An inverse technique for retrieving higher mode phase velocity and mantle structure

Eléonore Stutzmann and Jean-Paul Montagner
Laboratoire de Sismologie, Institut de Physique du Globe de Paris, 4 Place Jussieu Tour 2A, F-75252 Paris Cedex 05, France

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SUMMARY
We present a new inverse method for retrieving the mantle structure, using the fundamental and higher modes surface waves. The fundamental mode is only sensitive to structures down to 400–500 km but higher modes have a depth resolution down to 1000–1500 km. So, they are well suited to the study of the transition zone between 400 and 1000 km depth.

A data set of seismograms corresponding to events all located in a small area and recorded at a single station is selected. The inversion of the seismograms is divided into two steps. First, the difference between the real spectra and the sum of the higher modes synthetic spectra is inverted for retrieving the phase velocities of the first higher modes. In a second step, these phase velocities are inverted at depth in order to retrieve the average mantle structure between the receiver and the epicentral area. This structure is modelled as a transversely isotropic medium with a vertical symmetry axis. Because of the non-linearity of the problem, it is necessary to iterate the procedure and to recalculate the Earth eigenfunctions and the new synthetic spectra at each iteration.

Numerical tests on synthetic data have been performed and they demonstrate that we have a good resolution at depth down to 1500 km. Some results obtained on real data recorded on GEOSCOPE network are also presented. This method can now be applied systematically to real data.

Key words: earth mantle, higher modes, inversion, surface waves, synthetic seismograms, tomography.

1 INTRODUCTION
The large number of available data have allowed the improvement of the precision of the Earth models in the upper mantle. These models have been generally determined using body waves or fundamental mode of surface waves. But the use of these kinds of data does not enable us to have a good resolution in the depth range between 400 and 1000 km. However, this transition zone is of primary importance in global seismology. For instance, a good knowledge of the earth tomography in this zone would allow us to discriminate between a convection in the whole mantle and a convection in two superimposed cells (Elsasser, Olson & Marsh 1979).

The tomographic models derived from body waves are either truncated at low degree (degree 6 for Dziewonski 1984) or are represented by small boxes (Inoue et al. 1990). But their lateral resolution is rather poor in the upper 1000 km. The use of long period $S$ waves and/or multiple $S$ waves (Tanimoto 1990; Su & Dziewonski 1991; Masters & Bolton 1991), which are modelled by normal mode summation, have improved the depth resolution. However, large areas remain poorly sampled by body waves, mainly beneath the oceans.

Surface waves provide a better coverage of the earth (Woodhouse & Dziewonski 1984; Nataf, Nakanishi & Anderson 1984, 1986; Tanimoto 1989; Roul, Romanowicz & Montagner 1990; Montagner & Tanimoto 1991; Zhang & Tanimoto 1991). The fundamental mode is not appropriate for the study of the transition zone because it is only sensitive to the shallowest 400 km in depth. But this transition zone can be studied using the higher modes whose maximum of sensitivity corresponds to deeper depths (Takeuchi & Saito 1972). For instance, at a period of 50 s the second higher mode is sensitive to mantle structure down to 500 km and the third one down to almost 1000 km. Time variable filtering (Cara 1973) allows us to extract the fundamental mode and in some cases a particular higher
mode when its observed group velocity curve is well isolated on the energy diagram (Roult & Romanowicz 1984). But, the problem with higher modes is that they have very close group velocities which make them travel as a whole wave packet. Consequently, the separation of overtones is difficult in some frequency ranges and it is necessary to deal with the complete waveform in order to use the amplitude information.

Different approaches have been proposed to retrieve the mantle structure from higher modes. About 15 years ago, Nolet (1975), Nolet & Panza (1976) and Cara (1976, 1978) proposed an elegant method which consists of using several seismograms recorded in an array of stations and originating from a given earthquake. By stacking the different seismograms, they obtain an U-C diagram which makes it possible to separate the different higher modes. This method was later improved by Mitchel (1980) and Okal & Jo (1987) to allow the use of a sparser network, by taking in account informations from the source mechanism. In a second step, they invert at depth the higher mode phase velocity dispersion curves for retrieving the depth distribution of the $S$-wave velocity. However, the method is limited because of the scarcity of available arrays of stations at the surface of the earth (only in north America and Europe).

