Gaussian Source One-Boson-Exchange Potential between Two Octet Baryons

Isamu Arisaka,1 Kimiko Nakagawa,2 Shoji Shinmura3 and Masanobu Wada2

1Chiba Institute of Technology, Narashino 274-0023, Japan
2College of Science and Technology, Nihon University, Funabashi 274-8501, Japan
3Faculty of Engineering, Gifu University, Gifu 501-1193, Japan

(Received January 18, 2005)

The nonstatic baryon-baryon potential arising from the Gaussian meson source function, $G_i \exp (-\Lambda_i^2 r^2/4)$, is constructed by exchanging the scalar, pseudoscalar and vector meson nonets, where $G_i$ is the strength and $\Lambda_i$ the size parameter of source $i$ ($i = 1, 2$). This potential is referred to as GSOBEP. It has no singularity at the origin. In this potential, the radial function behaves like the Yukawa function in the outer region ($r \geq 1$ fm), and like the Yukawa function multiplied by the error function in the inner region ($r \leq 1$ fm). The Gaussian source is assumed to originate from the statistical distribution of valence quarks and many sea quarks. The potential parameters consist of the $SU(3)$ parameters, such as the singlet and octet coupling constants, $g_1$ and $g_8$, the coupling ratio $\alpha = g_F/(g_D + g_F)$, the singlet-octet mixing angle $\theta$ for each meson, and the effective source size parameter, expressed as $\Lambda = \Lambda_1 \Lambda_2 / \sqrt{\Lambda_1^2 + \Lambda_2^2}$. The potential parameters are determined by the $\chi^2$-fitting to the nucleon-nucleon, hyperon-nucleon and hyperon-hyperon data. The fit is quite good on the whole.

§1. Introduction

It is known that there are several one-boson-exchange potentials (OBEP) derived from the meson theory with a $\delta$ source function. Such potentials are singular at the origin. In order to avoid such a singularity, various mechanisms have been introduced, such as a soft core with a cutoff function,1,2 a hard core,3 a form factor,4,5 and an exchange kernel of the quark model.6,7 From the three-range approach (the so-called Taketani methodology), the one-pion-exchange potential (OPEP) has been established in the region I ($r \geq 2$ fm), the one-boson-exchange potential (OBEP) and its nonstatic effects play important roles in the region II (1 fm $\leq r \leq 2$ fm), and the phenomenological core potential is introduced in the region III ($r \leq 1$ fm). The Funabashi-Gifu (FG) potential1 is constructed to reproduce the nucleon-nucleon ($NN$), hyperon-nucleon ($YN$) and hyperon-hyperon ($YY$) data from the point of view of the above-mentioned approach, where $N = (p, n)$ and $Y = (\Lambda, \Sigma^{0, \pm}, \Xi^{0, -})$.

There is a great deal of information regarding the core of the potential at short ranges from nucleon-nucleon scattering experiments, the statistical properties of the quark model,8,9 etc. In particular, it has been shown that the radial Gaussian dependence at short ranges can be derived from the “structural core” picture of the quark model,10,11 in which baryons and mesons are composite particles composed of quarks and antiquarks, as $|B\rangle = |qqq\rangle$ and $|M\rangle = |q\bar{q}\rangle$, where $q$ represents a $u, d$
or $s$ quark.

Because of the composite nature of baryons, we introduce an extended source in the form of a Gaussian distribution of the valence quarks ($u_v,d_v,s_v$) and many sea quarks ($u_s\bar{u}_s,d_s\bar{d}_s,s_s\bar{s}_s$), writing $\rho(r) = G_i \exp \left(-\Lambda_i^2 r^2/4\right)$, where $G_i$ is the strength of the source, and $\Lambda_i$ is the source size parameter. (Here $i = 1$ and 2 correspond to source 1 and 2.) Such baryon structure is supported by the EMC effects.

We construct the OBEP arising from Gaussian sources, which is referred to as GSOBEP. Preliminary calculations are given in Ref. 12). The scalar mesons [S, including the singlet ($\sigma$) and octet ($a_0,f_0,K^0_0$)], pseudoscalar mesons [P, including the singlet ($\eta'$) and octet ($\pi,\eta,K$)] and vector mesons [V, including the singlet ($\omega$) and octet ($\rho,\phi,K^*$)] are taken into account. These mesons are the same as those considered in the FG potential.

At the present time, the scalar mesons are not yet established, and therefore we prefer to choose $\sigma$ (600 MeV, $\Gamma = 450$ MeV) for $f_0$ (400–1200 MeV, $\Gamma = 600$–1000 MeV), following the Particle Data Group 96 (PDG)\(^{13}\) and a recent study of the $\sigma$ meson.$^{14}\)

Effects of the width of broad mesons, $\sigma$ (600 MeV, $\Gamma = 450$ MeV) and $\rho$ (780 MeV, $\Gamma = 150.7$ MeV) have been incorporated by employing the broad meson propagator as the dispersion integral.$^{15}$ In our potential, the mass integration has been carried out in the OBEP with a broad meson. Contrastingly, the two-pole approximation is realized using a sum of Yukawa functions in the Nijmegen potential.$^{3}\)

In the GSOBEP model, the radial dependence of the potential behaves like the Yukawa function, i.e., $Y(r) = e^{-m r}/r$, in the outer region ($r \geq 1$ fm), and like the Yukawa function multiplied by the (complementary) error function, $Y(r) \cdot \text{Erfc}(z)$ (where $\text{Erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$, $z = -\Lambda r/2 + m/\Lambda$), in the inner region ($r \leq 1$ fm), where $m$ is the meson mass and $\Lambda = \Lambda_1\Lambda_2/\sqrt{\Lambda_1^2 + \Lambda_2^2}$ is the effective source size parameter. There GSOBEP is finite at the origin, and for this reason, it extends the applicability of the OBEP approach.

The $SU(3)$ parameters are the singlet coupling $g_1$, the octet coupling $g_8$, the coupling ratio $g_F/(g_D + g_F) = \alpha$, and the singlet-octet mixing angle $\theta$ for the scalar, pseudoscalar and vector meson nonets. There are two types of coupling for the vector meson nonet, the electric ($V(e)$) and magnetic ($V(m)$) ones. Thus there are sixteen parameters in the $SU(3)$ framework. However, as discussed below, six constraints on the $SU(3)$ parameters are considered. Finally, there are ten free $SU(3)$ parameters. For the scalar mesons, all parameters are free. For the pseudoscalar meson, (1) the coupling constant for $\pi$ is fixed as $f_\pi/\sqrt{4\pi} = g_8 p/\sqrt{4\pi} = 0.27$ from the analysis of the $NN$ data, (2) the singlet-octet mixing angle is given by $\theta_\pi = -23.92^\circ$ from the linear Gell-Mann Okubo (GMO) mass formula, and (3) the coupling ratio is fixed as $\alpha_\rho = 0.355$ by the value found in semileptonic decays.$^{16}$ For the vector mesons, (4) the electric coupling constant is given by $g^n = g_V$, with $\alpha^n = 1.0$, due to the universality assumption,$^{17}$ (5) the magnetic coupling constant is given by $g^m = g_V + f_V$, with free $\alpha^m$, and (6) the mixing angle is fixed as $\theta^n = \theta^m = 36.44^\circ$, from the linear GMO mass formula.

Employing the Gaussian source, we introduce the size parameter including the
baryon core \((\Lambda_B)\) and the meson corrections \((\Lambda_M)\). The size parameter of the baryon core is expressed in terms of the strangeness \((S)\) dependence, considering the \(S\) dependence of the baryon mass data, \(m_Y \approx m_N (1 - 0.2S)\) and \(\Lambda_B = \Lambda_N (1 + \beta S)\), where \(S = 0\) \((N)\), \(S = -1\) \((Y = \Lambda, \Sigma)\), \(S = -2\) \((Y = \Xi)\), and \(\Lambda_N = \sqrt{6}/\sqrt{\langle R^2 \rangle}\) is the size parameter of the nucleon core from the root-mean-square radius of the nucleon, \(\sqrt{\langle R^2 \rangle}\). The source size parameters for each meson at the sources 1 and 2 are defined as \(A_{1,2} = \Lambda_B + \Lambda_M\), \(\Lambda_B = \Lambda_N (1 + \beta S)\) and \(\Lambda_M = \Lambda_{ij} (i = 1, 8\) and \(j = S, P, V)\). There are eight source size parameters \((\Lambda_N, \beta, \Lambda_{ij})\). The \(SU(3)\) parameters and the source size parameters are referred to as the GSOBEP parameters. There is a total of \(10 + 8 = 18\) free GSOBEP parameters.

GSOBEP in the outer region is determined by the Yukawa-type OBEP, and that in the inner region is determined by the error functions arising from the Gaussian meson sources. The nature of the repulsive core and the relation between the strength of the source and the meson-baryon coupling constant are yet unknown. In this study, the strength of the source corresponds to the meson-baryon coupling constant.

In this paper, employing the multi-channel Schrödinger equations, we treat \(NN\) scattering, deuteron properties, and the following \(YN\) and \(YY\) reactions

1. the single channel \(\Sigma^+ p \rightarrow \Sigma^+ p\);
2. the coupled channels \(Ap \rightarrow (Ap, \Sigma^+ n, \Sigma^0 p)\) and \(\Sigma^- p \rightarrow (\Sigma^- p, \Sigma^0 n, Ap)\);
3. the coupled channels \(\Lambda \Lambda \rightarrow (\Lambda \Lambda, \Sigma^0 \Sigma^0, \Xi^0 n, \cdots)\).

