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## DISCUSSION

### Erhard Krempl<sup>1</sup>

#### Part I—Equations Development and Analytical Procedures

The author has contributed to the modeling of inelastic behavior of metals by considering creep, short-time plasticity and anelasticity together. In this respect the paper differs significantly from others in the metallurgy oriented literature where one mode of deformation, e.g., creep, is investigated and detailed mechanisms are postulated which may or may not be operative with other tests. The author's concept represents a significant step forward in the development of realistic constitutive equations. While reading the papers the discussor noticed the following items.

It is stated that "physical mechanisms" are the key to finding the best material model. Rather than using physical mechanisms directly (dislocation density growth, interaction with grain boundaries and impurities, integration over the polycrystal) intuitive statements about necessary variables are made. The same insights help in the determination of the necessary constants of the mathematical equations. However, these statements appear not always in accord with physical or mathematical reality.

Before equation (5) it is stated that "for low  $R$  the yield strength is essentially the stress required to cause a measurable  $\dot{\epsilon}$ ." Two questions arise: What about creep below the yield stress that is of great engineering concern ( $\dot{\epsilon}$  is not constant there) and why can  $R$  be set zero in equation (5) considering the highly nonlinear function involved?

In steady-state creep  $\dot{\epsilon} = \text{constant}$  and the discussor would expect that  $D$  and  $R$  should attain their constant steady-state values  $D_{ss}$  and  $R_{ss}$ , respectively. This is implied in the author's theory through equations (2), (3), (8), and (9). The mathematical model should reproduce this postulated physical fact. However, substitution of  $\dot{\epsilon} = \text{constant}$  into equations (26) and (27) does not give  $\dot{R} = 0$  and  $\dot{D} = 0$ , respectively.

In reference [21] of Part II the discussor has found that significant interactions between creep and plasticity can exist. This phenomenon was called nonclassical history dependence and it was shown that this phenomenon required special mathe-

matical forms of the constitutive equations. Equations (25) through (28) are basically rate-type equations and no yield-type condition is apparent. The discussor therefore wonders whether MATMOD has been used to reproduce the phenomena depicted in Figs. 12(b), 12(d), 14(a), 17(c) (Bauschinger Effect in Creep), 17(e) (cyclic softening in creep) and Fig. 18(c) (difference of the 12.5 ksi creep curve for first and third test) of reference [21] of Part II.

#### Part II—Application to Type 304 Stainless Steel

In Part I the exponent 1.5 in equations (10) and (25) was determined from the data in Fig. 1. As far as the discussor can see the same exponent was used for stainless steel. Why is it not necessary to have the data displayed in Fig. 1 of Part I if one applies the model to other materials than the Fe-3 percent Si?

A comparison of Figs. 7(a) and 7(b) shows that the strain hardening in the real material is strongly cycle dependent whereas a nearly constant strain-hardening characteristic is displayed in Fig. 7(b). It appears therefore that the MATMOD is limited to materials that show insignificant influence of prior deformation on the strain hardening. If further appears that in simulating the tests in Fig. 7(a) an equation must have been used which describes the growth of the elastic limit with cycles (a yield condition). Such an equation is not contained in equations (25)-(28) of Part I. In the cyclic case equation (6) of Part I does not appear to be useful since a measurable strain rate is imposed, see Figs. 7(a) and 7(b). Further,  $R$  changes with cycles as shown in Fig. 6 of Part I and therefore the hypothesis leading to equation (6) of Part I does not appear to be valid for Fig. 7(b).

### Author's Closure

I wish to thank Professor Krempl for his detailed consideration of the papers.