A second approach consists of inverting the complete waveform. Woodhouse & Dziewonski (1984) and Takimoto (1989) employ time-domain waveform fitting using a normal mode formalism and invert simultaneously a large number of seismograms to provide a global tomographic model of the upper mantle. But they only make use of the phase information, not of the amplitude information. Other studies utilize the complete waveform to obtain gross average of regional earth structure. Though these methods can be applied to one single seismogram, they have generally relied on stacking over an array of stations and/or earthquakes in order to achieve adequate sampling of the different higher modes branches. Nolet, Van Trier & Huisman (1986) and Nolet (1990) use an iterative inverse algorithm to fit the waveform in the time domain, and the complexity of the model increases within the iteration. Lerner-Lam & Jordan (1983) compute cross-correlograms between data and monomode synthetic seismograms and use a non-linear inverse method to fit real and synthetic cross-correlograms.

In general, the direct waveform inversion remains rather complex due to the non-linearity of the problem and because frequency-dependent data errors cannot be incorporated into the inversion. Cara & Lévêque (1987) and Lévêque, Cara & Rouland (1991) have improved Lerner-Lam & Jordan’s method by filtering the cross-correlogram and inverting the envelope and the phase of the filtered cross-correlogram. Their method employs an inversion technique which is better able to incorporate external constrains on the data space and the model space, but it introduces additional parameters (attenuation and seismic moment) which may be poorly resolved.

In this paper, we propose a new method which takes advantage of the stacking techniques on one hand and of modern inversion techniques on the other hand. By dividing the inversion of surface-wave data into essentially two steps—phase velocity perturbations of higher modes branches, followed by inversion for the model—we gain insight into which factors control the depth resolution, and we are able to introduce more plausible constraints into the inversion. This new method can be applied to a set of seismograms recorded at one single seismic station and originating from earthquakes located in a small area, but corresponding to various depths, which improves the resolution. This enables us to extend the retrieval of higher modes to a relatively greater number of paths at the surface of the earth, and to provide a good spatial coverage.

2 THE METHOD

The aim of this method is to match long period seismograms for periods ranging between 70 and 250 s. This method is general and can be applied to any component of the seismogram, that means the vertical, radial and tangential component. Generally, an observed long-period seismogram is described as the sum of the fundamental mode ($n=0$) and the higher modes ($n \geq 1$). But, for spheroidal modes, the classical nomenclature of higher modes [Gilbert & Dziewonski 1975 (model 1066 A)] is not appropriate in the period range of interest. So, for the vertical and radial components, we have reorganized the higher modes according to Okal’s classification (Okal 1978). The Stoneley modes were removed and a new radial order $n$ has been defined: for a given new radial order $n$, phase and group velocities are made continuous. The higher modes thus defined correspond to the “R modes” proposed by Okal (1978). This kind of problem does not arise for toroidal modes.

For a given recorded seismogram, we can calculate the corresponding synthetic seismogram by normal mode summation. We use the formalism defined by Woodhouse & Girnius (1982). In spherical coordinates $(r, \theta, \phi)$, the seismic displacement at a receiver $(r_s, \theta_s, \phi_s)$ for a source located at $(r_o, \theta_o, \phi_o)$ is given by:

$$u_{po}(r_s, \theta_s, \phi_s, t) = \sum_{n=0}^{\infty} \sum_{l=0}^{n} \sum_{m=-n}^{n} \mathbf{S}_{nlm}^{n}(r_o) \mathbf{R}_{nlm}^{n}(r_s) \exp \left(i \omega t \right)$$

where

- $\mathbf{S}_{nlm}^{n}$ is the source function.
- $\mathbf{R}_{nlm}^{n}$ is the receiver function.
- $i \omega$ is the eigenfrequency.
- $r_s$ is the vector corresponding to the receiver located at the point $(r_s, \theta_s, \phi_s)$.
- $r_o$ is the vector corresponding to the source located at the point $(r_o, \theta_o, \phi_o)$.
- $n, l, m$ are respectively the radial, angular and azimuthal orders.

