) MeV/nucleon. The use of the optical limit means that the optical potential of Eq. (2.5) is given by a double-folding potential. For the neutron-target system, we use the optical potential\textsuperscript{22}) to obtain the neutron-$^{208}$Pb $S$-matrix $S_{nT}^N$ at 68 MeV/nucleon, but we again calculate $S_{nT}^N$ for other cases in the optical limit of the Glauber theory.

The bound-state wave functions and the nuclear $S$-matrices ($1 - \text{Re}S$ and $\text{Im}S$) are fitted using a combination of Gaussians. This enables us to obtain all the needed matrix elements analytically. The fitting is very good in general, except in the case of the core-Pb $S$-matrix at 68 MeV/nucleon.

4.2. Use of OPW

Here, we present some examples that demonstrate the importance of using the OPW for the continuum states. Figure 1 displays the nuclear elastic breakup probability $P_{-n}^N$ given by Eq. (2.28) for the $^{11}$Be+$^{12}$C system at 68 MeV/nucleon. The dashed curve represents the results calculated with the first term on the right-hand side of Eq. (2.28). The solid curve represents the probability obtained with OPW. The orthogonality of the PW to the ground state is most important, and it accounts for almost all of the reduction of the probability.

Fig. 1. One-neutron nuclear elastic breakup probability for $^{11}$Be+$^{12}$C at 68 MeV/nucleon as a function of the impact parameter. The solid curve corresponds to the calculation with the OPW, while the dashed curve is the result calculated with the first term on the right-hand side of Eq. (2.28).
Fig. 2. Dipole strengths of $^{11}\text{Be}$ and $^{19}\text{C}$ as functions of the relative energy. The open circles represent the results of the numerical calculation given in Ref. 7). The dotted curve is the result obtained with the PW, while the dashed and solid curves were obtained using the OPW which excludes the $0p_{1/2}$ orbit and both the $0p_{1/2}$ and $0p_{3/2}$ bound orbits, respectively.

Next, we compare in Fig. 2 the dipole strength functions of $^{11}\text{Be}$ and $^{19}\text{C}$ calculated with the OPW [Eq. (3.4)] and the PW [Eq. (3.2)]. The dotted curve plots the results of the calculation with PW, while solid curve plots the results of that with the OPW, which excludes the bound $0p_{1/2}$ and $0p_{3/2}$ states. The bound $0s_{1/2}$ state does not contribute to the $E1$ transition. The dashed curve represents the result calculated by excluding only the $0p_{1/2}$ state.

In the case of $^{11}\text{Be}$ the dipole strength function calculated with the PW gives a much larger strength than that calculated with the OPW, particularly at small energies. They predict similar strengths at higher energies. The $0p_{1/2}$ excited state has a large $E1$ matrix element with the $1s_{1/2}$ ground state, because it has a very small separation energy and therefore a spatially extended structure. For this reason the exclusion of this bound state from the PW yields a significant reduction in the strength function. The $0p_{3/2}$ bound state has an $E1$ matrix element with the ground state, but the exclusion of this state produces only a small effect, because the matrix element is rather small. The dipole strength function with the OPW is compared to that calculated by using continuum eigenstates of $H_{0\text{C}}$. The continuum eigenstates are naturally orthogonal to the bound-state solutions, and therefore this comparison is useful to judge the utility of the OPW as the continuum wave functions in the breakup process. It is seen that the dipole strength function obtained by subtracting the transitions to the bound states is very similar to that obtained in Ref. 7), except for a slight difference at the peak around 0.3 MeV.

In the case of $^{19}\text{C}$, the difference between the dipole strengths obtained with the OPW and the PW is very small. In this case, both the $0p_{1/2}$ and $0p_{3/2}$ states are deeply bound and have small $E1$ matrix elements with the ground state. This explains the difference between the roles of the OPW for $^{11}\text{Be}$ and $^{19}\text{C}$.
4.3. Use of full recoil

To simplify the calculations, the no-recoil limit is often employed. This approximation ignores the difference between the impact parameters of the projectile and the core. That is, the nuclear S-matrices, $S_{N}^{nT}(b + \frac{Ac}{Ap}s)$ and $S_{CT}^{N}(b - \frac{1}{Ap}s)$, are replaced with $S_{nT}^{N}(b + s)$ and $S_{CT}^{N}(b)$, respectively. The no-recoil approximation affects the relative energy distribution as well as the integrated elastic cross section, but it does not change the integrated inelastic cross section $\sigma_{n}^{inel}$, which is obtained by integrating Eq. (2.31) over the impact parameter $b$. The validity of no-recoil approximation is tested by comparing its results to those obtained with a full calculation that takes into account the difference between the impact parameters.