The experimental data adopted here are the \(NN\) phase shifts obtained from the latest database, SAID, compiled by Arndt et al.,\(^{18}\) 57 \(YN\) data,\(^{19} - 24\) which contain the total cross sections, integrated cross sections and differential cross sections, and the \(\Lambda \Lambda\) data.\(^{25}\) In addition to these data, there are the \(Ap\) phase shifts needed to reproduce the low energy parameters for the \(1S_0\) and \(3S_1\) states and the data of the inelastic capture ratio \(r_R\) for the \(\Sigma^- p\) channel and the \(\Lambda\)-hypernuclei.\(^{26} - 29\) For the strangeness \(-2\) system, the experimental data concerning the \(\Lambda \Lambda\) interaction are limited to the ground states of the double-\(\Lambda\) hypernuclei\(^{30} - 36\) and neutron stars.\(^{37}\)

The contribution of the anti-symmetric spin-orbit potential is examined in the phase shifts of the \(\Sigma^+ p\) and \(Ap\) reactions, and it is concluded that it has a strong effect on the \(\Lambda N\) reaction.

Using the simplex method, we determine the GSOBEP parameters through a \(\chi^2\)-fitting of the \(NN, YN\) and \(YY\) data.

In §2, theoretical remarks regarding the GSOBEP are made. The characteristics and details of the GSOBEP are described in §3. The \(SU(3)\) parameters, the meson-baryon coupling constants and the source size parameters are discussed and the obtained values are presented in §4. Calculations and discussion concerning the \(NN, YN\) and \(YY\) systems are given in §5.

§2. Potential arising from Gaussian sources

Let us consider a scalar meson field \(\Phi\) and its source 1 located at \(r_1\) with density \(\rho_1\). In the static limit, \(\Phi\) originating from this source is a solution of the Klein-Gordon equation

\[
(-\nabla^2 + m^2) \Phi(r') = G_1 \rho_1 (r' - r_1),
\]

(2.1)
where $m$ is the meson mass and $G_1$ is the strength of source 1. Here, $\rho_1$ possesses a scalar invariant property, and it is assumed to be time independent.

Suppose that each source has a Gaussian distribution centered at $r_i$ of the form

$$\rho_i(x_i) = \frac{1}{(2\pi)^{3/2} \sigma_i^3} \exp \left( -\frac{x_i^2}{2 \sigma_i^2} \right),$$

where

$$x_i = r' - r_i, \quad \Lambda_i = \frac{\sqrt{2}}{\sigma_i}. \quad (i = 1, 2)$$

The Fourier transform of Eq. (2.1) is given by

$$\Phi(k) = \frac{1}{(2\pi)^{3/2}} \int dr' \exp(-ik \cdot r') \Phi(r'),$$

$$\rho_1(k) = \frac{1}{(2\pi)^{3/2}} \int dx_1 \exp(-ik \cdot x_1) \rho_1(x_1),$$

$$= \frac{1}{(2\pi)^{3/2}} \exp(-k^2/\Lambda_1^2).$$

The Fourier transform of Eq. (2.1) is given by

$$(k^2 + m^2) \Phi(k) = G_1 \exp(-ik \cdot r_1) \rho_1(k).$$

Then we can write $\Phi(r')$ using Eqs. (2.5) and (2.6) as follows:

$$\Phi(r') = \frac{G_1}{(2\pi)^3} \int dk \frac{\exp(-k^2/\Lambda_1^2) \exp(ik \cdot x_1)}{k^2 + m^2}.$$ 

To carry out the calculation of Eq. (2.7), we first write

$$\Phi(x_1) = \frac{G_1}{(2\pi)^2} \frac{1}{ix_1} \int_0^\infty dk \frac{k[\exp(ikx_1) - \exp(-ikx_1)] \exp(-k^2/\Lambda_1^2)}{k^2 + m^2}.$$
where \( x_1 = |r' - r_1| \) and \( k = |k| \). Then, to evaluate this integral analytically, we rewrite it using the formula

\[
\exp(-k^2/\Lambda_1^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} ds \exp(-s^2/2 + i\sqrt{2}ks/\Lambda_1). \tag{2.9}
\]

Then we obtain

\[
\Phi(x_1) = G_1 \left( \frac{1}{4\pi} \right) \frac{1}{4ix_1} \int_{-\infty}^{\infty} dse^{-s^2/2} \int_{-\infty}^{\infty} dk \left( \frac{1}{k + im} + \frac{1}{k - im} \right) \left\{ \exp \left[ i(\sqrt{2}s/\Lambda_1 + x_1)k \right] - \exp \left[ i(\sqrt{2}s/\Lambda_1 - x_1)k \right] \right\}
\]

\[
= G_1 \left( \frac{1}{4\pi} \right) \frac{1}{4ix_1} \left\{ \int_{-\Lambda_1x_1/\sqrt{2}}^{\infty} dse^{-s^2/2}(2\pi i) \exp[-(\sqrt{2}s/\Lambda_1 + x_1)m] \right.
\]

\[
\left. + \int_{-\Lambda_1x_1/\sqrt{2}}^{\Lambda_1x_1/\sqrt{2}} dse^{-s^2/2}(2\pi i) \exp[-(\sqrt{2}s/\Lambda_1 - x_1)m] \rightangle
\]

\[
\left. - \int_{-\Lambda_1x_1/\sqrt{2}}^{\infty} dse^{-s^2/2}(2\pi i) \exp[-(\sqrt{2}s/\Lambda_1 - x_1)m] \right)
\]

\[
= G_1 \left( \frac{1}{4\pi} \right) \frac{e^{(m/\Lambda_1)^2}}{2} \left\{ \frac{e^{-mx_1}}{x_1} \int_{-\Lambda_1x_1/\sqrt{2}}^{\infty} ds \exp[-(s/\sqrt{2} + m/\Lambda_1)^2] \right.
\]

\[
\left. - \frac{e^{mx_1}}{x_1} \int_{\Lambda_1x_1/\sqrt{2}}^{\infty} ds \exp[-(s/\sqrt{2} + m/\Lambda_1)^2] \right\}
\]

\[
= G_1 \left( \frac{1}{4\pi} \right) \frac{e^{(m/\Lambda_1)^2}}{2} \left[ \frac{e^{-mx_1}}{x_1} \int_{z}^{z_+} e^{-t^2} dt - \frac{e^{mx_1}}{x_1} \int_{z}^{z_+} e^{-t^2} dt \right], \tag{2.10}
\]

where we have

\[
t = \frac{s}{\sqrt{2}} + \frac{m}{\Lambda_1}, \quad dt = \frac{ds}{\sqrt{2}} \quad \tag{2.11a}
\]

with

\[
z = -\frac{\Lambda_1}{2}x_1 + \frac{m}{\Lambda_1}, \quad z_+ = \frac{\Lambda_1}{2}x_1 + \frac{m}{\Lambda_1}. \tag{2.11b}
\]

Next, we use the complementary error function,

\[
\operatorname{Erfc}(z) = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-t^2} dt, \tag{2.12}
\]

and we then obtain the meson field from Eq. (2.10) as

\[
\Phi(x_1) = \frac{G_1}{4\pi} \frac{e^{(m/\Lambda_1)^2}}{2} \left[ \frac{e^{-mx_1}}{x_1} \operatorname{Erfc}(z) - \frac{e^{mx_1}}{x_1} \operatorname{Erfc}(z_+) \right]. \tag{2.13}
\]
The meson theory with an extended source is applicable to the entire range including the origin. The meson field given in Eq. (2.13) is regular at the origin $x_1 = 0$:

$$
\Phi(0) = \frac{G_1}{4\pi} m \left[ -e^{(m/\Lambda_1)^2} \text{Erfc} (m/\Lambda_1) + \frac{1}{\sqrt{\pi}} \frac{\Lambda_1^2}{m} \right].
$$

(2.14)

Here, the following approximations of the error functions near the origin ($x_1 \approx 0$) have been used.

$$
\text{Erfc}(z) \approx \text{Erfc}(m/\Lambda_1) + \frac{2}{\sqrt{\pi}} e^{-(m/\Lambda_1)^2} \left[ \frac{A_1 x_1}{2} + \frac{m}{A_1} \left( \frac{A_1 x_1}{2} \right)^2 + \cdots \right],
$$

(2.15a)

$$
\text{Erfc}(z_+ ) \approx \text{Erfc}(m/\Lambda_1) - \frac{2}{\sqrt{\pi}} e^{-(m/\Lambda_1)^2} \left[ \frac{A_1 x_1}{2} - \frac{m}{A_1} \left( \frac{A_1 x_1}{2} \right)^2 + \cdots \right],
$$

(2.15b)

where the relations $\partial \text{Erfc}(z)/\partial x_1 = (\Lambda_1/\sqrt{\pi}) e^{-z^2}$ and $\partial \text{Erfc}(z_+)/\partial x_1 = -(\Lambda_1/\sqrt{\pi}) e^{-z^2}$ are explicitly expressed.