To perform this computation, we assume that the source parameters and the instrumental response are well defined. The eigenfunctions and eigenfrequencies are calculated for a reference Earth model $p_0$. The chosen reference Earth model corresponds to a spherical transversely isotropic medium with a vertical symmetry axis. It is characterized by five independent parameters $(A, C, F, L, N)$ plus density. That means that we neglect the coupling between Rayleigh and Love waves, and also, among Rayleigh or Love waves, the coupling between two different dispersion branches that correspond to two different modes $n$ and $n'$ (Lognonné & Romanowicz 1990).

Then, when calculating separately each synthetic seismogram which corresponds to a fixed mode (radial order $n$
fixed), the total synthetic seismogram is the sum of all these monomode seismograms and its Fourier transform can be expressed as follows:

$$u'_n(t_s, t_r, \omega) = \sum_{n=0}^{\infty} B_{jn}(\omega) \cdot \exp \{ i \beta_{jn}(\omega) \}$$

where the index $j$ denotes the number of the earthquake, $B_{jn}$ and $\beta_{jn}$ are respectively the amplitude and the phase of the $n$th monomode synthetic seismograms for the model $p_n$. The Fourier transform of the corresponding recorded data is:

$$u'_n = A_j(\omega) \cdot \exp \{ i \phi_j(\omega) \}$$

where $A_j$ and $\phi_j$ are respectively the amplitude and the phase of the phase of the recorded seismogram.

Let us now consider the real earth model $p$. This model deviates from the reference model $p_0$ by a perturbation $\delta p = p - p_0$. This perturbation $\delta p$ modifies the phase $\beta_{jn}$ but also the amplitude $B_{jn}$ of each monomode synthetic seismogram calculated for the model $p_0$. Phase perturbations can be related to phase velocity perturbations whereas amplitude perturbations are connected to eigenfunctions variations. These two effects correspond to first-order dependence relations. So, both of them are important but they are very difficult to take into account simultaneously. Providing this, we separate the inversion into two steps, and an iterative procedure is designed. It will make it possible to retrieve the same time the perturbed model $\delta p$ and the higher mode dispersion curves $\delta c_n$.

The first step consists in retrieving the phase velocity delay, $\delta c_n$, between the real earth model, $p$, and the reference model, $p_0$. We use the asymptotic expansion of eq. (1) for large angular order $l$. This development has been performed by Kanamori & Given (1972). In a first approximation, we consider that, for a fixed radial order $n$, there is only a phase velocity difference between a real monomode seismogram and the corresponding synthetic monomode seismogram. A first-order Taylor development of the phase velocity is performed. So, the Fourier transform of the total synthetic seismogram, $u'_n$, corresponding to an earth model $p$, is, for a frequency $\omega$:

$$u'_n(t_s, t_r, \omega) = \sum_{n} B'_{jn} \cdot \exp \left( \beta_{jn} + \frac{i \omega \cdot \Delta_j \cdot \delta c_n}{(c_{on})^2} \right)$$

where:

- $\Delta_j$ is the epicentral distance for the earthquake $j$.
- $c_{on}$ is the phase velocity for the mode $n$ and the reference model $p_0$.
- $\delta c_n$ is the phase velocity difference that we want to retrieve: $\delta c_n = c_n - c_{on}$.
- $B'_{jn}$ is the amplitude of the monomode synthetic seismogram $n$, corresponding to the real model $p$.

The phase term $\frac{i \omega \Delta_j c_n}{c_{on}^2}$ is not small, so it is impossible to expand the exponential term at first order. In this first step, the amplitude $B'_{jn}$ is assumed to be equal to the amplitude $B_{jn}$ which corresponds to the reference model $p_0$. The computation of the higher modes synthetic seismograms up to the radial order $n = 10$ enables us to know the relative excitation of each mode. Consequently, we can assess for which modes the phase velocity determination will be reliable. In general, we only invert the phase velocities up to the mode $n = 3$ or $n = 4$.

