For a reference dimension of 2.5 ft impeller diameter

\[ (M_0)_L = \frac{6000 \times 2.5}{60 \sqrt{(1.26 \times 32.2 \times 0.90 \times 95.6 \times 530)} = 0.487 } \]

\[ (M_0)_H = 0.487 \times 1.02 = 0.497 \]

If the air-inlet temperature is 80°F

\[ \frac{(RPM) \text{ air}}{2.5} = 0.497 \times 60 \sqrt{(1.4 \times 32.2 \times 53.3 \times 540)} = 4330 \]

Hence we conduct the air test at 4330 rpm, calculate the data in terms of \( \varphi \), \( \psi \), and \( \eta \); then convert these figures using the gas properties to the given condition for the final head-capacity-efficiency curves. In this example, the difference between design and test values of \( R_m \) is sufficient to consider a separate correction for Reynolds-number effects according to reference (3). It is to be noted that in this case, the speed coefficients are unequal by the small amount of 2.0 per cent, even though the \( k \)-values for the design gas and the test gas are substantially different.

Fig. 5 has been prepared to illustrate the approximate relations of test speeds to design speed for various gases, assuming that it is intended to conduct the test with air at inlet temperature of 80°F. Gases having a Class I or II relationship with air are included. The approximate nature of the curves arises from possible variations in \( k \) and \( Z \) for the various gases when their behavior is outside the perfect gas range.

**Conclusion**

In conclusion, the paper has presented the parameters of equivalence, conditions and classifications of correlation, and numerical examples of their use. Limitations of correlation procedures are discussed. It is hoped that these concepts can be studied by an appropriate group for the purpose of adapting them for use in future test codes for gas compressors.

**Acknowledgment**

The author wishes to acknowledge gratefully the contributions of F. C. Gilman, L. N. Tao, and C. A. Macaluso, all of the Wroughton Corporation, whose comments and criticisms aided in the preparation of this paper.

**BIBLIOGRAPHY**


**Discussion**

W. A. Clark. The author has presented an excellent paper on equivalent performance parameters. However, the writer would like to present some criticism and some additional information on this subject.

To begin with, the whole basis of an equivalent gas test depends upon the head coefficient \( \psi \), and the compressor efficiency \( \eta \), remaining the same for the design gas and the test gas. The paper proposes that if the ratio of the various parameters is kept close to unity, then \( \psi \) and \( \eta \) will remain the same. However, no proof from actual tests has been presented. It is the writer's opinion that proof of this last fact should be established definitely by actual test before any recognition by the Power Test Code is given.

The paper barely touches upon water-cooled blowers and rightfully so. It is stated that where water cooling is present the heat flows must be controlled to provide identity of temperature ratios between significant points. From a practical testing standpoint this is nearly impossible because of the usual types of internal cooling employed, together with the fact that the heat flows desired must be predetermined. Then, if the blower performance is not exactly as designed, the determined relative heat flows are in error. In a blower with many stages a small heat-flow error in the first few stages will become one of large consequence in the latter stages. About the only way to obtain the true performance of a water-cooled blower is to test with the design gas at the design-inlet conditions and speed and maintain the design cooling conditions. In all cases the cold temperature difference between inlet-gas and inlet-water temperatures should be maintained. For most cases this can be accomplished by use of a closed-loop test setup.

One small error in the calculation of Example 2 should be pointed out. The ratio of \( \frac{(M_0)_{\psi}}{(M_0)_{\psi}} \) is in error. This ratio actually should be 1.07 instead of 1.04, which shows the speed coefficients unequal by 3.5 per cent instead of 2.0. This will make the determined air speed become 4390 rpm in place of the given 4330 rpm.

The main item to be pointed out is that the given paper covers a very limited number of blowers as far as equivalent air testing is concerned. When a design gas is heavier than air it is usually impossible to run a blower on air at a speed in excess of the rated speed. On the other hand, a very light gas, hydrogen in particular, for which there are many blowers being built today, will result in such a large reduction in speed that the power measurement becomes inaccurate. This in turn requires a horsepower correction to the design condition which is impracticable.

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1. Test Engineer, Ingersoll-Rand Company, Phillipsburg, N. J.
From a designer's standpoint and from a manufacturing standpoint, it is much more desirable to test a blower at full speed or at least within 10 per cent of the rated speed. In order to do this and still maintain an equivalent performance, the use of an equivalent gas mixture can be selected and used in a closed-loop test circuit. This, of course, appears to be a costly process; however, the additional cost usually can be justified, especially in the case of a motor-driven blower where the speed cannot be varied without a special test setup. Also, in the case of a blower with a high design-inlet pressure, a closed loop will provide the means to adjust the inlet density to produce a desired power measurement.