Figures 3 and 4 display the one-neutron nuclear elastic and inelastic breakup probabilities for $^{11}\text{Be}$ incident on $^{12}\text{C}$ and $^{208}\text{Pb}$ at 68 MeV/nucleon as functions of the impact parameter. The solid curve corresponds to the full calculation, while the dashed curve corresponds to the calculation in the no-recoil limit, while the solid curve is the result with the full recoil effect.

**Fig. 3.** One-neutron nuclear elastic breakup probabilities for $^{11}\text{Be} + ^{12}\text{C}$ and $^{11}\text{Be} + ^{208}\text{Pb}$ at 68 MeV/nucleon as functions of the impact parameter. The dashed curve corresponds to the calculation in the no-recoil limit, while the solid curve is the result with the full recoil effect.

**Fig. 4.** The same as Fig. 3, but for inelastic breakup probabilities.
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dashed curve corresponds to the no-recoil calculation. It is seen that the probability calculated with the no-recoil approximation shifts to the larger impact parameter region. One noteworthy point is seen in the elastic breakup probability for the $^{208}$Pb target: The no-recoil limit gives a considerably larger probability than the full recoil calculation.

To demonstrate the effect of the no-recoil limit on the Coulomb breakup, we present in Fig. 5 the Coulomb breakup probability per unit energy at $E_k=0.3$ MeV for $^{11}$Be+$^{208}$Pb at 520 MeV/nucleon. There the Coulomb breakup amplitude of Eq. (2.17) is used. The solid curve corresponds to the calculation with full recoil, while dashed curve corresponds to that with no-recoil, in which only the nuclear S-matrices are approximated. The dotted curve is the result with the nuclear potentials $V_{NT}^N$ and $V_{CT}^N$ set to zero. Therefore the difference between the solid and dotted curves shows how the Coulomb breakup probability defined in this paper is affected by the nuclear force. The no-recoil limit results in some difference from the full model, but it is limited to a small region of impact parameter values. Because the Coulomb breakup probability extends to far distances, it turns out that the no-recoil limit is good in the case of the Coulomb breakup.

The integrated elastic breakup cross sections, both nuclear and Coulomb, for a $^{11}$Be projectile are listed in Table I, together with the inelastic breakup cross section. That table also includes the cross sections calculated with the no-recoil limit. The no-recoil effect leads to approximately a 20% overestimation of the nuclear elastic breakup cross section for a $^{208}$Pb target, but it gives no change for other cases, in agreement with Ref. 11). As seen in the row with asterisk, the calculated elastic breakup cross section $\sigma_{el}^{1n}$ at 520 MeV/nucleon is slightly smaller than the experimental value, e.g., by about 6 mb for the $^{12}$C target. However the calculated

![Fig. 5. Elastic breakup probability per unit energy for $^{11}$Be+$^{208}$Pb at 520 MeV/nucleon as a function of the impact parameter. The energy $E_k$ of the neutron relative to the $^{10}$Be core is set to 0.3 MeV. The dashed curve represents the calculation in the no-recoil limit for the nuclear S-matrices, while the solid curve is the result with full recoil. The dotted curve was obtained by switching off the nuclear potentials $V_{NT}^N$ and $V_{CT}^N$.](https://academic.oup.com/ptp/article-abstract/112/6/1013/1911559)
Table I. One-neutron integrated breakup cross sections in units of mb calculated for a $^{11}\text{Be}$ projectile. The elastic breakup cross section $\sigma_{\text{el}}$ consists of the Coulomb ($\sigma_{\text{el}}^\text{C}$) and nuclear ($\sigma_{\text{el}}^\text{N}$) contributions as well as the Coulomb-nuclear interference term (not shown). The values in parentheses are the cross sections calculated using the no-recoil limit. The results listed in the rows with an asterisk were obtained by limiting the integration over the relative energy up to $E_k=5.6$ MeV in order to allow for comparison with the experimental results:\textsuperscript{25} a) $26.9\pm1.4$, b) $605\pm30$, c) $477\pm32$ mb.

<table>
<thead>
<tr>
<th>Target</th>
<th>Energy (MeV/nucleon)</th>
<th>$\sigma_{\text{el}}^\text{C}$</th>
<th>$\sigma_{\text{el}}^\text{N}$</th>
<th>$\sigma_{\text{el}}^\text{N}$</th>
<th>$\sigma_{\text{el}}^\text{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$</td>
<td>68</td>
<td>103</td>
<td>14</td>
<td>80 (80)</td>
<td>123</td>
</tr>
<tr>
<td></td>
<td>520</td>
<td>34</td>
<td>5</td>
<td>29 (27)</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>520$^*$</td>
<td>21$^a)$</td>
<td>5</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>68</td>
<td></td>
<td></td>
<td>219 (260)</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>520</td>
<td>570</td>
<td>455</td>
<td>115 (140)</td>
<td>304</td>
</tr>
<tr>
<td></td>
<td>520$^*$</td>
<td>553$^b)$</td>
<td>455$^c)$</td>
<td>98</td>
<td></td>
</tr>
</tbody>
</table>