In order to obtain reliable parameters $\delta c_n$, it is necessary to use several seismograms. We choose a set of seismograms, recorded at a single station and whose epicentres are located in a small area. We make the assumption that the paths corresponding to the different seismograms are close enough to neglect lateral variations of the model. Therefore, the size of the area must be smaller than the wavelength. For each earthquake, we calculate the corresponding synthetic seismogram. Then, we compute a first estimation of the phase velocity delays for different modes.

The computation of the phase velocity deviations $\delta c_n$ is performed by minimizing the difference between eqs (3) and (4). The phase velocity deviations are computed simultaneously in the whole period range for the different modes $n$ that are present in the recorded seismogram. Because of the non-linearity of the complex exponential function in eq. (4), we use the iterative generalized inverse method proposed by Tarantola & Valette (1982). The non-linear inverse problem presented here can be written as $d = g(p)$, where the data $d$ denote the seismogram spectra for the different earthquakes and the parameters $p$ correspond to the phase velocity differences $\delta c_n$ for the different modes $n$. The data $d$ and the parameters $p$ are defined in the whole frequency range of interest. The non-linear inverse algorithm defined by Tarantola & Valette (1982) can be expressed as follows in the parameters space:

$$p_{k+1} = p_k + (G^T C_{dk}^{-1} G_k + C_{pp})^{-1} (G^T C_{dc}^{-1} [d - g(p_k)] - C_{pp} (p_k - p_0))$$

where:

- $p_k$ corresponds to parameters at iteration $k$.
- $p_0$ corresponds to initial parameters.
- $C_{pp}$ is the $a priori$ parameters covariance matrix.
- $C_{dc}$ is the data covariance matrix.
- $G_k$ is the matrix of Frechet derivative of equation $d = g(p)$. It can be easily derived from eq. (4).

As the equation $d = g(p)$ is non-linear, it is necessary to iterate the algorithm. At each iteration, we have to check the convergence of the algorithm. The cost function $S_k$ is defined as follows:

$$S_k = \frac{1}{2} \| (g(p_k) - d) \|^2 C_{dc}^{-1} (g(p_k) - d) + (p_k - p_0)^T C_{pp}^{-1} (p_k - p_0)$$

This function has to satisfy the relation $S_k \leq S_{k-1}$ where $S_{k-1}$ is the cost function at the iteration $k - 1$. The iterative algorithm is stopped when the following relation is verified:

$$\| p_k - p_{k-1} \| < \varepsilon$$

The problem is non-linear but the reduction of the variance in the $a priori$ covariance matrix $C_{pp}$ and eventually the increase of data errors can constrain the convergence of the inversion algorithm. Another way of stabilizing the solution consists in introducing, for a given mode $n$, a Gaussian covariance function for parameters corresponding to neighbouring frequencies. This covariance function corresponds to the fact that the phase velocity differences are not independent from one frequency to another. Moreover, because $u'_n$ is the result of constructive and destructive
interference of several modes, it is necessary to introduce a smoothing effect at different periods especially when the interference between modes is destructive and the inversion poorly constrained.

Because the problem is non-linear, it is not possible to determine the a posteriori error on the parameters $\delta c_n$. But, the computation of the a posteriori covariance matrix after the first iteration can give an idea of the error on the parameters, though its expression is exact only in the linear case. This matrix has the following expression at the first iteration:

$$C_p = (G_lCG_l^T + C_c)^{-1}.$$  \hspace{1cm} (8)

The use of several earthquakes makes our technique equivalent to a stacking method. Indeed, the earthquakes have different source parameters, in particular different depths, and they can be sensitive to different modes at different frequencies. By using the information contained in each earthquake, we use the constructive interference of the earthquakes, which is the principal of the stacking methods.

In a second step, we invert phase velocities versus depth to find an approximation of the average earth model along the path between the earthquakes area and the receiver.

The use of several seismograms, without any restriction on the depth of the earthquakes, provides a good depth resolution. The relations between phase velocities and model parameters can be linearized, by using partial derivative relations calculated by Dziewonski & Anderson (1981). We calculate a new earth model $p_1$ with the generalized inverse method proposed by Tarantola & Valette (1982). Then, we can calculate the eigenfunctions that correspond to the new model $p_1$ and the new synthetic seismograms.

