Migration of synthetic seismograms for crustal structures with random heterogeneities

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SUMMARY
Fast frequency–wavenumber migration is used to migrate synthetic seismograms, calculated by a hybrid method for crustal models with complex 2-D scattering structures in the lower crust. Scatterers with predominantly horizontal or vertical orientation and scatterers without preferred orientation (isotropic scatterers) are investigated; horizontal scatterers or lamellae have frequently been suggested for the lower crust on the basis of modern reflection seismological experiments. In none of the cases studied here are the scattering structures imaged correctly. The reason is mainly multiple scattering which can produce coherent arrivals in the seismogram sections, and hence coherent signatures in the migrated sections, which have no relation to structure. Imaging is generally acceptable for horizontal scatterers, but for isotropic or vertical scatterers the migrated signatures are also horizontal and thus do not represent reality. It is concluded that the highly detailed line drawings, which are popular in crustal reflection seismology, are less reliable than believed, as far as the internal structure of scattering zones and the scatterer orientations are concerned. Horizontal or subhorizontal structures in the lower crust may be less common than assumed.

The paper also briefly addresses a few methodical aspects of fast frequency–wavenumber migration for depth-dependent background structure, in particular the concept of sounding beams, their width and the construction of a migrated section from several sounding beams.

Key words: crustal structure, random media, seismic migration, synthetic seismograms.

1 INTRODUCTION
Modern high-resolution seismic experiments in the framework of projects such as COCORP, BIRPS, ECORS or DEKORP have given evidence that the lower crust has often a greater density of reflecting and diffracting structures than the upper crust or the depth range immediately below the Moho. A few recent papers with such evidence are by Brown (1991), Meissner, Wever & Sadowiak (1991), Sadowiak, Wever & Meissner (1991) and Dyment & Bano (1991); numerous other papers with similar results exist in the literature.

Studies such as those mentioned generally end up with line drawings of the common mid-point (CMP) section of a profile or of a migrated CMP section, and the correlated signatures in these sections are considered as one-to-one representations of reflectors or diffractors. In this way, line drawings are taken as geological cross-sections and interpreted as such. This is certainly correct if the subsurface is well structured with a few interfaces and faults as in sedimentary basins, but is it also correct in the case of a large number of more or less randomly distributed small-scale heterogeneities, as they are often assumed in the lower crust? In this case, the basic single-scattering assumption of migration may not hold, i.e. multiple scattering between heterogeneities may contribute strongly to the energy distribution in observed seismograms. Hence, it is of interest to investigate theoretically, how the seismic response of such a medium is migrated and to what extent the migrated section is an image of the true structure. These questions are addressed in this paper.

We treat these questions under the simplifying assumption
of 2-D wave propagation in acoustic media. Our crustal models have a horizontally layered background structure, upon which a random velocity and density fluctuation is superposed in the lower crust. The scatterers can have a horizontal or a vertical preferred orientation, or they can be isotropic in the sense that there is no preferred orientation. Synthetic seismograms for these crustal models are computed with a hybrid method (Emmerich 1989, 1992), which combines ray theory, finite-difference calculations in the lower crust and wavefield continuation through the layered, laterally homogeneous upper crust with the aid of a frequency–wavenumber \((\omega-k)\) filter. For the migration of the resulting single-shot seismogram sections we use the fast \(\omega-k\) migration method for single-shot data and a horizontally layered background medium (Temme & Müller 1986; Müller & Temme 1987), which is a generalization of the method proposed by Stolt (1978) for CMP data and a homogeneous background. Synthetic seismograms for laterally heterogeneous sediment and crustal models have also been calculated by Wenzel, Wulfe & Sandmeier (1987) who used a Fourier method, by Gibson & Levander (1988a, b) using a finite-difference technique, and by Raynaud (1988) applying single-scattering theory. Gibson & Levander (1988a) investigated additionally the reconstruction of a random structure by CMP migration of synthetic seismograms; our study deals partly with similar questions.