$\sigma_{\text{el}}^\text{C}$ value is in satisfactory agreement with experimental result for the $^{208}\text{Pb}$ target. Thus we find that the underestimation of $\sigma_{\text{el}}^\text{el}$ is due to the fact that the calculated nuclear breakup cross section ($\sigma_{\text{el}}^\text{N}$) is slightly too small. In §4.5, we estimate the contribution of the $^{11}\text{Be}$ resonance at $E_k=1.29$ MeV to $\sigma_{\text{el}}^\text{N}$ for the $^{12}\text{C}$ target case. It is found to be about 5 mb, which is just the amount needed to reproduce the $\sigma_{\text{el}}^\text{el}$ value for a $^{12}\text{C}$ target. Similarly, the resonance contribution is expected to improve the fit to $\sigma_{\text{el}}^\text{el}$ for a $^{208}\text{Pb}$ target. The values of $\sigma_{\text{el}}^\text{el}$ and $\sigma_{\text{el}}^\text{C}$ for the Pb target at 68 MeV/nucleon are not given in the table, because the behavior of the nuclear $S$-matrices makes it difficult to obtain accurate cross sections.

4.4. Breakup by a $^{208}\text{Pb}$ target

Figure 6 displays the result for the relative energy distribution for the elastic breakup of $^{11}\text{Be}$ on a $^{208}\text{Pb}$ target at a beam energy of 520 MeV/nucleon. Because the experiment confirms that the $^{10}\text{Be}$ core is in its ground state after the breakup, a comparison to the calculated distribution can be made directly. Both the Coulomb and nuclear breakup terms are included. The solid curve represents the full calculation with the OPW, while the dotted curve represents the result obtained with the PW. The dashed curve represents the contribution due to the nuclear breakup. It is seen that the nuclear breakup process alone accounts for the cross section at higher energies. The effect of the OPW significantly reduces the cross section at low relative energy, while it has a small contribution at higher relative energy. Our calculation reproduces the experimental data accurately beyond $E_k=0.6$ MeV, though it does not account for some resonant behavior around 1.5 MeV. A discrepancy between the calculational results and the experimental data appears around the peak of the distribution. This discrepancy seems to remain even when exact continuum states are employed instead of the OPW. It is an open question whether this discrepancy can be accounted for by appropriate spectroscopic factors or by other sophisticated models for $^{11}\text{Be}$.

Figure 7 displays the result for the relative energy distribution for the elastic
Fig. 6. Elastic breakup cross section for $^{11}\text{Be} + ^{208}\text{Pb}$ at 520 MeV/nucleon as a function of the relative energy. The solid and dotted curves represent the results with the OPW and the PW, respectively. The dashed curve corresponds to the calculation with the nuclear potential only. The experimental data are taken from Ref. 25).

$^{11}\text{Be} + ^{208}\text{Pb} \rightarrow ^{10}\text{Be}(g.s.) + n + ^{208}\text{Pb}$

$E = 520$ MeV/nucleon

$\frac{d\sigma^{el}_{\text{d}}}{{dE_k}}$ (mb/MeV)

$E_k$ (MeV)

Fig. 7. Elastic breakup cross section for $^{11}\text{Be} + ^{208}\text{Pb}$ at 68 MeV/nucleon as a function of the relative energy. The solid curve corresponds to the calculation with the OPW. The dotted curve is the result of the first-order perturbation theory with the PW. The experimental data are taken from Ref. 26).

$^{11}\text{Be} + ^{208}\text{Pb}$

$E = 68$ MeV/nucleon

$\frac{d\sigma^{el}_{\text{d}}}{{dE_k}}$ (b/MeV)

$E_k$ (MeV)

breakup of $^{11}\text{Be}$ on a $^{208}\text{Pb}$ target at 68 MeV/nucleon. The nuclear breakup contribution is included in the calculation. In the experiment, the $^{10}\text{Be}$ state is not identified, and therefore it may be the case that the experimental distribution contains a contribution from bound excited states of $^{10}\text{Be}$. The solid curve represents the calculations carried out with the OPW, while the dotted curve represents the result obtained with the first-order perturbation theory with PW. The difference between the solid and dotted curves reflects the combined effects of the use of the OPW and the higher-order terms included in the eikonal Coulomb phase. These effects are important to reduce the cross section, particularly near the peak.
distribution obtained with the OPW agrees very well with that calculated using the TDM.\(^7\) Though the theory employing the OPW reproduces the experimental distribution very accurately, it should be noted that the experiment may contain a contribution from the bound excited \(^{10}\)Be fragments. Concerning the discrepancy between the theoretical and experimental results, exactly the same comment as in the previous paragraph applies.