As the two steps of the calculation—the phase velocity and the depth inversion—are non-linear, it is necessary to iterate in order to retrieve a good fit between recorded data and synthetic seismograms. We compute the total $\chi^2$, corresponding to the $N$ data used in the inversion, as the sum of the discrete $\chi^2$ between a real data $u_j$ and the corresponding synthetic data $s_j$, over the discretized time interval $(t_0; t_1)$ (sum over $k$):

$$\chi^2 = \sum_{j=1}^{N} \chi_j^2 = \sum_{j=1}^{N} \sum_{k=1}^{M} \frac{(u_j^k - s_j^k)^2}{\sigma_j^2}.$$ \hspace{1cm} (9)

The iterative algorithm is stopped when the $\chi^2$ variation from one iteration to another becomes negligible. Fig. 1 summarizes the different steps of our method.

3 EXPERIMENT WITH SYNTHETIC DATA

The algorithm described in the previous section has been tested on synthetic data. We present here only two experiments. In the first one, the $S$-velocity anomaly is located in the uppermost mantle whereas in the second one, the anomaly is situated in the lower mantle. We will see how these velocity anomalies are retrieved with our inversion technique.

3.1 Upper mantle anomaly

In order to simulate a set of real data, we have considered a swarm of earthquakes located in the Molucca Islands and recorded at the GEOSCOPE station PPT (latitude: 17.57'S, longitude: 149.58'W). Five earthquakes are used in the inversion. Their coordinates are included in a box defined as follows: latitudes between $0^\circ$ and $2.5^\circ$N, longitudes between $126^\circ$ and $127^\circ$W and depths between 30 and 60 km. The epicentral distance is about 9400 km. “Real” seismograms have been synthetized by normal mode summation for an earth model which corresponds to the PREM model (Dziewonski & Anderson 1981) except between 20 and 220 km depth where there is an $S$-velocity difference of $-5$ per cent. We use the very long-period channel response and only the vertical component. We only consider the signal in the period range 70–250 s. Each seismogram is composed of three modes, the fundamental, the first and second higher modes ($n = 0, 1, 2$). The starting model is the PREM model. For each “real” seismogram, we calculate the corresponding monomode synthetic seismograms for $n = 0, 1, 2$ (Fig. 2). In the period range of interest, as the fundamental mode is well separated from the higher modes because of its slow group velocities, it can be treated separately with the same algorithm. That reduces the computation time because the matrices to invert become smaller. But above all, this separation prevents the higher modes from being contaminated by the highly energetic fundamental mode. In
practice, the fundamental mode amplitude can be twice as high as the amplitude of the sum of the higher modes. This is illustrated on Fig. 3, where the solid lines correspond to the spectra of the "real" data presented on Fig. 2, for group velocity windows corresponding to the fundamental mode (Fig. 3a) and to the higher modes (Fig. 3b). The shape of the spectra amplitude with its bumps and holes (Fig. 3b) is due to higher modes interferences. Each monomode synthetic seismogram is plotted with the symbol 0, 1, 2 for, respectively, the fundamental mode, the first and second higher modes. The sum of these spectra is plotted with a dashed line and we can see that there is a phase and also an amplitude difference with the corresponding "real" spectra.

As described in the previous section, at each iteration, the inversion is divided into two steps. The first step consists of retrieving an estimation of the different phase velocities of the modes. In the second step, these phase velocities are inverted versus depth. Before the second iteration, for each earthquake, the synthetic monomode seismograms corresponding to the new earth model are computed by taking the eigenfunctions variations into account. Two iterations of the algorithm are sufficient to retrieve the phase velocities of the different modes (Fig. 4) and the corresponding earth model (Fig. 5) with excellent accuracy. On Fig. 4, the solid lines
correspond to the inverted phase velocity differences after two iterations and the symbols 0, 1, 2 correspond to the exact phase velocity deviations respectively for the modes \( n = 0, 1, 2 \). The phase velocity deviations are retrieved with their exact values for the fundamental mode, and with a residual smaller than 0.002 km s\(^{-1}\) for the first and second higher mode, in the whole period range. The maximum residual (0.002 km s\(^{-1}\)) is observed at the two edges of the period range, where the spectra amplitude of the higher modes becomes close to zero as can be seen on Fig. 3(b).