Using the relations $\text{Erfc}(\infty) = 0$ and $\text{Erfc}(-\infty) = 2$, we can see that Eq. (2.13) takes the form of the Yukawa function at long ranges,

$$
\Phi(x_1) \approx \frac{G_1}{4\pi} e^{(m/\Lambda_1)^2} e^{-m x_1}.
$$

(2.16)

We next calculate the potential energy $V$ for two Gaussian sources. The potential is defined as the interaction energy between the meson field $\Phi(r')$, given by Eq. (2.7), and source 2, represented by $\rho_2$. Then we can obtain GSObEP as follows:

$$
V(r_1, r_2) = -\int dr' \Phi(r') G_2 \rho_2(r' - r_2)
$$

$$
= -\frac{G_1 G_2}{(2\pi)^{3/2}} \int dk \frac{\exp(-k^2/\Lambda_1^2) \exp(ik \cdot r) \rho_2(k)}{k^2 + m^2}
$$

$$
= -\frac{G_1 G_2}{(2\pi)^{3/2}} \int dk \frac{\exp(-k^2/\Lambda_1^2) \exp(-k^2/\Lambda_2^2) \exp(ik \cdot r)}{k^2 + m^2}
$$

$$
= -\frac{G_1 G_2}{(2\pi)^{3/2}} \int dk \frac{\exp(-k^2/\Lambda_1^2) \exp(-k^2/\Lambda_2^2) \exp(ik \cdot r)}{k^2 + m^2},
$$

(2.17)

where

$$
\rho_2(k) = \frac{1}{(2\pi)^{3/2}} \int dx_2 \exp(ik \cdot x_2) \rho_2(x_2) = \frac{1}{(2\pi)^{3/2}} \exp(-k^2/\Lambda_2^2),
$$

(2.18)

$$
\frac{1}{\Lambda_2^2} = \frac{1}{\Lambda_1^2} + \frac{1}{\Lambda_2^2}.
$$

(2.19)

The quantity $\Lambda$ is called the effective source size parameter. Here the relations $x_2 = r' - r_2$, $dx_2 = dr'$ and $r = x_1 - x_2$ have been used.

It is seen that in Eq. (2.17) the propagator, $1/(k^2 + m^2)$, is multiplied by two source functions, $e^{-k^2/\Lambda_1^2}$ and $e^{-k^2/\Lambda_2^2}$, as

$$
\frac{e^{-k^2/\Lambda_1^2} e^{-k^2/\Lambda_2^2}}{k^2 + m^2} = \frac{e^{-k^2/\Lambda^2}}{k^2 + m^2},
$$

(2.20)
Thus the potential energy Eq. (2.17) is obtained in the same manner as Eqs. (2.7) and (2.13), replacing $(x_1, A_1)$ with $(r, A)$ as follows:

$$V(r_1, r_2) = V(r) = -\frac{G_1 G_2}{4\pi} \frac{e^{(m/A)^2}}{2} \left[ \frac{e^{-mr}}{r} \text{Erfc}(z) - \frac{e^{mr}}{r} \text{Erfc}(z+). \right]$$  \hspace{1cm} (2.21)

$$z = -\frac{A}{2} r + \frac{m}{A}, \quad z_+ = \frac{A}{2} r + \frac{m}{A}, \quad r = |r| = |r_1 - r_2| = |x_1 - x_2|. \quad (2.22)$$

In our Gaussian source model, the strength of the source corresponds to the meson-baryon coupling constant with the factor $e^{(m/A)^2}$, and it is analytic at the origin $(r = 0)$:

$$V(0) = -\frac{G_1 G_2}{4\pi} m \left[ -e^{(m/A)^2} \text{Erfc}(m/A) + \frac{1}{\sqrt{\pi}} \frac{A}{m} \right]. \quad (2.23)$$

Using the relations $\text{Erfc}(\infty) = 0$ and $\text{Erfc}(-\infty) = 2$, in the outer region, the potential converges to the Yukawa potential

$$V(r) \approx -\frac{G_1 G_2}{4\pi} e^{(m/A)^2} e^{-mr}/r. \quad (2.24)$$

In Fig. 2, the Gaussian source potential, given in Eq. (2.21) is compared with the Yukawa-type potential given in Eq. (2.24). The two potentials are nearly coincident in the region $r \geq 0.5$ fm, but, they have entirely different forms in the most inner region $r \leq 0.5$ fm. In particular, the potential obtained with the $\delta$ source is infinite at the origin.

The quark structure of the baryon is believed to be spread with a root-mean-square (r.m.s) radius $\sqrt{\langle R^2 \rangle}$, where $R$ is the radius of the source. The relation between the source size parameter $A_S$ and $\sqrt{\langle R^2 \rangle}$ is given by the density distribution $\rho(r) = \frac{A_S^3}{(4\pi)^{3/2}} \exp(-A_S^2 r^2/4)$. With this, we obtain

$$\langle R^2 \rangle = \int_0^\infty r^2 \rho(r) 4\pi r^2 dr = \frac{6}{A_S^2},$$

Fig. 2. The potentials given by the Gaussian and $\delta$ sources, where $G_1 G_2/4\pi = 1.0$, $m = 600$ MeV= 3.0 fm$^{-1}$ and $A = 2000$ MeV$/\sqrt{2} = 7.17$ fm$^{-1}$ are used.
\[
A_S = \frac{\sqrt{6}}{\langle R^2 \rangle}.
\]

§3. Baryon-baryon potential in coordinate space

We now construct the nonstatic GSOBEP with the nonet mesons exchange. The meson-baryon interaction Lagrangians at the vertex 1–3 are expressed as follows:

\[
L_S = -g_{S}^{13}\bar{\Psi}_{3}\Psi_{1}\Phi_{S}, \quad \text{(scalar meson)} \tag{3.1}
\]

\[
L_P = -\frac{iF_{P}^{13}}{m_{s}}\bar{\Psi}_{3}\gamma_{5}\gamma_{\mu}\Psi_{1}\partial_{\mu}\Phi_{P}, \quad \text{(pseudoscalar meson)} \tag{3.2}
\]

\[
L_V = -ig_{V}^{13}\bar{\Psi}_{3}\gamma_{\mu}\Psi_{1}\Phi_{V}^{\mu} - \frac{F_{V}^{13}}{4M_{s}}\bar{\Psi}_{3}\sigma_{\mu\nu}\Psi_{1}\Phi_{V}^{\mu\nu}, \quad \text{(vector meson)} \tag{3.3}
\]

\[
\Phi_{V}^{\mu\nu} = \partial_{\mu}\Phi_{V}^{\nu} - \partial_{\nu}\Phi_{V}^{\mu}, \quad \sigma_{\mu\nu} = [\gamma_{\mu}, \gamma_{\nu}]/2i, \tag{3.4}
\]

where \(\Psi\) and \(\Phi\) are the baryon and meson fields, \(g\) and \(f\) are the meson-baryon direct and derivative coupling constants, \(\gamma_{5}\) and \(\gamma_{\mu}\) are the Dirac matrices, and \(m_{s}\) and \(M_{s}\) are the scaling masses of the meson and baryon, respectively. The interaction Lagrangians at the vertex 2–4 are given in the same forms as Eqs. (3.1)–(3.3) through exchange of the subscripts 1 and 2 and exchange of the subscripts 3 and 4.

The scalar and pseudoscalar meson fields \(\Phi\) constitute a solution of the Klein-Gordon equation

\[
\left(-\nabla^2 + \frac{\partial^2}{\partial t^2} + m^2\right)\Phi = G_{1}\rho_{1}, \tag{3.5}
\]

where \(m\) is the meson mass, \(G_{1}\) is the strength (coupling constant) and \(\rho_{1}\) is the source density assumed to be independent of time. The vector meson field is given by the following equation:39)

\[
\left(-\nabla^2 + \frac{\partial^2}{\partial t^2} + m^2\right)\Phi_{\mu} = \left(\delta_{\mu\nu} - \frac{1}{m^2}\partial_{\mu}\partial_{\nu}\right)G_{1}\rho_{1}. \tag{3.6}
\]

Here, \(\Phi\) and \(\Phi_{\mu}\) are obtained analogously to Eq. (2.13).

The \(SU(3)\) invariant Lagrangian is defined as

\[
L = -g_{1}^{(1)}\text{Tr}(\bar{\Psi}_{3}^{8}\Psi_{1}^{8})\Phi_{1}^{8} - g_{8}^{(1)}\text{Tr}(\bar{\Psi}_{3}^{8}\Phi_{1}^{8}\Psi_{1}^{8}) - g_{8}^{(2)}\text{Tr}(\bar{\Psi}_{3}^{8}\Psi_{1}^{8}\Phi_{8}^{8}), \tag{3.7a}
\]

\[
= -g_{S}\text{Tr}(\bar{\Psi}_{3}^{S}\Psi_{1}^{S})\Phi_{1}^{S} - g_{D}\left[\text{Tr}(\bar{\Psi}_{3}^{S}\Phi_{1}^{S}\Psi_{1}^{S}) + \text{Tr}(\bar{\Psi}_{3}^{S}\Psi_{1}^{S}\Phi_{8}^{8})\right] - g_{F}\left[\text{Tr}(\bar{\Psi}_{3}^{S}\Phi_{8}^{8}\Psi_{1}^{S}) - \text{Tr}(\bar{\Psi}_{3}^{S}\Psi_{1}^{S}\Phi_{8}^{8})\right], \tag{3.7b}
\]

where \(\Psi^{8}\) is the octet baryon, and \(\Phi_{1}^{8}\) and \(\Phi_{8}^{8}\) are the singlet and octet mesons, respectively. The quantities \(g_{1}^{(1)}, g_{8}^{(1)}\) and \(g_{8}^{(2)}\) are the singlet and octet coupling constants, defined in terms of \(g_{D}, g_{F}\) and \(\alpha\) as follows:

\[
g_{1}^{(1)} = g_{S} = g_{1}, \tag{3.8a}
\]

\[
g_{8}^{(1)} = g_{D} + g_{F} = \sqrt{2}g_{8}, \quad g_{8}^{(2)} = g_{D} - g_{F} = \sqrt{2}(1 - 2\alpha)g_{8}, \tag{3.8b}
\]
\[ \alpha = \frac{g_F}{g_D + gF}. \] (3.8c)

For example, the relations between the meson-baryon coupling constants and the \( SU(3) \) coupling constants are defined for the pseudoscalar mesons\(^1\) as follows:

\[ f_{pp\pi} = g_8, \quad f_{pp\eta} = -g_1 \sin \theta + \frac{1}{\sqrt{3}}(-1 + 4\alpha)g_8 \cos \theta, \] (3.9a)

\[ f_{pp\eta'} = g_1 \cos \theta + \frac{1}{\sqrt{3}}(-1 + 4\alpha)g_8 \sin \theta, \] (3.9b)

\[ \tan^2 \theta = \frac{m_8 - m_\eta}{m_\eta' - m_8}, \quad m_8 = \frac{1}{3}(4m_K - m_\pi). \] (3.9c)

For the vector and scalar mesons, respectively, the following translations are made: \( \pi \to \rho, a_0; \ K \to K^+, K_0^*; \ \eta \to \phi, f_0; \ \eta' \to \omega, \phi. \)

The quantities \( g_1, g_8, \alpha \) and \( \theta \) are called the \( SU(3) \) parameters. The relations between the meson-baryon coupling constants, \( g_S, f_P, g_V \) and \( f_V \), and the \( SU(3) \) parameters are given in Appendix A of Ref. 1).