Figure 8 displays the relative energy distribution for the elastic breakup of \(^{19}\)C on a \(^{208}\)Pb target at a beam energy of 68 MeV/nucleon. The binding energy of the halo neutron in the ground state of \(^{19}\)C is taken to be 0.53 MeV.\(^{27}\) The solid curve corresponds to the complete calculation carried out by using both the OPW and the full recoil treatment. The difference between the results obtained with the PW and the OPW is negligible, as in the case of Fig. 2. Our calculation overestimates the cross section in the peak region. The result is very similar to that obtained in Ref. 6) with a TDM calculation.

4.5. **Breakup by a \(^{12}\)C target**

One advantage of the present approach is that there is no difference in its formulation for the case of heavy and light targets or medium and high energy reactions. Contrastingly, when partial wave expansions are used to obtain the projectile wave function including the breakup channels, more partial waves have to be taken into account. This leads to slow convergence, particularly at high incident energy. Figure 9 displays the relative energy distribution for the elastic breakup of \(^{11}\)Be on a \(^{12}\)C target at a beam energy of 520 MeV/nucleon. The solid curve is the result which includes both the Coulomb and nuclear breakup contributions. The dotted curve represents the cross section due to the Coulomb breakup, while the dashed curve represents the contribution due to the nuclear breakup. It is interesting that the Coulomb breakup is not completely negligible, even for such a light target as \(^{12}\)C: The Coulomb breakup contributes an amount to the relative energy distribution that
is about 50% of the nuclear contribution near its peak region. In total, the Coulomb breakup contributes about 15% of the integrated elastic breakup cross section, as seen from Table I.

The experimental values are larger than the theoretical values in the region $E_k = 1-3$ MeV. This may be due to the existence of some resonances which are not accounted for in the present description of the continuum states. To confirm this possibility, we approximated a $5/2^+$ resonance state at $E_k = 1.29$ MeV as a $0d_{5/2}$ bound state. This may be permissible as its width is narrow, $\Gamma = 100 \pm 20$ keV. The cross section obtained with the nuclear breakup is found to be 5.2 mb, which roughly agrees with the needed cross section around $E_k = 1.3$ MeV.

§5. Summary

The eikonal approximation is valid for medium and high energy reactions. To apply it to the breakup reaction of halo-nuclei, however, one has to resolve the well-known divergence problem of the Coulomb dissociation cross section. This problem is solved by replacing the divergent term with a first-order perturbation term. With this eikonal model, one can describe the nuclear and Coulomb breakup reactions of a halo nucleus and compare to experiment without introducing any ad hoc parameters like a cutoff impact parameter for the Coulomb breakup. All the breakup amplitudes are evaluated accurately.

In this paper we have analyzed the neutron-core relative energy distribution of the breakup cross sections of $^{11}$Be and $^{19}$C projectiles at beam energies of 68 and 520 MeV/nucleon. The target nuclei considered are $^{12}$C and $^{208}$Pb. These projectile nuclei are halo nuclei described by a $1s_{1/2}$ neutron coupled to the $0^+$ core nucleus. However, they exhibit different responses to the Coulomb breakup. The first excited
state of $^{11}\text{Be}$ is $\frac{1}{2}^-$, and it has a strong $E1$ transition with the ground state, but this transition has no active contribution to the Coulomb breakup cross section, as the excited state is bound. By contrast, $^{19}\text{C}$ has no excited states that have large $E1$ matrix elements with the ground state.

In the eikonal description of the breakup process, the neutron-core interaction is ignored, and therefore the continuum states are usually approximated by plane waves. As the continuum state must be orthogonal to any bound states of the projectile, we have modified the plane wave to satisfy this orthogonality constraint. This constraint is necessary to exclude such false breakup processes as the transition from the ground state to the first excited state in $^{11}\text{Be}$, which are included if the continuum is described by plane waves. The use of orthogonalized plane waves for continuum states enables us to obtain the relative energy distribution in fair agreement with experiment. The agreement is particularly good at higher relative energies, where the nuclear breakup process becomes important, but it is still unsatisfactory at such small relative energies that the breakup distribution reaches a maximum. For a more detailed analysis, it is important to extend the present theory by allowing core excitations and by describing the ground state of the projectile as a configuration of coupled channels.

We have examined the validity of the no-recoil limit. It changes the distribution of the breakup cross section as a function of the impact parameter, but it causes an almost negligible change in the integrated cross section, except in the case of the elastic breakup cross section for a $^{208}\text{Pb}$ target.

An extension of the present approach to two-neutron halo nuclei would be interesting. By ignoring the neutron-core interaction as well as the neutron-neutron interaction, the two-neutron wave function after the reaction can easily be expressed in terms of appropriate $S$-matrix operators. It is a challenge to test how well the continuum states obtained by solving the three-body Schrödinger equation are approximated by a product of modified plane waves.

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**References**