The paper is organized as follows. Section 2 gives a brief summary of fast \(\omega-k\) migration and demonstrates for simple synclinal reflectors that this migration method works successfully. In Section 3 synthetic seismograms are presented for velocity fluctuations in the lower crust which are described by a double-exponential autocorrelation function; the parameters in this function are the standard deviation and the horizontal and vertical correlation distance. The seismograms are migrated in Section 4, and the migrated cross-sections are compared with the given cross-sections of the velocity fluctuations. For the latter, filtered versions are used, with emphasis on higher wavenumbers. It turns out that there is only limited agreement between the two types of cross-sections. Conclusions for data interpretation in crustal reflection seismology are drawn in Section 5.

### 2 FAST \(\omega-k\) MIGRATION OF SINGLE-SHOT DATA

#### 2.1 Overview of the method

As described in more detail by Temme & Müller (1986) and Müller & Temme (1987), the two main steps of \(\omega-k\) migration of single-shot data are (1) downward continuation of the observed wavefield at the earth's surface \((z=0)\) in the frequency–wavenumber domain, and (2) imaging, i.e. picking the value of the downward continued wavefield at the imaging time of each cross-section point \((x, z)\). This time is the travelt ime from the source to the surface at the cross-section point. In Müller & Temme (1987, eq. 6) the following frequency–wavenumber representation of the migrated section \(M(x, z)\) is given:

\[
M(x, z) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{U}(k, \omega) S(k, z, \omega) e^{iF(x, z, k, \omega)} d\omega dk,
\]

\(F(x, z, k, \omega) = \omega t(x, z) + kx + \Psi(k, z, \omega).\) (2)

\(\tilde{U}(k, \omega)\) is the Fourier transform of the observed wavefield \(U(x, t)\) at \(z=0\), \(t(x, z)\) is the imaging time, \(S\) is an amplitude function and \(\Psi\) a phase integral, both of which will not be reproduced here. The essential step of fast \(\omega-k\) migration is the transformation of the double integral (1) into a 2-D Fourier integral over the wavenumbers \(A\) and \(B\), corresponding to the spatial coordinates \(x\) and \(z\). This transformation can be achieved by linearization of the function \(F\) in eq. (2) with respect to \(x\) and \(z\). From the linearization, the transformation equations from \((k, \omega)\) to \((A, B)\) follow. Then the Fourier integral

\[
M(x, z) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{M}(A, B) e^{i(Ax + Bz)} dA dB
\]

is obtained with the Fourier transform \(\tilde{M}(A, B)\) depending mainly on the functions \(\tilde{U}\) and \(S\) in eq. (1) and on the Jacobian of the variable transformation.

As a consequence of the linearization of \(F\) in eq. (2), \(M(x, z)\) calculated via (3) is correct or focused only along one of the segments of a sounding ray with preselected sounding angle \(\phi_s\) at the shot (Fig. 1). This segment runs through a preselected layer of the background structure. The velocity of this layer has to be chosen as the migration velocity \(v_m\); velocity differences in the overburden with respect to \(v_m\) are taken into account through a phase factor in \(\tilde{M}\). The mappings from \((k, \omega)\) to \((A, B)\) and vice versa include the parameters \(\phi_s\) (or the corresponding angle \(\phi_m\) in the preselected layer, see Fig. 1) and \(v_m\). Details of the theory and of the numerical calculation of the Fourier integral (3) can be found in the two papers quoted. The computing time for one sounding ray is of the order of 1 per cent of the computing time of recursive \(\omega-k\) migration, which is a generalization of the phase-shift method of Gazdag (1978) for single-shot wavefields and a layered background structure.

The sounding ray is the centre of the sounding beam (shaded in Fig. 1), which is the cross-section area where the migrated section (3) is practically correct. In Appendix A a

![Figure 1. Fast \(\omega-k\) migration for single-shot data is exact in a preselected layer along a sounding ray with preselected angle \(\phi_s\) at the shot (related, according to Snell's law, to the angle \(\phi_m\) in the layer).](https://academic.oup.com/gji/article-abstract/113/1/225/731608)
theoretical expression for the width of the sounding beam at an arbitrary point on the sounding ray is derived.

In the case of small-scale objects in the subsurface one sounding ray may be sufficient for reconstruction. If larger scale structures are present, several sounding rays have to be used, and the correct sectors of the migrated sections are fitted together. In practice, the sectors are limited by rays with an angular difference at the shot of 4–20°, depending on the velocity–depth function and on frequency; such sectors usually have a width which is less than the sounding beamwidth according to eq. (A2) of Appendix A. The fitting procedure requires ray tracing for the sector boundaries and is very simple; no smoothing of migration traces at the boundaries is needed.