In the center-of-mass system, the momenta of the initial and final states of the two baryons are denoted by \((p, -p)\) and \((p', -p')\), respectively. With these momenta and \( k = p' - p, q = (p' + p)/2 \), the general form of the \( p \)-space GSOBEP is expressed as follows:

\[ V_{GSOBE}(p', p) = V_0(p', p) + i(S \cdot k \times q)V_1(p', p) + (\sigma_1 \cdot k)(\sigma_2 \cdot k)V_2(p', p) \\
+ (\sigma_1 \cdot q)(\sigma_2 \cdot q)V_3(p', p) + (\sigma_1 \cdot k \times q)(\sigma_2 \cdot k \times q)V_4(p', p) \\
+ (\sigma_1 \cdot \sigma_2)V_5(p', p) + i(S \cdot k \times q)V_6(p', p), \] (3.10)

where

\[ V_i(p', p) = V_i(k^2, q^2, (k \times q)^2)\tilde{\Delta}(k^2), \quad S = (\sigma_1 + \sigma_2)/2, \quad S_\bot = (\sigma_1 - \sigma_2)/2, \] (3.11)

\[ \tilde{\Delta}(k^2) = \frac{e^{-k^2/\Lambda^2}}{k^2 + m^2 - (E_3 - E_1)^2} \]

\[ = \frac{1}{k^2 + m^2 - (\Delta M)^2} + \frac{(k \cdot q)^2}{M_{AV}^2} \left[ \frac{1}{[k^2 + m^2 - (\Delta M)^2]^2} \right] e^{-k^2/\Lambda^2} \]

\[ \equiv \tilde{\Delta} + \tilde{\Delta}_R. \] (3.12a)

\[ \tilde{\Delta} = \frac{e^{-k^2/\Lambda^2}}{k^2 + \mu^2}, \quad \tilde{\Delta}_R = \frac{(k \cdot q)^2}{M_{AV}^2} \frac{e^{-k^2/\Lambda^2}}{[k^2 + \mu^2]^2}, \] (3.12b)

\[ \Delta M = M_3 - M_1, \quad \mu^2 = m^2 - (\Delta M)^2, \]

\[ M_{AV} = 2M_r = 2M_1M_3/(M_1 + M_3), \quad \Lambda = \Lambda_1\Lambda_2/\sqrt{\Lambda_1^2 + \Lambda_2^2}. \] (3.12c)

Here, \( m \) and \( \mu \) are the meson mass and effective meson mass, \( M_{AV} \) is the average baryon mass, defined by the reduced mass \( M_r \), and \( \Lambda \) is the effective size parameter.
The modified propagators $\tilde{\Delta}$ and $\tilde{\Delta}_R$ give the ordinary potential $V$ and retarded potential $U$, respectively.

In order to perform the Fourier transform of the $p$-space potential into the $r$-space potential, we keep the terms up to first order in $k^2/M^2$, $q^2/M^2$ and $(k\cdot q)^2/M^4$ in the expression for $V_i(p', p)$. The approximate $p$-space GSOBEP, $V_i$ and $U_i$ (with $i = S, P, VV, VM, VT$) are given in Appendix B of Ref. 1), but there $\Delta(k^2)$ is replaced by $\tilde{\Delta}(k^2)$.

The basic functions for the ordinary and retarded GSOBEP are

$$
\frac{1}{(2\pi)^3} \int \frac{dk \exp(-k^2/\Lambda^2) \exp(ik \cdot r)}{k^2 + \mu^2} = \frac{\mu}{4\pi^2} \left[ \frac{e^{-\mu r}}{\mu r} \text{Erfc}(z) - \frac{e^{\mu r}}{\mu r} \text{Erfc}(z_+) \right] = \frac{\mu}{4\pi} Y_0(r),
$$

(3.13)

$$
\frac{1}{(2\pi)^3} \int \frac{dk \exp(-k^2/\Lambda^2) \exp(ik \cdot r)}{(k^2 + \mu^2)^2} = \left( -\frac{1}{2\mu} \frac{\partial}{\partial \mu} \right) \frac{1}{(2\pi)^3} \int \frac{dk \exp(-k^2/\Lambda^2) \exp(ik \cdot r)}{k^2 + \mu^2} = \frac{1}{8\pi \mu} R_0(r),
$$

(3.14)

$$
Y_0(r) = e^{(\mu/\Lambda)^2} \left[ \frac{e^{-\mu r}}{\mu r} \text{Erfc}(z) - \frac{e^{\mu r}}{\mu r} \text{Erfc}(z_+) \right],
$$

(3.15)

$$
E_0(r) = e^{(\mu/\Lambda)^2} \left[ e^{-\mu r} \text{Erfc}(z) + e^{\mu r} \text{Erfc}(z_+) \right],
$$

(3.16)

$$
R_0(r) = E_0(r) - 2 \left( \frac{\mu}{\Lambda} \right)^2 Y_0(r),
$$

(3.17)

where

$$z = -\frac{\Lambda}{2} r + \frac{\mu}{\Lambda} = -\frac{\Lambda}{2\mu} x + \frac{\mu}{\Lambda} = -ax + b,
$$

(3.18a)

$$z_+ = \frac{\Lambda}{2} r + \frac{\mu}{\Lambda} = \frac{\Lambda}{2\mu} x + \frac{\mu}{\Lambda} = ax + b,
$$

(3.18b)

$$a = \frac{\Lambda}{2\mu}, \quad b = \frac{\mu}{\Lambda}.
$$

(3.18c)

In general, the GSOBEP can be expressed as a sum of the ordinary potential $V$ and retarded potential $U$ as follows:

$$V_{GSOBE} = [V(r) + U(r)] P,
$$

(3.19)

$$V(r) = V_C + S_{12} V_T + (L \cdot S)V_{LS} + (L \cdot S_-)V_{ALS}
+ W_{12} V_W + L^2 V_{LL} + \nabla^2 V_p,
$$

(3.20a)

$$U(r) = U_C + S_{12} U_T + (L \cdot S)U_{LS} + W_{12} U_W + L^2 U_{LL} + \nabla^2 U_p,
$$

(3.20b)
where
\[ \overrightarrow{\nabla}^2 f \equiv (\nabla^2 f + f \nabla^2), \]
and \( P = 1 \) for the non-strange mesons, \( P = -P_x P_y = -(-1)^L (1 + \sigma_1 \cdot \sigma_2) / 2 \) for the strange mesons, and
\[ S_{12} = \frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - (\sigma_1 \cdot \sigma_2), \quad (3.22a) \]
\[ W_{12} = \frac{1}{2} [(\sigma_1 \cdot L)(\sigma_2 \cdot L) + (\sigma_2 \cdot L)(\sigma_1 \cdot L)] - \frac{1}{3} (\sigma_1 \cdot \sigma_2)L^2. \quad (3.22b) \]

We next introduce a reduction parameter \( \lambda \) for the higher nonstatic potential terms, as
\[ \lambda (V_{W} + V_{LL} + V_{p}) + \lambda (U_{W} + U_{LL} + U_{p}). \] In this \( r \)-space GSOBEP, all terms of order \( p^2/M^2 \) are retained and the higher-order terms are omitted. We set \( \lambda = 0.5 \) so as to improve this approximation, referring to the results of the \( p \)-space calculation.\(^{40} \)

The quantities defined below are used in the potential functions:
\[ x = \mu r, \]
\[ E = e^{-x}, \quad E_+ = e^{x}, \]
\[ Y = \frac{e^{-x}}{x}, \quad Y_+ = \frac{e^{x}}{x}, \]
\[ X = \left(1 + \frac{1}{x^2}\right) \frac{e^{-x}}{x}, \quad X_+ = \left(-\frac{1}{x} + \frac{1}{x^2}\right) \frac{e^{x}}{x}, \]
\[ Z = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x}, \quad Z_+ = \left(1 - \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{x}}{x}, \]
\[ W = \frac{Z}{x^2}, \quad W_+ = \frac{Z_+}{x^2}. \]

The potential functions \( Y_1 - E_3 \) are derived from \( Y_0 \) and \( E_0 \) using following relations:
\[ \delta = \frac{1}{r} \frac{\partial}{\partial r} = \mu x \frac{\partial}{\partial x}, \quad \nabla^2 = r^2 \delta^2 + 3 \delta, \]
\[ Y_{i+1} = \frac{1}{\mu^2} \nabla^2 Y_i, \quad E_{i+1} = \frac{1}{\mu^2} \nabla^2 E_i + 2Y_i, \quad (3.30a) \]
\[ X_0 = -\frac{1}{\mu^2} \delta Y_0, \quad X_i = \frac{1}{\mu^2} \delta Y_i \quad (i = 1, 2), \quad (3.30b) \]
\[ Z_0 = -\frac{x^2}{\mu^2} \delta X_0, \quad Z_i = \frac{x^2}{\mu^2} \delta X_i \quad (i = 1, 2). \quad (3.30c) \]