On Fig. 5, the S-velocity anomaly after inversion is well located at depth, but its amplitude slightly extends at the two edges of the layer, because we use in the depth inversion a Gaussian covariance function, to correlate the S velocities for neighbouring depths. The synthetic seismogram corresponding to the model after inversion compared to the "recorded" data shows a negligible residue (Fig. 6).

To test the importance of the eigenfunctions recomputation within the iterative algorithm, we have compared a synthetic seismogram resulting from our inversion and a synthetic seismogram corresponding to the same final earth model but for which we consider only the phase velocity difference with the PREM model. That means that this synthetic seismogram is calculated with relation (4) by using the dispersion curves presented on Fig. 4. Fig. 7 presents the amplitude of the spectra in both cases. It shows that a good agreement in amplitude between the "recorded" and synthetic seismograms can only be obtained when the eigenfunctions are recomputed. This effect is especially visible on the higher modes. In the algorithm, the amplitude

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Figure 3. Spectra of the seismograms presented on Fig. 2 for group velocity windows corresponding to the fundamental mode (3a) and to the higher modes (3b).
3.2 Lowermost mantle anomaly

In the previous section, the method has provided good results with seismograms corresponding to the superposition of three modes. In this new experiment, we use an additional mode, the third one and so, the seismograms correspond to four modes, \( n = 0 \) to \( n = 3 \). The source parameters, the location of the earthquakes and the recording station are the same as previously. The five "recorded" seismograms are calculated for a model whose difference with the PREM model is located in the transition zone, with an \( S \)-velocity anomaly of +3 per cent in the depth range 670–1318 km. The starting model is again the PREM model. One of the five earthquakes and the corresponding monomode synthetic seismograms calculated for the PREM model are presented on Fig. 8. For the fundamental mode, there is practically no difference between the "real" and synthetic seismogram as can be seen on Fig. 8 and also on the spectra Fig. 9. This is because the \( S \)-velocity anomaly is too deep to affect the fundamental mode.
As previously, after two iterations, the $S$-velocity anomaly and the phase velocity anomalies for each mode branch are retrieved with accuracy. The phase velocity derivations are presented on Fig. 10. The fundamental mode is retrieved almost exactly, with a maximum residual of 0.003 km s$^{-1}$. The first and the second higher are well retrieved with a residual smaller than 0.009 km s$^{-1}$. On the other hand, the phase velocity deviations obtained for the third higher mode are not as good. This is due to the fact that this third higher mode is only excited in the frequency range 0.008–0.013 Hz. The residual on this mode is the smallest near the maximum of its spectra amplitude and reaches 0.045 km s$^{-1}$ at low frequency, where the spectra amplitude is close to zero. But, even when the phase velocity differences are not retrieved with accuracy, we believe that they should be taken into account in the depth inversion because the more modes we have, the better the anomaly is located at depth. Observations of modes with small excitation are implicitly given less weight in the inversion, since the excitation of the different modes is introduced in the algorithm by calculating an a priori covariance matrix on the phase velocities which depends on the relative amplitude of each mode for the different frequencies.

The $S$-velocity anomaly after inversion, presented on Fig.

Seismograms corresponding to the sum of the modes n=0, 1, 2, 3

Figure 8. One of the five "data" and the corresponding synthetic seismograms used in the inversion. Case of the S-velocity anomaly located in the lower mantle (model B).