Fig. 2 presents a first example of fast $\omega-k$ migration. The subsurface model is a syncline, separating two homogeneous

![Figure 2](https://academic.oup.com/gji/article-abstract/113/1/225/731608)

**Figure 2.** Single-shot migration for a syncline: (a) seismogram section; (b) recursive $\omega-k$ migration; (c) and (d) fast $\omega-k$ migrations for the sounding angle -40 and 20°, respectively; (e) three sounding beams (for sounding angles -50, 0, 40°); and (f) complete migrated section, consisting of nine beams.
media. Its reflection response for a shot above the centre (Fig. 2a) is migrated recursively (Fig. 2b) and with the fast $\omega-k$ technique for the sounding angles $-40$ and $20^\circ$. The migrated sections in Figs 2(c) and (d) are correct in the neighbourhood of the sounding rays, as follows from comparison with the curved line which represents the syncline. Three sounding beams, each $20^\circ$ wide, are shown in Fig. 2(e), and the complete migrated section, consisting of nine beams, is given in Fig. 2(f). The syncline is modelled at least as well as with recursive $\omega-k$ migration.

2.2 Migration example

Another example of the fast $\omega-k$ migration technique will be described in more detail, because it is closely related to the main topic of this paper: migration of responses of the lower crust (Fig. 3). The crustal model consists of four layers, and in the third layer a synclinal reflector is embedded. It may be imagined as being a thin layer with properties different from those of layer 3, but actually it is modelled by 600 closely spaced line diffractors. The shot is located at offset zero, and the dominant frequency of the radiated $P$-wave is 5 Hz. Further parameter values are: offset increment $\Delta x = 94.5 \text{ m}$ (526 receivers), time increment $\Delta t = 15.62 \text{ ms}$ (512 samples), depth increment $\Delta z = 39.1 \text{ m}$ (512 depths). The synthetic seismograms in Fig. 3(b), calculated using the method in Appendix B, show the reflection from the syncline, characterized by multiple arrivals, and the reflection from the bottom of layer 3. The other reflections of the model have been suppressed. The maximum offset, 24 km, guarantees that the syncline response is complete.

The migrations, fast and recursive, of the seismogram section are shown in Figs 3(c) and (d). In the case of fast migration nine sounding rays with the sounding angles $\phi_s = 0, 4, \ldots, 28, 32^\circ$ at the shot are used, and focusing is on layer 3, i.e. the migration velocity is $v_m = v_s = 6.0 \text{ km s}^{-1}$. The syncline image and the bottom-of-layer-3 image extend to an offset of almost 12 km, in accordance with the receiver-profile length 24 km. The agreement of fast and recursive migration is very good. The computing time for the former was 9 per cent of the time for the latter.

Occasionally we observe in the composite migrated sections of fast $\omega-k$ migration artifacts such as those in Fig. 3(c) close to the shotpoint at depths of about 1–3 km. Usually they stem from a sounding ray with large sounding angle $\phi_s$, and their location may be influenced by aliasing in the space domain. In the migration of synthetic seismograms these artifacts can normally be recognized as artifacts. In the case of field data some caution may be required. For instance, one would avoid or reduce aliasing by extending the data by zeros and zero traces, or one would avoid large sounding angles altogether.

3 CRUSTAL STRUCTURES WITH RANDOM HETEROGENEITIES

In this section, we describe four of the crustal models that we have investigated (Fig. 4) and discuss the corresponding synthetic-seismogram sections (Fig. 5) which will later be migrated.

To the left in Fig. 4 the background velocity–depth function of all models is shown. The velocity in the upper crust varies from 4.5 to 6.0 km s$^{-1}$, then a velocity inversion follows, and the lower crust which extends from 15 to 25 km has a constant background velocity of 6.3 km s$^{-1}$. There is a discontinuity at the top of the lower crust called, for brevity, Conrad in the following, with a jump from 5.5 to 6.3 km s$^{-1}$. At the Moho with depth 25 km the velocity increases from 6.3 to 8.34 km s$^{-1}$. A constant-velocity background in the lower crust is required by the migration method; a linearly increasing background velocity, for instance, is not allowed. According to many refraction studies the velocity in the lower crust increases roughly from 6 to 7 km s$^{-1}$. This variation is small enough to allow a representation, for migration purposes, by one homogeneous layer. The densities of the background model follow from the velocities by Birch’s law $\rho = 0.3788\nu + 0.252$ ($\nu$ in km s$^{-1}$, $\rho$ in g cm$^{-3}$).