The quantities \( Y_0 - E_3 \) are given as follows:
\[ Y_0 = \frac{e^{-a^2 x^2}}{2} [Y \cdot \text{Erfc}(z) - Y_+ \cdot \text{Erfc}(z_+)], \quad (3.31a) \]
\[ Y_1 = Y_0 - \frac{4 a^3}{\sqrt{\pi}} e^{-a^2 x^2}, \quad (3.31b) \]
\[ Y_2 = Y_1 + \frac{8 a^5}{\sqrt{\pi}} (3 - 2a^2 x^2) e^{-a^2 x^2}. \quad (3.31c) \]
\[
X_0 = \frac{e^{b^2}}{2}[X \cdot \text{Erfc}(z) - X_+ \cdot \text{Erfc}(z_+)] - \frac{2a}{\sqrt{\pi}} \frac{1}{x^2} e^{-a^2 x^2}, \quad (3.32a)
\]
\[
X_1 = -X_0 + \frac{8a^5}{\sqrt{\pi}} e^{-a^2 x^2}, \quad (3.32b)
\]
\[
X_2 = X_1 + \frac{16a^7}{\sqrt{\pi}} (-5 + 2a^2 x^2) e^{-a^2 x^2}. \quad (3.32c)
\]
\[
\begin{align*}
Z_0 &= \frac{e^{b^2}}{2}[Z \cdot \text{Erfc}(z) - Z_+ \cdot \text{Erfc}(z_+)] - \frac{2a}{\sqrt{\pi}} \frac{(3 + 2a^2 x^2)}{x^2} e^{-a^2 x^2}, \quad (3.33a) \\
Z_1 &= Z_0 - \frac{16a^5}{\sqrt{\pi}} a^2 x^2 e^{-a^2 x^2}, \quad (3.33b) \\
Z_2 &= Z_1 + \frac{32a^7}{\sqrt{\pi}} a^2 x^2 (7 - 2a^2 x^2) e^{-a^2 x^2}. \quad (3.33c)
\end{align*}
\]
\[
W_0 = \frac{Z_0}{x^2}, \quad (3.34a)
\]
\[
W_1 = \frac{Z_1}{x^2}. \quad (3.34b)
\]
\[
\begin{align*}
E_0 &= \frac{e^{b^2}}{2}[E \cdot \text{Erfc}(z) + E_+ \cdot \text{Erfc}(z_+)], \quad (3.35a) \\
E_1 &= E_0 - \frac{2a}{\sqrt{\pi}} e^{-a^2 x^2}, \quad (3.35b) \\
E_2 &= E_1 + \frac{4a^3}{\sqrt{\pi}} (1 - 2a^2 x^2) e^{-a^2 x^2}, \quad (3.35c) \\
E_3 &= E_2 + \frac{8a^5}{\sqrt{\pi}} (-3 + 12a^2 x^2 - 4a^4 x^4) e^{-a^2 x^2}. \quad (3.35d)
\end{align*}
\]

The behavior of the basic potential functions of GSOBEP for the central, tensor and spin-orbit potentials, \(Y_0(x), Z_0(x)\) and \(X_0(x)\) for the vector meson (vector coupling) with \(\mu = 780\) MeV, \(A = 2000/\sqrt{2} = 1414\) MeV \((A_1 = 2000\) MeV), are plotted in Fig. 3. These potential functions are finite at the origin, and are smooth in all regions. Also, higher-order \(Y_i(x), Z_i(x)\) and \(X_i(x)\) have the same characteristics, except for their signs.

The analyticity of the potential functions of the GSOBEP at the origin is demonstrated by using an approximate formula for the error function, Eq. (2.15), from which we have
\[
Y_0(0) = -e^{b^2} \text{Erfc}(b) + \frac{2a}{\sqrt{\pi}}, \quad (3.36)
\]
Gaussian Source One-Boson-Exchange Potential

Fig. 3. The behavior of $Y_0$, $Z_0$ and $X_0$.

\[ X_0(0) = \frac{e^{b^2}}{3} \text{Erfc}(b) - \frac{2a}{\sqrt{\pi}} \frac{1 - 2a^2}{3}, \]  
\[ (3.37) \]

\[ Z_0(0) = 0. \]  
\[ (3.38) \]

Below we list the GSOBEP, in which we use the following:

\[ M = (M_1M_2M_3M_4)^{1/4}, \quad c = 2(\mu/\Lambda)^2. \]  
\[ (3.39) \]

(1) Scalar meson exchange:

\[ V_S = \mu \frac{g_{13}g_{24}}{4\pi} \left[ - \left( Y_0 - \frac{\mu^2}{4M^2}Y_1 \right) - \lambda \frac{1}{2M^2} \hat{\nabla}^2 Y_0 \right. \]
\[- \left. (L \cdot S) \frac{\mu^2}{2M^2} X_0 - (L \cdot S) \frac{\mu^2}{4M^2} (M_2M_4 - M_1M_3) X_0 \right], \]  
\[ (3.40a) \]

\[ U_S = \mu \frac{g_{13}g_{24}}{4\pi} \frac{\mu^2}{2M_4^2} \left\{ \frac{1}{4} [E_2 - 2Y_1 - c(X_1 + Z_1 + 4W_0)] \right. \]
\[- \left. - \lambda \frac{1}{2\mu^2} \hat{\nabla}^2 [E_1 + c(X_0 - Z_0)] - \lambda L^2 (X_0 - cW_0) \right\}. \]  
\[ (3.40b) \]

(2) Pseudoscalar meson exchange ($m_s = m_\pi = 139.570$ MeV):

\[ V_P = \mu \frac{f_{13}f_{24}}{4\pi} \frac{\mu^2}{m_s^2} \left\{ \frac{(\sigma_1 \cdot \sigma_2)}{3} \left[ Y_1 + \frac{\mu^2}{4M^2} (Y_2 - 2X_1 + 4W_0) - \lambda \frac{1}{M^2} \hat{\nabla}^2 X_0 \right. \right. \]
\[- \left. \left. - \lambda L^2 \frac{\mu^2}{2M^2} W_0 \right] \right\} + \frac{S_{12}}{3} \left[ Z_0 + \frac{\mu^2}{8M^2} (-X_1 + 2Z_1 + 5W_0) \right] \]
\[ + \lambda W_{12} \frac{\mu^2}{2M^2} W_0 \}, \]  
\[ (3.41a) \]

\[ U_P = \mu \frac{f_{13}f_{24}}{4\pi} \frac{\mu^2}{2M_4^2} \frac{\mu^2}{m_s^2} \left\{ \frac{(\sigma_1 \cdot \sigma_2)}{3} \left[ - \frac{E_3}{4} + 2X_1 + Z_1 + 2W_0 \right. \right. \]
\[- \left. \left. + c \left( \frac{X_2}{4} + \frac{Z_2}{4} + W_1 \right) \right] + \lambda \frac{1}{2\mu^2} \hat{\nabla}^2 (E_2 + 2X_0 - 2Z_0 - c(X_1 + Z_1)) \right\}. \]
- $\lambda L^2(X_1 + 2W_0 + cW_1) - S_{12} \left[ E_3 + X_1 - Z_1 - 8W_0 \right]$
+ $c \left( 2X_2 - Z_2 - 16W_1 + \frac{24X_1}{x^2} - \frac{120W_0}{x^2} \right)\right]
+ \lambda W_{12} \left[ 2W_0 + c \left( \frac{2X_1}{x^2} - \frac{10W_0}{x^2} \right) \right] \right\}.
\end{equation}

(3) Vector meson exchange (vector coupling):

\begin{equation}
V_{VV} = \mu \frac{g_{13g_{24}}}{4\pi} \left[ Y_0 + \frac{\mu^2}{2M^2} Y_1 + (\sigma_1 \cdot \sigma_2) \frac{\mu^2}{6M^2} Y_1 - S_{12} \frac{\mu^2}{12M^2} Z_0 - \lambda \frac{1}{2M^2} \nabla^2 Y_0 \right]
- (L \cdot S) \frac{3\mu^2}{2M^2} X_0 - (L \cdot S_-) \frac{\mu^2}{4M^4} (M_2 M_4 - M_1 M_3) X_0 \right]
- \mu \frac{g_{13g_{24}}}{4\pi} \frac{\mu^2}{2M^2} \left[ \frac{1}{4} \left[-(E_2 - 2Y_1) + c(X_1 + Z_1 + 4W_0) \right]
+ \lambda \frac{1}{2\mu^2} \nabla^2 [E_1 + c(X_0 - Z_0)] + \lambda L^2(X_0 - cW_0) \right]
- \mu \frac{g_{13g_{24}}}{4\pi} \frac{\mu^2}{2M^2} \left[ \frac{1}{4} \left[-(E_2 - 2Y_1) + c(X_1 + Z_1 + 4W_0) \right]
+ \lambda \frac{1}{2\mu^2} \nabla^2 [E_1 + c(X_0 - Z_0)] + \lambda L^2(X_0 - cW_0) \right].
\end{equation}