The use of a gas mixture in a closed loop has been used many times with moderate success. In one case a comparison of actual test results showed the head coefficients to duplicate each other, but the efficiencies, although fairly close, showed a difference. This difference was attributed to the fact that the air test at such a large speed did not afford as accurate a power measurement as with the full-speed test.

For the question of how to determine an equivalent gas mixture for a particular blower, the derivation of a formula is being given along with examples of its use. The formula is designed for determining an equivalent gas mixture to produce a design-volume ratio at design speed and intake temperature of a blower.

**DERIVATION OF FORMULA TO DETERMINE AN EQUIVALENT GAS MIXTURE TO PRODUCE A RATED VOLUME RATIO AT RATED SPEED AND INTAKE TEMPERATURE OF A BLOWER**

Rated volume ratio is defined as the ratio of the inlet volume rate divided by the discharge volume rate. As the inlet and discharge areas, or volumes, are fixed by the blower design, this ratio also could be called the rated velocity ratio. In order to have this ratio the same for the test mixture as for the rated gas, the following must be true

\[ \frac{V_1}{V_2} = \left( \frac{p_2}{p_1} \right)^{1/n} \]

from

\[ p^{1/n} = pV \]

then \( R^{1/n} \) of the design gas = \( R^{1/n} \) of the mixture gas

**Definitions of Terms**

- \( k = c_p/c_v \): By definition
- \( n = \) polytropic exponent
- \( \eta = \) polytropic efficiency
- \( \frac{n}{n-1} = \frac{k}{k-1} \times \eta \) By definition
- \( M = \) molecular weight
- \( X = \) fractional part—by volume
- \( R = \) \( p_v/p_i \): Compression ratio
- \( b = \frac{n}{n-1} \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] \) By definition
- \( W = \) impeller peripheral velocity, fps
- \( \mu = \frac{1716 \times T \times b}{\Sigma W^2 \times sp\text{gr}} \): Coefficient of pressure rise

**Subscripts**

- \( d = \) design conditions
- \( m = \) test gas mixture
- 1 = 1st gas of mixture
- 2 = 2nd gas of mixture

**Derivation**

\[ M_n = X_1M_1 + (1.00 - X_1)M_2 \]

Assuming \( \mu \) of the blower will be the same with the mixture as with the design gas

\[ \mu_d = \mu_m \]

\[ \frac{b_d}{M_d} = \frac{b_m}{M_m} \text{ or } M_m = \frac{b_m}{b_d} M_d \]

By substitution

\[ b_m \cdot M_d = X_1M_1 + 1.00 M_2 - X_1M_2 \]

as

\[ b = \frac{n}{n-1} \left[ \left( \frac{p_2}{p_1} \right)^{n-1} - 1 \right] \]

By substitution

\[ \frac{n_m}{n_d - 1} \left[ \left( \frac{R_{m}}{R_{d}} \right)^{n_m - 1} - 1 \right] \]

\[ M_d = X_1M_1 - X_1M_2 + M_2 \]

Assuming \( \eta \) of the blower will be the same with the mixture as with the design gas and

\[ \frac{n}{n-1} = \frac{k\eta}{k-1} \]

\[ \frac{n-1}{n} = \frac{k-1}{k\eta} \]

\[ \frac{1}{n} = \frac{k-1}{k\eta} \]

By substitution

\[ \frac{k_n \eta}{k_m \eta} \left[ \left( \frac{1 - k_d^{-1}}{1 - k_m^{-1}} \right) ^{k_m-1} - 1 \right] \]

\[ M_d = X_1(M_1 - M_2) + M_2 \]

To use this formula, the properties of the substitute gases must be known. Then by trial and error, various \( k \)-values of the substitute mixture must be tried and the equation solved for \( X \), the fractional volume of the substitute gases. With the fractional volumes known, the \( k \)-value can be computed to show whether the first \( k \)-value assumption was correct.
EXAMPLE TO SHOW USE OF FORMULA

Mixture Gases

<table>
<thead>
<tr>
<th>Design gas</th>
<th>Air (Gas 1)</th>
<th>Helium (Gas 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>6.0</td>
<td>28.96</td>
</tr>
<tr>
<td>$k$</td>
<td>1.4</td>
<td>1.67</td>
</tr>
</tbody>
</table>

The mixture gases were picked such that one gas has a higher molecular weight than the design gas and the other a lower molecular weight. As air is abundant and its use eliminates tedious purging processes, it was selected as the heavy gas. Because of the inflammable hazards connected with hydrogen it would not be permitted in most shops. Therefore the only other gas with a mol. weight less than 6.0 is helium.

On the basis of the molecular weights it can be seen that a larger volume of helium will have to be used and thus the $A$-value will be closer to 1.67 than to 1.4 for the test mixture.

For the purpose of the example, however, a mixture value of 1.5 will be tried.