In the lower crust, a 2-D velocity fluctuation $\delta v(x, z)$ is superposed on the background velocity. It is constructed from a double-exponential autocorrelation function,

$$\phi(x, z) = e^\nu \exp \left( -|x|/x_{\text{corr}}, |z|/z_{\text{corr}} \right),$$

where $x$ and $z$ are not the spatial coordinates, but the corresponding lags; $\nu$ is the background velocity, $\epsilon$ the standard deviation of the relative velocity fluctuations, and $x_{\text{corr}}$ and $z_{\text{corr}}$, the correlation distance in the horizontal and vertical direction, respectively. Roughly speaking, $x_{\text{corr}}$ and $z_{\text{corr}}$ are typical scatterer dimensions; $x_{\text{corr}} > z_{\text{corr}}$ means horizontally elongated scatterers, $x_{\text{corr}} < z_{\text{corr}}$ vertically elongated scatterers, and $x_{\text{corr}} = z_{\text{corr}}$ means scatterers without preferential orientation (isotropic scatterers). The construction of a velocity or slowness fluctuation from an autocorrelation function such as (4) is, by now, a standard technique in scattering theory; some details can be found in Müller, Roth & Korn (1992) and Roth & Korn (1993).

The velocity fluctuations that we have studied are shown in Fig. 4 for $\epsilon = 5$ per cent. The correlation distances are $x_{\text{corr}} = 1.8 \text{ km}$ and $z_{\text{corr}} = 0.3 \text{ km}$ (thin horizontal scatterers, Fig. 4 top left), $x_{\text{corr}} = 1.8 \text{ km}$ and $z_{\text{corr}} = 0.5 \text{ km}$ (thick horizontal scatterers, top right), $x_{\text{corr}} = z_{\text{corr}} = 0.21 \text{ km}$ (isotropic scatterers, bottom left), and $x_{\text{corr}} = 0.21 \text{ km}$ and $z_{\text{corr}} = 1.8 \text{ km}$ (vertical scatterers, bottom right). Note that with a standard deviation of 5 per cent extreme velocity fluctuations exceeding 10–15 per cent are connected. The relative density fluctuations in the lower crust are proportional to the relative velocity fluctuations with a proportionality factor of 0.76.

In the synthetic-seismogram sections in Fig. 5 correspond to a single shot of explosive type at the surface and at the horizontal coordinate zero. Shotpoint distances range from $-9$ to 21 km with the increment $\Delta x = 60 \text{ m}$ (512 receivers); only every eighth seismogram is shown. The seismograms were calculated by the hybrid method for a finite-difference box which extends horizontally from $-9$ to 21 km and vertically from 15 to 25 km. Note that the velocity fluctuations in Fig. 4 are shown for about half the horizontal range, $-5$ to

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**Figure 3.** Synclinal reflector in the middle crust: (a) crustal model; (b) synthetic vertical-component seismograms for a shot at offset zero; (c) fast $\omega-k$ migration; and (d) recursive $\omega-k$ migration.
Layer 1, $v_1 = 3.5 \text{ km/s}$

Layer 2, $v_2 = 4.7 \text{ km/s}$

Layer 3, $v_3 = 6.0 \text{ km/s}$

Layer 4, $v_4 = 7.2 \text{ km/s}$

Syncline
Figure 4. Velocity fluctuations in the lower crust. Top left: thin horizontal; top right: thick horizontal; bottom left: isotropic; bottom right: vertical scatterers. The horizontal and vertical correlation distances are given in the text and in Fig. 5, the standard deviation is ± 5 per cent. The velocity-depth function to the left is the background velocity.
10 km; this part of the model mainly determines the seismogram section. For details of the hybrid method the reader is referred to Emmerich (1989, 1992).