(4) Vector meson exchange (mixing coupling) ($M_s = M_p = 938.272$ MeV):

\begin{equation}
V_{VM} = \mu \frac{g_{13g_{24}}}{4\pi} \frac{\mu^2}{2M_s} \left[ \frac{\mu}{2\sqrt{M_2 M_4}} Y_1 + \frac{\sigma_1 \cdot \sigma_2}{3} \frac{\mu}{\sqrt{M_1 M_3}} Y_1 - \frac{S_{12}}{6} \frac{\mu}{\sqrt{M_1 M_3}} Z_0 \right]
- (L \cdot S) \frac{2\mu}{M} X_0 - (L \cdot S_-) \frac{2\mu}{M} X_0 \right]
+ \mu \frac{g_{13g_{24}}}{4\pi} \frac{\mu^2}{2M_s} \left[ \frac{\mu}{2\sqrt{M_1 M_3}} Y_1 + \frac{\sigma_1 \cdot \sigma_2}{3} \frac{\mu}{\sqrt{M_2 M_4}} Y_1 - \frac{S_{12}}{6} \frac{\mu}{\sqrt{M_2 M_4}} Z_0 \right]
- (L \cdot S) \frac{2\mu}{M} X_0 - (L \cdot S_-) \frac{2\mu}{M} X_0 \right],
\end{equation}

$U_{VM} = 0.$

(5) Vector meson exchange (tensor coupling) ($M_s = M_p = 938.272$ MeV):

\begin{equation}
V_{VT} = \mu \frac{g_{13g_{24}}}{4\pi} \frac{\mu^2}{4M_s^2} \left[ \frac{\mu^2}{4M^2} Y_2 + \frac{\sigma_1 \cdot \sigma_2}{3} \left( 2Y_1 + \frac{\mu^2}{2M^2} Y_2 \right) \right]
- \frac{S_{12}}{3} \left[ Z_0 + \frac{\mu^2}{8M^2} (2Z_1 + 3X_1 - 15W_0) \right]
- \frac{S_{12}}{3} \left[ Z_0 + \frac{\mu^2}{8M^2} (2Z_1 + 3X_1 - 15W_0) \right].
\end{equation}
1301

\( + (L \cdot S) \frac{3 \mu^2}{2 M^2} X_1 - (L \cdot S) \frac{\mu^2 (M_2 M_4 - M_1 M_3)}{4 M^4} X_1 \\
+ \lambda W_{12} \frac{3 \mu^2}{2 M^2} W_0 \),

\[ (3.44a) \]

\[ U_{VT} = \mu f_{13} f_{24} \frac{\mu^2}{4 \pi} \frac{\mu^2}{2 M^2} \langle L \cdot S \rangle - \frac{E_3}{3} \left[ \frac{E_3}{2} + 2 Z_1 + 4 X_1 + 4 W_0 \\
+ \frac{c}{2} (X_2 + Z_2 + 4 W_1) + \lambda \frac{1}{\mu^2} \sqrt{2} (E_2 + 2 X_0 - 2 Z_0 - c (X_1 + Z_1)) \\
- 2 \lambda L^2 (X_1 + 2 W_0 + c W_1) + \frac{S_{12}}{12} [E_3 + X_1 - Z_1 - 8 W_0] \\
+ c \left( 2 X_2 - Z_2 - 16 W_1 + \frac{24 X_1}{x^2} - \frac{120 W_0}{x^2} \right) \right] - \lambda W_{12} \left[ 2 W_0 + c \left( \frac{2 X_1}{x^2} - \frac{10 W_0}{x^2} \right) \right]. \]

\[ (3.44b) \]

§4. The \( SU(3) \) parameters and source size parameters

4.1. The \( SU(3) \) parameters and the source size parameters

In order to construct the GSOBEP, the masses of the meson nonets are adopted from the values of PDG\(^{13}\) and the results of a recent study of the \( \sigma \) meson\(^{14}\) listed in Table I. These values are the same as in our previous paper.\(^1\) At present, the \( \sigma \) (600 MeV) meson is considered as a possibility.

In the \( SU(3) \) framework, there are sixteen \( SU(3) \) parameters, \( g_1, g_8, \alpha \) and \( \theta \) for the scalar, pseudoscalar and vector (electric and magnetic coupling) mesons. In this study, the six constraints mentioned in §1 for the pseudoscalar and vector meson nonets are imposed. Thus there are ten free \( SU(3) \) parameters.

The source size parameters, as mentioned in §1, can be regarded as corresponding to the r.m.s radius of the statistical distribution of the valence and sea quarks, \( \sqrt{\langle R^2 \rangle} \), where \( R \) is such that it satisfies \( \Lambda_N = \sqrt{6}/\sqrt{\langle R^2 \rangle} \). From the estimation of the core radius \( r_C \approx 2\sqrt{\langle R^2 \rangle} = 0.5 \) fm, we obtain \( \Lambda_N = \sqrt{6}/\sqrt{\langle R^2 \rangle} \approx 2000 \) MeV. We searched for source size parameters satisfying \( \Lambda_N \approx 2000 \) MeV and \( |\Lambda_{ij}| \leq 500 \) MeV.

The values obtained for the \( SU(3) \) parameters and \( \Lambda_{ij} \) are listed in Table II, where the values of the \( SU(3) \) parameters with asterisks have been determined with

<table>
<thead>
<tr>
<th>scalar mesons</th>
<th>pseudoscalar mesons</th>
<th>vector mesons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>( \eta' )</td>
<td>( \omega )</td>
</tr>
<tr>
<td>( a_0^\pm, a_0^0 )</td>
<td>( \pi^\pm )</td>
<td>( \rho^0, \rho^\pm )</td>
</tr>
<tr>
<td>( \pi^0 )</td>
<td>( \eta )</td>
<td>( \phi )</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>( K_0^\pm )</td>
<td>( K_0^{\pm} )</td>
</tr>
<tr>
<td>( K_0^{a0}, K_0^{\pm} )</td>
<td>( K_0^{a0}, K_0^a )</td>
<td>( K_0^{a0}, K_0^{a0} )</td>
</tr>
</tbody>
</table>

\( \sqrt{\langle R^2 \rangle} = 0.5 \) fm, we obtain \( \Lambda_N = \sqrt{6}/\sqrt{\langle R^2 \rangle} \approx 2000 \) MeV. We searched for source size parameters satisfying \( \Lambda_N \approx 2000 \) MeV and \( |\Lambda_{ij}| \leq 500 \) MeV.

The values obtained for the \( SU(3) \) parameters and \( \Lambda_{ij} \) are listed in Table II, where the values of the \( SU(3) \) parameters with asterisks have been determined with
Table II. The $SU(3)$ parameters and source size parameters. (The values with * are fixed.)

<table>
<thead>
<tr>
<th>$j$</th>
<th>$g_j/\sqrt{4\pi}$</th>
<th>$g_s/\sqrt{4\pi}$</th>
<th>$\alpha$</th>
<th>$\theta$(deg)</th>
<th>$A_{ij}$(MeV)</th>
<th>$A_{ij}$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>2.81821</td>
<td>1.39734</td>
<td>0.85158</td>
<td>39.57</td>
<td>23.27</td>
<td>-454.66</td>
</tr>
<tr>
<td>$P$</td>
<td>0.13345</td>
<td>0.27*</td>
<td>0.355*</td>
<td>-23.92*</td>
<td>316.32</td>
<td>478.63</td>
</tr>
<tr>
<td>$V^{(e)}$</td>
<td>3.75211</td>
<td>0.63543</td>
<td>1.00*</td>
<td>36.44*</td>
<td>482.03</td>
<td>-496.74</td>
</tr>
<tr>
<td>$V^{(m)}$</td>
<td>2.11674</td>
<td>4.41941</td>
<td>0.68061</td>
<td>36.44*</td>
<td>482.03</td>
<td>-496.74</td>
</tr>
</tbody>
</table>

Table III. The source size parameters ($A_1, A_2$) for various reactions and ($A_N, \beta$).

<table>
<thead>
<tr>
<th>reactions</th>
<th>source size parameters</th>
<th>$A_N, A_Y$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NN \rightarrow NN$</td>
<td>$A_1 = A_2 = A_N + A_{ij}$</td>
<td>$A_N = 1815.11$</td>
</tr>
<tr>
<td>$NY \rightarrow NY$</td>
<td>$A_1 = A_N + A_{ij}, A_2 = A_Y + A_{ij}$</td>
<td>($\beta = 0.03227$)</td>
</tr>
<tr>
<td>$NY \rightarrow YN$</td>
<td>$A_1 = A_2 = (A_N + A_Y)/2 + A_{ij}$</td>
<td>$A_{\Sigma} = A_\Lambda = 1756.54$</td>
</tr>
<tr>
<td>$YY \rightarrow YY$</td>
<td>$A_1 = A_2 = A_Y + A_{ij}$</td>
<td>$A_{\Xi} = 1697.96$</td>
</tr>
</tbody>
</table>

Table IV. Baryon-baryon-meson coupling constants. ($m$ indicates the meson.)