Several of the points in his discussion (i.e., in the fifth paragraph, the question of interactions between creep and plasticity

<sup>1</sup>Professor of Mechanics, Dept. of Mechanical Engineering, Aeronautical Engineering and Mechanics, Rensselaer Polytechnic Institute, Troy, N.Y.

and in the last paragraph, the question of the origin of the growing "elastic limit" in the cyclic test simulated in Fig. 7(b) can be answered by re-examining the role of the isotropic hardening variable,  $D$ . Both "creep" strains and "short-time plastic" strains are affected by, and in turn cause changes in, the value of  $D$ . Thus  $D$  is the major means by which the two "types" of strain communicate and interact. Also, according to equation (25) of Part I, for a given  $\dot{\epsilon}$ , temperature, and  $R$ , the required stress  $\sigma$  depends directly on  $D$ ; thus it is the gradual changes in  $D$  that control the growth of the elastic limit in Fig. 7(b) of Part II. This was pictured schematically in Fig. 6 of Part I. Similarly (with respect to the discussor's question about the applicability of equation (5)), for the particular situation of a series of tests all at the same  $\dot{\epsilon}$ , same temperature, and low  $R$ , the stress to cause plastic flow (i.e. the yield strength) varies directly with  $D$ . Thus the value of  $D$ , as measured by the yield strength, indicates the severity of the isotropic hardening which was introduced by the prior warm-working.  $R$  can be neglected in this situation only because its values are probably small compared to the large differences in yield strength among the various tests.

Some of the interaction effects shown by the discussor in reference [21] of Part II have been simulated in reference [1] of the papers. The effects of stress changes during creep (discussor's Fig. 12(b), 12(d)) are shown in Fig. 3.14 of reference [1]. The renewed primary creep occurring following a zero-stress recovery period (discussor's Figs. 12(b), 13(d)) is simulated in Figs. 3.22 and 3.23 of reference [1].

Other interaction effects mentioned by the discussor have not yet been shown specifically, but the proposed model should simulate them. The discussor's Figs. 12(c), 14(a), and 18(c) show that prestraining diminishes the amount of primary creep. The proposed model should do the same, as discussed on the fourth page of Part I.

The two remaining interaction effects (cyclic softening in creep and Bauschinger effect in creep) are as yet unexplored with this model.

With respect to the exponent of 1.5 in equation (25), reference [1] gives the warm-working data for type 304 stainless steel showing that an exponent of 1.5 is *plausible* but by no means *definitive* for this material. Obtaining a proper relationship between  $D_{ss}$  and  $\sigma_{ss}$  for type 304 is probably complicated by dislocation-solute interactions, as also discussed in reference [1]. But in any event the data of Fig. 10 of Part II does give us an opportunity to check the exponent of 1.5 and show that it is approximately correct.

With respect to the effect of prior straining on the rate of strain hardening, changes in the value of the kinematic hardening variable,  $R$ , govern the slope of the stress-strain curves in the cyclic test of Fig. 7(b). Since  $H_1$  in equation (26) is a permanent material constant, it is quite true that for this situation (in which changes in  $R$  dominate the local strain hardening) the strain hardening will be independent of prior history. But this will be true only for situations of small strain amplitude. At large strains, the rate of strain hardening will be more dependent on changes in  $D$  and will, therefore, be more history-dependent.

Finally,  $\dot{R}$  and  $\dot{D}$  do go to zero at steady state in equations (26) and (27).<sup>5</sup> For any particular  $\dot{\epsilon}_{ss}$  and  $T$ , we can set  $\dot{R} = 0$  in equation (26) and solve for  $R_{ss}$ , and then substitute this  $R_{ss}$  into equation (27), setting  $\dot{D}$  equal to zero and solving for  $D_{ss}$ . We could then substitute these  $R_{ss}$  and  $D_{ss}$  into equation (25) and find  $\sigma_{ss}$ , the steady-state flow stress. Hence at steady-state,  $\dot{\epsilon}$ ,  $\sigma$ ,  $R$ , and  $D$  do attain their constant, steady-state values  $\dot{\epsilon}_{ss}$ ,  $\sigma_{ss}$ ,  $R_{ss}$ , and  $D_{ss}$  and  $\dot{R} = \dot{D} = 0$ . Of course the particular values which are attained will depend on the imposed conditions.

<sup>5</sup>It should be noted that the first term on the right side of equation (26) is  $H_1\dot{\epsilon}$ ; there was a typographical error in the preprinted paper.