The time increment is $\Delta t = 10.16 \text{ ms}$ (1024 samples), and a simple source pulse with a dominant frequency of 7 Hz is used. This value is lower than the dominant frequencies typical for DEKORP or similar projects, 15 to 20 Hz, but it had to be taken because of storage limitations of the computer used. The dominant wavelength in the lower crust is 0.9 km. The ratio of dominant wavelength to scatterer thickness $z_{\text{comp}}$ in the case of Fig. 4 (top left) is about 3 and agrees approximately with the ratios of wavelength to scatterer or lamellae thickness, which have been proposed for the lower crust, e.g. by Sandmeier & Wenzel (1986).

The first arrival in the seismogram sections of Fig. 5 is the Conrad reflection. It is coherent but shows some amplitude variations due to laterally variable velocity contrasts across the Conrad discontinuity. The Moho reflection at about 9 s
looks much different: it has pronounced amplitude and traveltime fluctuations and often consists of more than one arrival. These complications are due to interaction of both the downgoing and the upgoing wave with the heterogeneities in the lower crust. The Moho reflection is particularly complicated in the case of vertical scatterers. Multiple scattering between the elongated structures in this case is more efficient than for horizontal scatterers, because the angle of incidence is closer to 90°, whereas for horizontal scatterers it is closer to zero; reflection coefficients for large angles of incidence are generally larger than those for low angles of incidence.

The responses of the lower crust in Fig. 5, between about 6 and 9 s, also differ markedly from model to model, both in strength and coherence. As expected, the energy level for the vertical scatterers is much lower than the energy level for the other scatterers, and arrivals coherent over several traces are found only rarely. Coherence is also rather weak for the isotropic scatterers. In both cases there appears to exist strong interference of single and multiple scattering. The coherence length in the horizontal-scatterer cases is generally a few kilometres and seldom exceeds 5 km. The response of a point diffractor would be strong over a much longer range; this follows from calculations like those for Fig. A2 in Appendix A. Hence, even the coherent lower crust arrivals in Figs 5(a) and (b) will to some extent be interference phenomena rather than responses of isolated scatterers, and their migration may not give an image close to the true structure. This is investigated in more detail in the next section.

4 MIGRATION AND TRUE STRUCTURE

Migration of the seismogram sections in Fig. 5 for the four types of random structures is performed as follows. Focusing is on the lower crust, i.e. the migration velocity is \( v_m = 6.3 \text{ km s}^{-1} \). Six sounding rays are used, with sounding angles \( \phi_s = -20, -10, 0, 10, 20 \) and \( 30^\circ \) at the shot. The spatial increments are \( \Delta x = 60 \) m (512 receivers from about \(-9 \) to \(21 \) km) and \( \Delta z = 30 \) m (2048 depths). The migrated sections are shown in Figs 6(a)–9(a); only the distance range from \(-5 \) to \(10 \) km is given, which is mainly responsible for the seismograms in Fig. 5. Every fourth trace is plotted.

The amplitude distribution in the migrated sections
Figure 7. The same as Fig. 6 for the thick horizontal scatterers (seismogram section in Fig. 5b). Same scales as in Fig. 6.

repeats, by and large, the pattern in the seismogram sections. A conspicuous feature are the bowl-shaped (elliptic) signatures in the lower crust and at the Moho, in particular in Fig. 8(a) for the isotropic and in Fig. 9(a) for the vertical scatterers. This shape is due to the small width of the coherent reflection/diffraction responses in the seismogram sections (Figs 5c and d) and resembles the migration impulse response. Therefore, it is rather unlikely that the corresponding migration signatures represent true structure. Another feature in Fig. 9(a), is pronounced apparent structure below the Moho, where in reality there is no velocity variation. This apparent structure would also be present and extend to somewhat greater depths, if migration would focus on the layer below the Moho ($v_m = 8.34\text{ km s}^{-1}$). This structure is no surprise, in view of the seismogram section in Fig. 5(d) and the single-scattering assumption inherent in all migration techniques, but it is not real and only a consequence of pronounced multiple scattering, as explained earlier.

The remaining discussion in this section concerns the migration signatures inside the lower crust. Here it is necessary to have an objective version of the reflective parts of the velocity fluctuations. The velocity fluctuations themselves, which are shown in Figs 6(b)–9(b) in the form of depth functions at fixed values of horizontal distance, do not serve this purpose. There is no correlation of the positive velocity fluctuations (which are emphasized in the plots by black colour) with the positive (black) parts of the migrated sections; there would also be no correlation for the negative velocity fluctuations. This lack of correlation is expected, because the reflective parts of the structure are the places where the velocity gradient is high and not the places where the velocity fluctuations agree in sign.