<table>
<thead>
<tr>
<th>$m$</th>
<th>coupling</th>
<th>$\Sigma^+ \Sigma^+ m(p\Sigma^+ m)$</th>
<th>$\Lambda Am(\Lambda \Sigma^+ m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$3.40906$</td>
<td>$2.32496$</td>
</tr>
<tr>
<td>$a_0$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$1.39734$</td>
<td>$2.37989$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$-0.29880$</td>
<td>$-1.61066$</td>
</tr>
<tr>
<td>$K_0^*$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$(-2.18079)$</td>
<td>$(-1.38954)$</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$0.09544$</td>
<td>$0.04045$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$0.27000$</td>
<td>$0.19170$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$0.11396$</td>
<td>$0.23793$</td>
</tr>
<tr>
<td>$K$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$(-0.26656)$</td>
<td>$(0.11073)$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$3.67223$</td>
<td>$3.01850$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$0.64112$</td>
<td>$-0.34751$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$0.17459$</td>
<td>$-0.11513$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$0.63543$</td>
<td>$1.27086$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$3.78398$</td>
<td>$4.74493$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$5.95499$</td>
<td>$3.73364$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$-1.34327$</td>
<td>$-2.22868$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$3.62157$</td>
<td>$2.28258$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$-2.69608$</td>
<td>$-1.02418$</td>
</tr>
<tr>
<td>$K^*$</td>
<td>$g/\sqrt{4\pi}$</td>
<td>$(-1.10060)$</td>
<td>$(-0.89863)$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$(-4.92417)$</td>
<td>$(-1.35899)$</td>
</tr>
<tr>
<td>$f/g$</td>
<td>$f/\sqrt{4\pi}$</td>
<td>$(4.47409)$</td>
<td>$(1.51228)$</td>
</tr>
</tbody>
</table>

the above-mentioned six constraints. For the baryon-baryon reactions, the source size parameters and ($A_N, \beta$) are given in Table III. The effective source size parameter, $\Lambda = A_1 A_2/\sqrt{A_1^2 + A_2^2}$ corresponds to the cutoff parameter of the Nijmegen soft-core OBEP.\textsuperscript{5}

The meson-baryon coupling constants were calculated using our $SU(3)$ parameters with the relations given in Appendix A of Ref. 1). The values obtained for the coupling constants between $p$, $\Sigma^+$, $\Lambda$ and the mesons are listed in Table IV.
4.2. Comparison with the coupling constants of GSOBEP and other models

From the quark model prediction, the coupling relations for the vector mesons are as follows:

\[ g_{\Sigma\Sigma\omega} = g_{\Lambda\Lambda\omega} = \frac{2}{3} g_{NN\omega}, \quad g_{\Sigma\Sigma\phi} = g_{\Lambda\Lambda\phi}, \quad g_{NN\phi} = 0, \quad (f/g)_{NN\rho} \approx 5. \]  (4.1)

The obtained values are as follows:

\[ g_{\Sigma\Sigma\omega} = g_{\Lambda\Lambda\omega} \approx \frac{4}{5} g_{NN\omega}, \quad g_{\Sigma\Sigma\phi} = g_{\Lambda\Lambda\phi}, \quad g_{NN\phi}/\sqrt{4\pi} \approx -1.34, \quad (f/g)_{NN\rho} \approx 6.0. \]  (4.2)

Note that, in particular, the value \( g_{NN\phi}/\sqrt{4\pi} \approx -1.34 \) is somewhat large compared with those of the quark model prediction. The coupling ratio \((f/g)_{NN\rho}\) plays an important role in the study of the electromagnetic form factor of the nucleon, and it is similar to that obtained with the quark model.

The main part of the repulsive core originates from the \( \omega \) meson exchange. Regarding the \( \omega \) coupling constant, we have

\[ g_{NN\omega}^2/4\pi = 13.49(8.68), \quad (f/g)_{NN\omega} = 0.17(0.31), \]  (4.3)

where the values used in the Nijmegen soft-core OBEP (NSC89) appear in parentheses. The value of \( (f/g)_{NN\omega} \) is small, and this shows that the direct coupling gives the main contribution. For the pseudoscalar \( K \) meson exchange, we have

\[ f_{p\Lambda K}^2/4\pi = 0.071(0.077), \quad f_{p\Sigma K}^2/4\pi = 0.012(0.005). \]  (4.4)

These values are similar to those from the \( K^\pm N \) forward dispersion relation given in parentheses. For the vector \( K^* \) meson, we have

\[ (f/g)_{p\Lambda K^*} = 4.474(4.564), \quad (f/g)_{p\Sigma K^*} = 1.512(2.143), \]  (4.5)

where the values for the FG potential appear in parentheses. It is seen that there is a large discrepancy in the value of \( (f/g)_{p\Sigma K^*} \).

For the \( YY\pi \) coupling, we have

\[ f_{A\Sigma\pi}^2/4\pi = 0.0404(0.0261), \quad f_{B\Sigma\pi}^2/4\pi = 0.0367(0.072), \]  (4.6)

where the values from Ref. 38) are given in parentheses. The values in NSC89 are \( f_{A\Sigma\pi}^2/4\pi = 0.0411 \) and \( f_{B\Sigma\pi}^2/4\pi = 0.0373 \), which are the same as those of GSOBEP.

\section*{§5. Calculations and discussion}

5.1. \( NN \) results

The calculated phase shifts are shown in Figs. 4, 5, 6 and 7 for the singlet even \((1E)\), singlet odd \((1O)\), triplet even \((3E)\) and triplet odd \((3O)\) states, respectively. The experimental values with error bars are taken from the latest single-energy phase shift analysis of Arndt et al.\textsuperscript{18} The phase shifts for the \( 1E \) state are given in Fig. 4. We searched for the parameter values for the \( pp \) system. However, the values for the \( 1S_0 \) state were calculated for both the \( pp \) and \( pn \) systems. We find that the fitting is very good for the phase shifts of the \( 1S_0 \) and \( 1D_2 \) states.
Table V. $NN(1S_0)$ low energy parameters in fm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calc.</th>
<th>Exp. $^{41}, 42$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_s(pp)$</td>
<td>$-7.7602$</td>
<td>$-7.823\pm0.0023$</td>
</tr>
<tr>
<td>$r_s(pp)$</td>
<td>$2.648$</td>
<td>$2.794\pm0.01$</td>
</tr>
<tr>
<td>$a_s(np)$</td>
<td>$-20.065$</td>
<td>$-23.748\pm0.010$</td>
</tr>
<tr>
<td>$r_s(np)$</td>
<td>$2.646$</td>
<td>$2.75\pm0.05$</td>
</tr>
</tbody>
</table>

The calculated low energy parameters, the scattering length $a_s$ and the effective range $r_s$ are shown in Table V. These parameters for the $pp$ system are in good agreement with the experimental values, $^{41}$ but there are some discrepancies for the $np$ system. $^{42}$

The phase shifts for the $1O$ state are given in Fig. 5. The fitting is found to be poor for the $1P_1$ state at high energies. The phase shift for the $1F_3$ state is omitted because of a lack of phase shift analysis data. $^{18}$

For the $3E$ states, the obtained phase shifts are shown in Figs. 6 and 7. Excellent fits for the $3S_1$ and $3D_1$ states are obtained. The mixing parameter $\epsilon_1$ is also in good agreement. The result for the $3D_3$ state at high energies is not so good.

The triplet-even low energy parameters, such as the scattering length $a_t$ and the effective range $r_t$, and the deuteron properties, such as the binding energy $E_B$, the quadratic moment $Q_d$, and the $D$-state probability $P_D$, are listed in Table VI. These
values reproduce the experimental data very well. Our value of $P_D$ is somewhat large compared with those obtained from other models.

For the $^3O$ states, the obtained phase shifts are in good agreement with the experimental values shown in Figs. 8 and 9. We find an excellent fit for the $^3P_J$ states. Especially, the $^3P_2$ state is important for the superfluid properties in the high energy region.\cite{43) GSOBEP yields large positive values of the $^3P_2$ phase shift in the region 300–500MeV. These values are similar to those given in Ref. 43). The $^3F_J$ phase shifts and $\epsilon_2$ are shown in Fig. 9 and found to be close to the experimental data.

5.2. YN results

Using the obtained GSOBEP, we calculated the integrated cross sections, the differential cross sections and the inelastic capture ratios. Here we compare the results to the experimental data. We obtain good agreement on the whole. In addition, we calculated the $S$- and $P$-wave phase shifts in the $YN$ scatterings and the $S$-wave phase shifts in the $YY$ scatterings. These results give some insight regarding the properties of light hypernuclei\cite{44) and double hypernuclei. We find that our potential is qualitatively consistent with the light hypernuclei and recent data for double hypernuclei.\cite{25} It should be noted that our new potentials give considerably better descriptions in the $YN$ and $YY$ channels than the previous FG potential, in spite of the fact that there are fewer parameters in the short-range part. The effective OBEP proposed recently by Tominaga and Ueda\cite{45) gives results similar to ours.

(1) $Ap$ scattering

The $Ap$ elastic scattering total cross sections up to $p_A = 1$ GeV/$c$ are shown
in Figs. 10 and 11 with the experimental data by the Heidelberg,\textsuperscript{19} Maryland\textsuperscript{20} and Berkeley\textsuperscript{21} groups. A good fit to the low energy \( \Lambda p \) data is provided by our GSOBEP. The large cusp of about 45 mb at the \( \Sigma^+ n \) threshold energy shown in Fig. 11 is due to the enhancement in the \( ^3S_1 \) state and the strong \( ^3S_1 - ^3D_1 \) tensor coupling.