1/2 hour

3.3 Study of bad source parameters influence

For the two numerical tests presented above, we have tested the influence of bad source parameters determination. We have used the same "real" data as previously but the corresponding synthetic seismograms have been modified. First, the synthetic seismograms are computed with an error of ±5° on the strike, dip and slip axis, for each of the five earthquakes used in the inversion. In the first experiment, for the case of an S-velocity anomaly located in the uppermost mantle, the $\chi^2$ reduction, that is $\frac{\chi^2_{\text{initial}} - \chi^2_{\text{final}}}{\chi^2_{\text{initial}}}$, is 80 per cent. In the second experiment, for the case of an S-velocity anomaly located in the lowermost mantle, the $\chi^2$ reduction is 80 per cent. The effect of a change in source geometry is actually about the same in both cases, but the initial variance is much smaller in the second experiment. Since the real and total synthetic seismograms are renormalized in our procedure, these experiments indicate...
that the different mode branches undergo different amplitude changes in response to a change in source geometry, but overall the effect is small compared to the initial phase differences due to the velocities anomalies.

We have then tested the influence of a depth mislocation. The synthetic seismograms are computed with a depth mislocation of ±10 km for each of the earthquakes used in the inversion. Here again, the $\chi^2$ reduction is greater than 80 per cent for the two experiments. The conclusion of these tests is that these two kinds of errors provide variations on the seismograms amplitudes but their influence on the phase of the seismograms is limited. So, the results of the inversion (phase velocity dispersion curves and $S$-velocity model) remain reliable in the case of such errors on the earthquakes source parameters.

Another parameter that may be a source of error in our algorithm is the seismic moment $M_o$, as determined, for example by the CMT method. This problem is overcome in our method by normalizing the real and the corresponding total synthetic seismogram to 1. In effect, each real spectrum

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Figure 9. Spectra of the seismograms presented on Fig. 8 for group velocity windows corresponding to the fundamental model (9a) and to the higher modes (9b).

Figure 10. Phase velocity deviations after two iterations. Case of the $S$-velocity anomaly located in the lower mantle (model B).
is divided by its maximum amplitude. The corresponding synthetic seismograms are normalized by dividing each monomode synthetic seismogram by the maximum value of the total corresponding synthetic seismogram. This normalization has also the advantage that the same weight is given for each earthquake whatever its amplitude is.

Finally, we have computed "real" data which contained all the modes up to the radial order \( n = 10 \). The \( \chi^2 \) reduction after inversion is greater than 89 per cent for the two experiments. In fact, we can eliminate most of the signal corresponding to the fourth and higher modes by using an appropriate group-velocity window since these modes generally arrive before the mode-wave group \( n = 0-3 \). And in any case, these higher modes are generally much less energetic.

4 APPLICATION OF THE METHOD TO REAL DATA

Finally, the method has been applied to a set of real data. We selected four seismograms originating from the Molucca Islands (latitudes between 0° and 3°N, longitudes between 124° and 128°E and depths between 25 and 150 km) and recorded at the SSB GEOSCOPE station (latitude: 45.28°N, longitude: 4.54°E). The average epicentral distance is about 12,000 km. On Fig. 14, we present one of the four seismograms and the corresponding total synthetic seismogram. In this example, the modes 0 to 3 are used in the inversion. The real and synthetic seismograms are normalized.

On Fig. 15, we present the phase velocity deviations versus to those corresponding to the PREM model and on Fig. 16, the S-velocity model obtained after two iterations. The phase velocities anomalies and the S-velocity anomaly are positive. These results are consistent with the fact that the SSB–Molucca Islands paths cross a large percentage of continents and then we can expect these paths to be fast.

On Fig. 17, we compare one of the real seismograms and the corresponding synthetic seismogram obtained after inversion, each seismogram being normalized. The phase of the seismogram is correctly explained. There are still unmodelled amplitude residuals that could be the result of a mislocation of the earthquake, bad CMT determination (source geometry and magnitude) or attenuation differences... The coefficient \( \chi^2 \) has been computed to compare the synthetic and real seismogram. The \( \chi^2 \) reduction is 83 per cent after inversion.
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Figure 13. Comparison between "real" spectra and synthetic spectra that correspond to the inverted model and are computed: (13a) without eigenfunctions recomputation (13b) after eigenfunctions recomputation. Left: for a group velocity window corresponding to the fundamental mode. (Between 2.7 and 4.0 km s\(^{-1}\).) Right: for a group velocity window corresponding to the higher modes. (Between 4.0 and 7.5 km s\(^{-1}\).)