As a consequence we need an appropriate representation of the velocity gradient. We have chosen to work with the vertical derivative of velocity alone and to disregard the horizontal derivative. The reason is that steep-angle reflections are mainly produced at locations with strong vertical gradient; locations with strong horizontal gradient which occur particularly frequently in the model with vertical scatterers contribute less to the seismograms in the time interval from 6 to 9 s and should, therefore, not be represented. The true vertical-derivative cross-sections (Figs 6c–9c) contain very high vertical wavenumbers and differ
strongly in wavenumber content from the migrated sections. In Figs 6(d) - 9(d) filtered vertical derivatives are displayed; each true vertical-derivative trace has been convolved with a signal which follows from the source-time function by changing its duration $T$ into wavelength $\lambda$ according to $\lambda = v_s T/2$ (Müller & Temme 1987, eq. 19). The wavenumber content of the filtered cross-sections agrees closely with the wavenumber content of the migrated sections, such that a comparison is now meaningful.

For the horizontal scatterers (Figs 6 and 7) the agreement of both sections is partly good, but there are also differences. Coherent signatures are generally longer in the migrated sections than in the filtered-derivative sections. In particular, there are migrated signatures without counterpart in the filtered-derivative sections. For instance, an apparent Moho high in Fig. 7(a) at a horizontal distance of 5 km is connected with an 'oblique reflector' to the left at distances of 0-5 km and depths of 23-24 km which does not exist in Fig. 7(d). The cause of this feature is not clear. It could be due to multiple scattering, but it could also be another example of bowl-shaped signatures or migration artifacts related to the Moho.

For the isotropic and the vertical scatterers (Figs 8 and 9) there is practically no similarity between the migrated and the filtered-derivative section. According to the latter, the lower crust is full of small-scale scatterers without preferential orientation, which is correct for the isotropic scatterers, but not for the vertical scatterers. In both cases, the migrated section gives the impression of quite a number of subhorizontal elements with a length of 1-2 km, which is at variance with the true structure. A similar disagreement was found by Gibson & Levander (1988a) in the case of CMP migration for isotropic scatterers. Hence, the success of migration depends strongly on the orientation of scatterers or lamellae in the crust, and vertical or isotropic scatterers are much more difficult to detect than horizontal scatterers.

5 DISCUSSION AND CONCLUSIONS

The results of this paper have some bearing on the interpretation of seismic sections in crustal reflection seismology. The usual view is that coherent arrivals in CMP sections are seismic images of structural elements. A line
Figure 9. The same as Fig. 6 for the vertical scatterers (seismogram section in Fig. 5d). Same scales as in Fig. 6.

drawing is formed from these arrivals either manually or automatically, and each line element is migrated into a structural element with a ray-theoretical migration scheme or by simple time-to-depth conversion. An alternative procedure is to migrate the CMP section with a wave-theoretical method and to construct a line drawing from the coherent signatures in the migrated section. As in the first method, coherent signatures are considered as structural elements. Our study shows that there are problems with both procedures if crustal structure contains many small-scale heterogeneities.

First, the complexities of seismogram sections (here Fig. 5) and of the corresponding migrated sections (here Figs 6a–9a) are partly different. This is no surprise, but it implies that line drawings constructed from both section types would also differ. From a conceptual point of view, line drawings produced from migrated sections should be preferred.

Second, only in the case of horizontal scatterers or structures is there a certain agreement of the migrated section and the structures, but not in the case of isotropic or vertical heterogeneities. In the latter cases, the migrated sections would be interpreted in terms of short horizontal structures, in disagreement with reality.

Third, the seismic sections (here single-shot sections) show partly pronounced multiple-scattering effects which are transferred to the migrated sections and form coherent signatures without relation to the true structure. This is most prominent for isotropic or vertical scatterers, but it also occurs for horizontal structures.

On the background of these results a few recommendations for interpreting complex crustal reflection data can be formulated.