The \( S \)- and \( P \)-wave phase shifts are shown in Figs. 12 and 13, respectively. We find that the potential of the \( ^1S_0 \) state is more attractive than that of the \( ^3S_1 \) state.\textsuperscript{44} In fact, the GSOBEP gives a value of about 10 degree as the \( ^1S_0 - ^3S_1 \) phase shift difference at \( p_{\text{Lab}} = 200 \) MeV/c, which is large in comparison with those predicted by some other potential models.\textsuperscript{5,7} However, the result obviously depends on the model used. The GSOBEP is weakly repulsive for the \( ^1P_1, ^3P_0 \) and \( ^3P_1 \) states, and attractive for the \( ^3P_2 \) state. The scattering lengths and effective ranges of \( \Lambda p \ (^1S_0) \) and \( \Lambda p \ (^3S_1) \) are shown in Table VII.

\begin{table}[h]
\centering
\caption{\( \Lambda p \) effective range parameters in fm.}
\begin{tabular}{|c|c|}
\hline
 & Calc. & Calc. \\
\hline \( a_s \) & $-2.70$ & $-1.47$ \\
\hline \( r_s \) & $2.92$ & $3.52$ \\
\hline
\end{tabular}
\end{table}
(2) \( \Sigma^+ p \) scattering

The integrated cross sections for the \( \Sigma^+ p \rightarrow \Sigma^+ p \) reaction are shown in Fig. 14. The calculated cross sections agree with the experimental data of the Heidelberg group, except in the high energy region. The differential cross sections for the \( \Sigma^+ p \rightarrow \Sigma^+ p \) reaction at \( p_{\Sigma^+} = 170 \text{ MeV}/c \) and the KEK data at \( p_{\Sigma^+} = 400 \text{ MeV}/c \) are shown in Figs. 15 and 16, respectively. The KEK data fit well to our GSOBEP as well as the FG potential.

The \( S \)- and \( P \)-wave phase shifts are shown in Figs. 17 and 18, respectively. We find that the GSOBEP for the \( ^1S_0 \) state is attractive and that for the \( ^3S_1 \) state is somewhat repulsive. For the \( ^1P_1 \) and \( ^3P_2 \) states, the GSOBEP is attractive and for the \( ^3P_0 \) and \( ^3P_1 \), it is repulsive. The FG potential have the same tendency.

(3) \( \Sigma^- p \rightarrow \Sigma^- p \) scattering

The fit of the integrated cross sections of the Heidelberg group is shown in Fig. 19. The agreement is very good. We compare the differential cross sections at \( p_{\Sigma^-} = 160 \text{ MeV}/c \) with the Heidelberg data in Fig. 20, and find improvement.
over the result obtained with the FG potential. However, the agreement is poor at cos θ = ±0.5.

(4) Σ⁻⁻p → Λn scattering

We calculated the integral cross sections of Σ⁻⁻p → Λn and the differential cross sections at pΣ⁻⁻ = 160 MeV/c using GSOBEP and compare these results with the experimental data²³ in Figs. 21 and 22, respectively. Good fits are obtained for both sets of data.

(5) Σ⁻⁻p → Σ⁰n scattering

The calculated integral cross sections of Σ⁻⁻p → Σ⁰n are compared with the data of the Heidelberg group²³) The calculational result fits the experimental data, except at pΣ⁻⁻ = 110 MeV/c, as shown in Fig. 23.

(6) Inelastic capture ratio

The inelastic capture ratio at rest is defined as

\[ r_R = \frac{1}{4} \frac{\sigma_s(\Sigma^-p \rightarrow \Sigma^0n)}{\sigma_s(\Sigma^-p \rightarrow \Lambda n) + \sigma_s(\Sigma^-p \rightarrow \Sigma^0n)} + \frac{3}{4} \frac{\sigma_t(\Sigma^-p \rightarrow \Sigma^0n)}{\sigma_t(\Sigma^-p \rightarrow \Lambda n) + \sigma_t(\Sigma^-p \rightarrow \Sigma^0n)}, \]

(5.1)
where $\sigma_s$ and $\sigma_t$ are the cross sections for the $^1S_0$ and $^3S_1$ states, respectively. The calculated capture ratio at rest for $\Sigma^-p$ reactions with the experimental data$^{27)-29)}$ is given in Table VIII. We obtain good agreement, which is comparable to that obtained with some other potential models.$^{5),46)}$ These values are important to determine the $YNm$ and $YYm$ coupling constants.

(7) Contributions of the anti-symmetric spin-orbit potential

The mass differences among the octet baryons and the differences between coupling constants, $g_{13}f_{24} - g_{24}f_{13}$ lead to the anti-symmetric spin orbit potential $V_{ALS}$ in the $NN$ and $YN$ systems. In the $NN$ system, the mass difference between $p$ and $n$ is very small, and as a result $V_{ALS}$ is negligibly small. However, since the mass differences between $N$ and $Y$ are about 180–380 MeV, $V_{ALS}$ is expected to become important in the $YN$ systems. $V_{ALS}$ contribute to the same $L$ and $J$ of the singlet-triplet coupling, such as $^1LJ-^3LJ$ coupling. As an example, the mixing

\begin{table}[h]
\centering
\caption{Inelastic capture ratio $r_R$.}
\begin{tabular}{|c|c|}
\hline
Calc. & Exp. \\
\hline
0.479 & 0.33$\pm$0.05$^{27)}$ \\
0.479 & 0.474$\pm$0.016$^{28)}$ \\
0.465 & 0.465$\pm$0.011$^{29)}$ \\
\hline
\end{tabular}
\end{table}
parameter $\rho_1$, which arises from the $^1P_1$-$^3P_1$ coupling, for the $\Sigma^+p$ and $Ap$ reactions are shown in Fig. 24. We can see that $V_{ALS}$ plays an important role; i.e. it contributes attractively to the $Ap$ reaction but repulsively to the $\Sigma^+p$ reaction. In the hyperon reactions, the determination of the contribution of the anti-symmetric spin orbit potential is important, as is the spin orbit potential.

5.3. $YY$ results

Some comments are now given regarding the $\Lambda\Lambda(^1S_0)$ scattering (strangeness $S = -2$) in connection with $^6\Lambda\Lambda$He and the “Nagara” double $\Lambda$ event. Takatsuka et al. calculated the $^1S_0$ phase shifts of the $\Lambda\Lambda$ scattering using various potentials considering the “Nagara” double $\Lambda$ event and proposed the $\Sigma^-\Sigma^-$ and $\Xi^-\Xi^-$ phase shifts. We have calculated the $^1S_0$ phase shifts for the $\Lambda\Lambda$ scattering by solving the coupled-channel equations, reproducing the results of Ref. 12). We also calculated the $^1S_0$ phase shifts for the $\Sigma^-\Sigma^-$ and $\Xi^-\Xi^-$ scatterings. These results are shown in Fig. 25. The $\Lambda\Lambda$ phase shifts calculated without channel cou-
Gaussian Source One-Boson-Exchange Potential

Fig. 28. The retarded effects on the $^1S_0$ phase shifts for (a) $pp$, (b) $Ap$ and (c) $ΛΛ$ scatterings. Solid (dotted) lines are the results obtained with (without) the retarded potentials in GSOBEP.

dpling differ significantly from those with the coupling, as shown in Fig. 25. It is seen that the channel coupling is essential to provide large (attractive) $ΛΛ$ phase shifts.

In order to investigate the properties of hyperon matter, the attractive effect of the GSOBEP is important to determine the $^1S_0$ phase shifts. The potentials of $ΛΛ$, $Σ^−Σ^−$ and $Ξ^−Ξ^−$ in the $^1S_0$ states are shown in Fig. 26. The potential of $ΛΛ$ at long range is negative and fairly weak.

5.4. Retarded effects

The static parts of GSOBEP for the $ΛΛ$ ($^1S_0$), $Ap$ ($^1S_0$) and $pp$ ($^1S_0$) states are shown in Fig. 27. Comparing to the potentials for these states, we obtain $|V(ΛΛ)| < |V(Ap)| < |V(pp)|$ in the outer region (in which $r > 0.5$ fm). This tendency is similar to that obtained in Ref. 47).

Nonstatic effects, especially the retarded potentials $U$, are essential in our potential. The retarded potentials come from the off-energy shell of the propagators, and they seem to have large effects, especially for the $S$ states. The retarded effects are estimated using the difference between phase shifts calculated with and without $U$. The results for $pp$, $Ap$ and $ΛΛ$ in the $^1S_0$ states are shown in Fig. 28. We find the retarded effects are attractive and fairly strong.

§6. Summary

We constructed the $SU(3)$ symmetric GSOBEP with the exchange of the scalar, pseudoscalar and vector meson nonets. The GSOBEP in which we assume the statistical distribution of the source takes the form of the Yukawa function in the outer region and the Yukawa function multiplied by the error function in the core region, and it possesses soft behavior of the central, tensor, spin-orbit and quadratic spin-orbit potentials in the core region. It is a very important result that the extended source generates a soft core potential.

The GSOBEP constructed from Gaussian sources has no singularity near the origin and provides smooth potential functions. The GSOBEP can be treated from the core region to the long-range region on the same footing. The GSOBEP has ten free $SU(3)$ parameters and eight free source size parameters. The total number
of free parameters is smaller than those in most other potential models. Further, the source size parameters can be reduced to only two parameters, \( \Lambda \) and \( \beta \). The resultant GSOBEP reproduces the \( NN \), \( YN \) and \( YY \) experimental values very well.

We expect that very accurate \( YN \) and \( YY \) data will be obtained in the future, and we hope to apply our GSOBEP more precisely to reproduce such data concerning the strength (coupling constant) and size of the Gaussian source. We also expect that the mechanism of repulsive core will become clear through analysis of high energy data.

As the next step, the nonstatic effects including retarded and relativistic effects will be estimated by calculating properties of hypernuclei with the GSOBEP. In order to estimate the nonstatic effects of the GSOBEP more precisely, the GSOBEP in \( p \)-space will be applied to two-baryon systems by carrying out numerical calculations without any approximation.

Acknowledgements

We would like to thank Professor T. Takatsuka and Dr. R. Tamagaki for their continued interest and information regarding the \( YY \) systems. Also, we thank Professor S. Abe for his useful comments on the Gaussian source.

References

18) R. A. Arndt et al., SAID program, access via TELNET under clsaid.phys.vt.edu (login: SAID).
Gaussian Source One-Boson-Exchange Potential