**Trial No. 1, $k = 1.5$**

<table>
<thead>
<tr>
<th>Design</th>
<th>Mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.68</td>
</tr>
<tr>
<td>$k$</td>
<td>1.4</td>
</tr>
<tr>
<td>[ \frac{k - 1}{k} ]</td>
<td>2.38</td>
</tr>
<tr>
<td>[ \frac{k - 1}{k \eta} ]</td>
<td>0.4205</td>
</tr>
<tr>
<td>$R$</td>
<td>1.442</td>
</tr>
</tbody>
</table>

Using Equation [1]

\[
X_1 = 0.121 \text{ or } 12 \text{ per cent air} \quad X_2 = 88 \text{ per cent helium}
\]

To prove: If first assumption of $k = 1.5$ is correct

\[
\frac{k}{k - 1} \text{ mixture} = (\text{fractional volume gas 1}) \left( \frac{k}{k - 1} \right) \text{ gas 1}
\]

\[
+ (\text{fractional volume gas 2}) \left( \frac{k}{k - 1} \right) \text{ gas 2}
\]

\[
\frac{k}{k - 1} \text{ mixture} = 0.169 \times 3.500 + 0.831 \times 2.500
\]

\[
k = 2.629 \quad k = 1.60
\]

As this value does not check the 1.5 of first trial, a second trial must be made. It is now known that the correct $k$-value will be between 1.5 and 1.62.

**Trial No. 2, $k = 1.60$**

<table>
<thead>
<tr>
<th>Design</th>
<th>Mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.68</td>
</tr>
<tr>
<td>$k$</td>
<td>1.4</td>
</tr>
<tr>
<td>[ \frac{k - 1}{k \eta} ]</td>
<td>2.38</td>
</tr>
</tbody>
</table>

Using Equation [1]

\[
X_1 = 0.109 \text{ or } 16.9 \text{ per cent air} \quad X_2 = 83.1 \text{ per cent helium}
\]

To prove, if $k = 1.60$ is correct

\[
\frac{k}{k - 1} \text{ mixture} = (\text{fractional volume gas 1}) \left( \frac{k}{k - 1} \right) \text{ gas 1}
\]

\[
+ (\text{fractional volume gas 2}) \left( \frac{k}{k - 1} \right) \text{ gas 2}
\]

\[
\frac{k}{k - 1} \text{ mixture} = 0.169 \times 3.500 + 0.831 \times 2.500
\]

\[
k = 2.69 \quad k = 1.60
\]

This checks the second trial $k = 1.60$ is correct.

With this known, other properties of the mixture can be determined.

**Molecular Weight of Mixture**

\[
M_1 \text{ (air part of mixture)} = 0.169 \times 28.96 = 4.89
\]

\[
M_2 \text{ (He part of mixture)} = 0.831 \times 4.00 = 3.32
\]

Molecular weight of mixture $= 8.21$

**Compression Ratio for Test Using Mixture**

\[
\frac{n}{n - 1} \text{ (mix)} = \frac{k}{k - 1} \text{ (mix)} \times \eta = 2.669 \times 0.68 = 1.818
\]

\[
n \text{ mixture} = 1.818 \quad 0.818 = 2.22
\]

As

\[
\frac{1}{R_m} = \frac{1}{R_m^{m_{\text{mix}}}} = \frac{1}{R_m^{2.22}}
\]

\[
R_m = 1.602
\]

From the compression ratio and the molecular weight the desired inlet density can be predetermined to give the approximate power requirement.

**AUTHOR’S CLOSURE**

The author is gratified to have Mr. Clark’s comments and especially appreciates the illustration of the calculations for determining equivalent gas mixtures.

It is definitely agreed that closed loop testing is a practical means of determining compressor performance and in some cases it is the only means for obtaining a significant test, as Mr. Clark points out. The advantage of this method is that a test gas with a Class II or a Class I relation to the design gas can be used, and that the density may be adjusted to produce satisfac-
tory power measurement as well. It is the author's feeling that this method illustrates how significant tests can be performed following the principles of the correlation classifications outlined in the paper.

The question of how we may be sure that the head coefficient $\psi$ and the efficiency $\eta$ will be duplicated when the other parameters are duplicated seems to rest only on the matter of the completeness of the list of independent parameters. If the independent parameters of Table 1 are conceded to be complete, then it undoubtedly is true that $\psi$, $\eta$, and the other dependent parameters will be duplicated.

The author's tests on refrigeration compressors with both Freon-11 and Freon-114 indicate very good agreement on this matter. For volume ratios $Q_2/Q_1$ up to 3.5 the $\varphi-\psi$ and $\varphi-\eta$ curves for the two gases at the same volume ratios were not different by more than 1 per cent. The molecular weights were 137.38 and 170.93, respectively, and the speeds for equal volume ratios were different by 17 per cent.