(i) Line drawings are of limited value, if the corresponding crustal zone is strongly heterogeneous. It is not guaranteed that each line element represents a reflector, because multiple scattering can produce coherent arrivals in the seismogram section which have no counterparts in the subsurface. Also, the spacing of line elements in the vertical and horizontal direction is more a consequence of the frequencies used rather than a geometrical measure of the heterogeneities. Therefore, the line elements just show that there exists a zone of heterogeneities. The boundaries of this zone are more reliable than any internal structure which is suggested by a detailed line drawing.

(ii) Scatterer orientations can be determined from migrated sections or line drawings with less safety than is usually
assumed, because isotropic and vertical scatterers produce horizontal signatures like horizontal scatterers, although less pronounced. To come up with the popular statement that the lower crust contains primarily horizontal lamellae requires more than just a look at line drawings. It requires seismogram modelling and a search in the data for the very different effects on amplitudes and coherence lengths that different scatterer sizes and orientations can have. Possibly, horizontal or subhorizontal structures in the lower crust are not as frequent as currently assumed. This suspicion is supported by Gibson & Levander’s (1988b) investigation of wave coherence in the wide-angle range. They found high coherence, as in observations, not only for horizontal lamination in the lower crust, but also for isotropic scatterers.

(iii) Migration interprets multiple scattering close to one reflecting interface in terms of single scattering in a zone with several closely spaced interfaces. The circumstances under which this happens may be quite special, but one should be aware that such misinterpretations are possible and not exotic.

These comments are based on theoretical migration results for one single-shot seismogram section per model. We have performed neither poststack migration of a CMP section nor postmigration stack of many single-shot sections. Both stacking processes may possibly reduce multiple-scattering effects, and the final migration result may be closer to the true structure than the result of one single-shot migration. However, whether stacking really improves the situation, in particular when isotropic or vertical scatterers are present, remains to be seen.

In conclusion, the relation between complex structure in the lower crust and its seismic image is also complex. The popular line drawings give no true account of this complexity. A more adequate way of presenting the results of a crustal reflection experiment would be by a migrated section, supplemented by a simple, manually constructed line drawing which contains only the broad features of interpreted reflectors and scattering zones. With such line drawings the danger of data overinterpretation would also be reduced.

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**Figure A1.** Comparison of the true wavefront of the downgoing wave and a locally plane wave in the vicinity of the sounding ray. s and n are ray-centred coordinates of point P. t(s) is the traveltme of the locally plane wave, and t(s) + 0.5m(s)n² the traveltme of the true wavefront in the paraxial approximation (see text).
APPENDIX A: SOUNDING BEAMWIDTH

The sounding beamwidth is derived by a traveltime argument similar to the one for Fresnel zones or volumes. The linearization of \( F \) in eq. (2) implies linearization of the imaging time \( t_i(x, z) \) and the phase integral \( \Psi \). In a layered background medium \( \Psi \) is piecewise linear in \( z \), such that it is the linearization of \( t_i \) which determines the accuracy of the Fourier integral (3).

Linearization of \( t_i \) means replacement of the curved wavefront of the downgoing wave locally, i.e. close to the chosen sounding ray, by a plane wave (Fig. A1). The time difference between the two wavefronts at point \( P \) with the ray-centred coordinates \( s \) and \( n \), \( \delta t = 0.5m(s)n^2 \), can be determined by dynamic ray tracing; \( m(s) \) times the velocity is the curvature of the wavefront. In the case of a piecewise linear sounding ray with \( i \) segments \( s_i \), . . . , \( s_i \), whose sum is \( s \), one obtains

\[
m(s) = \left( \sum_{k=1}^{i} s_k \nu_k \frac{\cos \phi_k}{\cos \phi} \right)^{-1}.
\] (A1)

Figure A2. Line diffraactor in a homogeneous medium (diffraactor depth 5.5 km, \( P \)-velocity 5.0 km s\(^{-1} \)): (a) vertical-component seismogram section for a single shot vertically above the diffraactor; (b) recursive \( \omega-k \) migration; (c) to (f) fast \( \omega-k \) migrations for the sounding angles \( \phi_s = \phi_m = 0, 10, 20 \) and 30°. The true diffraactor position is marked by a dot.
where $v_k$ is the velocity along the $k$th segment and $\phi_k$ the angle between this segment and the $z$-axis.