5 DISCUSSION

For the fundamental mode, we have compared our method with the single seismogram method proposed by Suetsugu & Nakanishi (1985). These authors retrieve the phase velocity by computing the cross-correlation between a real seismogram and the corresponding synthetic seismogram of the fundamental mode. For this mode, we can use our method with only one seismogram because the phase velocity calculation is a well posed problem (as many
parameters as data). We have checked that we retrieve the same results with both methods when we use one single seismogram and that, when we use simultaneously several seismograms, the phase velocity that we get is the average of the phase velocities obtained for each seismogram separately.

We use the spectra of the seismograms as data, instead of secondary observables like cross-correlograms (Lerner-Lam & Jordan 1983; Cara & Lévêque 1987, Lévêque et al. 1991). Following Cara & Lévêque (1987) and Lévêque et al. (1991), we introduce an a priori covariance matrix on the parameters and on the data in the non-linear inverse algorithm (Tarantola & Valette 1982). The a priori covariance matrix on parameters enables us to control their variation between two iterations and stabilize the inversion. But, unlike these time-domain methods, we perform two inversions of surface wave data: the first for the phase velocity perturbations of the mode branches and the second for the velocity model. The method of Lerner-Lam & Jordan (1983) is adaptable for a similar two steps inversion although they have chosen to invert directly for the model. We prefer the additional flexibility allowed by first inverting for the phase velocity perturbations of each mode branch. This allows us to see more clearly the contribution (or equivalently the weight) of each mode branch to the determination of the model, and to introduce frequency-dependent data errors. The a priori data covariance matrix specifies the physical information that we have on the error in the data. In the first step of the algorithm, the data are the spectra of the seismograms and so, the data error corresponds to spectra of the noise level on the seismograms. In the second step of the algorithm (the

Figure 14. One of the five data and the corresponding synthetic seismograms used in the inversion.
inversion versus depth), the data are the phase velocities computed in the first step. We use the \textit{a posteriori} covariance matrix of the phase velocities (which is computed during the first step of the algorithm) as the \textit{a priori} data covariance matrix in the inversion versus depth. So, we can take into account the error in the phase velocities in their inversion at depth.

Because of the complexity of the interference pattern of higher modes, the solution is probably non-unique when only one seismogram is used. The simultaneous inversion of several seismograms at different frequencies improves the resolution of the inverted earth model by taking advantage of the constructive interference between the seismograms.

The eigenfunctions are recomputed at each iteration. This effect is not too important for the phase of the seismogram, but it has to be taken in account when the amplitude of the seismogram is involved, although this effect may be secondary with respect to the likely errors in the scaling of the seismic moment tensor.

Our method has been developed as an automatic procedure. It is now possible to apply it to numerous paths at the surface of the earth. This method has been adapted to use data in the period range between 70 and 250 s for the modes 0 to 3. Although the fourth and higher mode branches are also present in the seismograms, the group velocity windowing that we employ is generally able to isolate the modes 0 to 3 without complication, because the fourth and higher mode wave groups generally arrive before the desired wave group and with much less amplitude.

6 CONCLUSION

We propose a method for the computation of the fundamental and first higher modes phase-velocity disper-
sion curves and the determination of the mantle structure averaged over the source-receiver paths. A set of seismograms originating from a small size area and recorded at a single station is used. Two inversions are performed at each iteration: the phase velocity computation for each mode branch and the earth-model determination. The eigenfunctions are recomputed at each iteration to fit the seismograms amplitude. This method enables us to explain the information contained in the phase of the seismogram and the amplitude variations related to eigenfunction variations. Possible misdetermination of the seismic moment \( M_0 \) is taken into account by normalizing the seismograms. In the future, we will take into account the attenuation effect. It will be rather simple to introduce this into our algorithm, because we can compute the attenuation independently from the phase velocity.

The synthetic tests and the use of the method on a set of real data have shown promising results, and this method will now be applied in a more systematic manner to regional and global data. We are planning to apply it for the purpose of studying the structure of the transition zone.

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