For an accurate representation of $M(x, z)$ by the Fourier integral (3) the time difference $\delta t$ should not exceed a fraction $\kappa T$ of the dominant wave period $T$; $\kappa$ is discussed below. Hence,

$$n_{\text{max}}(s) = \left( \frac{2\kappa T}{m(s)} \right)^{1/2}$$

(A2)

is half the local sounding beamwidth. For a homogeneous medium eq. (A1) yields $m(s) = (v_1)^{-1}$, and (A2) simplifies to

$$n_{\text{max}}(s) = (2\kappa \lambda s)^{1/2},$$

(A3)

with the dominant wavelength $\lambda$. Eqs (A2) and (A3) imply a decrease of the sounding beamwidth with increasing frequency and an increase with the length of the sounding ray.

The numerical value of $\kappa$ cannot be taken from the considerations that are used for Fresnel zones ($\kappa = 0.25$ or 0.5), but has to be derived from migration results. One example is presented here, the case of a line diffractor in a homogeneous medium. In Fig. A2 the seismogram section, calculated by the simple method described in Appendix B, and various migration results are shown. Recursive $\omega-k$ migration serves as a reference. The parameters used in the calculation are as follows: offset increment $\Delta x = 94.5$ m (128 receivers), time increment $\Delta t = 7.81$ ms (512 samples), migration velocity $v_m = 5.0$ km s$^{-1}$, sounding direction $\phi_s = \phi_m = 0$, 10, 20 and 30°, depth increment $\Delta z = v_m \Delta t/2 = 19.5$ m (512 depths), dominant frequency of the incident wave 10 Hz.

The fast-migration result in Fig. A2 for $\phi_s = 0^\circ$, the correct sounding direction, agrees almost exactly with the recursive-migration result, and the migrated section for $\phi_s = 10^\circ$ is also very close. Significant smearing of the diffractor image and noticeable shifts to greater depths begin at about $\phi_s = 20^\circ$. Hence, in the case $\phi_s = 10^\circ$ the diffractor distance $n \approx 0.5$ km from the sounding ray is clearly less than $n_{\text{max}}$, whereas $n \approx 1$ km in the case $\phi_s = 20^\circ$ should be considered larger than $n_{\text{max}}$. Taking $n_{\text{max}} \approx 0.7$ km, eq. (A3) yields $\kappa = 0.1$. This value has also been found in other cases.

**APPENDIX B: A SIMPLE 2-D METHOD FOR SYNTHETIC SEISMOGRAMS**

The model consists of homogeneous, horizontal layers, in which laterally heterogeneous structures such as synclines, domes, horsts, faults etc. are embedded. These structures are approximated by closely spaced line diffractors. Synthetic seismograms are calculated for the primary reflections from the interfaces between the layers and for single scattering from the diffractors. The primary reflections are calculated by generalized ray theory, using a wavefront approximation; this requires two-point ray tracing in layered media which is without problems. The same method is used for wave propagation from the shot to a diffractor and from the diffractor to the receiver. The *diffraction strength* $G$ is angle-dependent and follows from the theory of Rayleigh scattering of $P$-waves by small inclusions in solid media (Wu & Aki 1985, eq. 9):

$$G(\theta) \sim \frac{1}{8} \cos \theta - \frac{1}{8}(1 + 2 \cos^2 \theta).$$

(B1)

$\theta$ is the diffraction angle with respect to the propagation direction of the incident wave. The numerical factors in eq. (B1) correspond to $v_p = v_s \sqrt{3}$ and $\delta \rho/\rho = 0.5(\delta v_p/v_p)$, where $v_p$ and $v_s$ are the velocities of $P$- and $S$-waves, $\rho$ is density, and $\delta v_p$ and $\delta \rho$ are perturbations characterizing the inclusion. From scattering theory only the radiation characteristic (B1) is taken, together with the fact that the diffracted pulse form is the second derivative of the incident pulse form. Absolute diffracted amplitudes are fixed at one value, e.g. at $\theta = \pi$, by prescribing an arbitrary fraction of the incident amplitude. Hence, the amplitude level of the diffractions relative to the level of the primary reflections can be chosen arbitrarily.

The main approximations of this method for synthetic seismograms, (1) representation of structure by line diffractors, (2) restriction to single scattering, and (3) arbitrarily fixed diffraction amplitudes, make it much less accurate than the hybrid method which is used in the main part of the paper. It is, however, much faster and has no limitations to the model size. Its main purpose is to provide seismograms for the test of migration techniques.