fect of positive density gradients in such centrifugal field could be vertified from Synge's [5] and Kurzweg's[5] work. The rate of entrainment, equation (30), clearly shows all such effects.

Finally, the increased rate of decay of tangential velocity with increased buoyancy could be explained as follows. The spread of the swirling plume increases with increasing buoyancy at a given initial swirl parameter. This rapid spread causes the tangential velocity to decay faster. The increase in axial velocity with increasing buoyancy is predominantly a buoyancy effect and is least related to tangential velocity decay. Lee's calculations show a reduction in the decay of tangential velocity at small downstream distances and an increase in decay of tangential velocities at large downstream distances with increasing buoyancy. If the buoyancy effect such as the second term in equation (32) is included in Lee's analysis, the results would show a monotonic increased rate of decay of tangential velocity with increased buoyancy.

[5] Kurzweg, U. H., "A Criterion for the Stability of Heterogeneous Swirling Flows," *Zeitschrift für angewandte Mathematik und Physik*, Vol. 20, 1969, pp. 141-143.

count accurately for such microstructural changes; for discussions and references see, for example, [1-5].[3]

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[3] Numbers in brackets designate References at end of Note.

On Finite Elastic-Plastic Deformation of Metals[1]

S. Nemat-Nasser.[2] The authors assume that the Helmholtz free energy for a class of materials which admit elastoplastic deformation, can be written as a function of total (Lagrangian) strain **E**, "plastic strain" $\mathbf{E}_p$, as well as work-hardening parameter κ, and temperature θ:

$$\psi = \hat{\psi}(\mathbf{E}, \mathbf{E}_p, \kappa, \theta). \tag{15}$$

Then the stress tensor is obtained from their equation $(16)_1$.

I wish to make a few remarks on this representation.

It is physically impelling to assume that free energy is affected by temperature and elastic strains. Moreover, if work-hardening occurs by entanglement of dislocations and other microstructural rearrangements, one may include (approximately) its effect by means of one or several additional parameters, such as κ in (15). The question is, having taken care of work-hardening already, could one assume that the free energy is, in general, an explicit function of *macroscopic* plastic strains.

If the answer is affirmative, then, in view of the fact that the plastic strain is an *independent* variable in representation (15), one could, for example, set the other independent variables at fixed values within the range of their variation, and then study the effect of plastic strain on the free energy. In particular, one can set $\theta = \theta_0$, $\kappa = 0$, and fix the total strain in the manner which corresponds to either a constant, or zero elastic strains. *Since work-hardening is taken to be zero,* one can envisage large plastic deformations in which layers of atoms slide over layers of atoms, without trapped dislocations (hence no locked-in elastic strains), resulting in practically no change in the free energy of the body; for example, finite plastic shear of a block of a perfect crystal, where the horizontal layers of atoms are shifted as a group, with no relative motion of atoms within the group.

It may be argued that plastic deformation causes microscopic structural changes, hence giving rise to *locked-in microscopic elastic* strains, and in this manner affect the free energy. This is a sound argument. But in a general case, one may need more (or sometimes less) parameters (hidden coordinates) in order to account accurately for such microstructural changes; for discussions and references see, for example, [1-5].[3]

[1] By P. M. Naghdi and J. A. Trapp, published in the March, 1974, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 41, No. 1, TRANS. ASME, Vol. 96, Series E, pp. 254-260.

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A Refined Theory for Laminated Anisotropic, Cylindrical Shells[1]

C. W. Bert.[2] The authors are to be highly commended for presenting a relatively accurate analysis applicable to quite thick cylindrical shells constructed of arbitrarily laminated composite materials. They are to be especially commended for comparing numerical results based on their analysis with those of a more accurate theory (elasticity) and a number of less accurate, but more simplified theories. However, the discusser would like to make a few comments.

First, if one starts from the orthogonal curvilinear-coordinate strain equations given in Love's book [1][3] or those in tensor form in Fung's book [2], one obtains equations which are identical to those of equations (5).

Second, when the strain relations, equations (4) and (5), are substituted into the definitions of the stress resultants and stress couples, equation (1), one obtains numerous higher-order terms. Even when one neglects the thickness-shear and thickness-normal deformation effects (i.e., when ψ_x, ψ_θ, ψ_z, and ϕ_z are all zero), there result terms of higher order than those containing the A_{ij}, B_{ij}, and D_{ij} stiffness coefficients as defined in equations (9). These terms contain coefficients of the form

$$F_{ij} = \int_{-h/2}^{h/2} C_{ij} z^3 dz \tag{1}$$

Specifically, in cylindrical shells, the following F_{ij} terms are present: F_{11}, F_{16}, and F_{66}. These terms were first included in the analysis by Cheng and Ho [3].

Third, there is a small point regarding the authors' use of Mindlin's value $(\pi/\sqrt{12})$ for the correction factors k_i in view of the fact that Mindlin's value was obtained from dynamic considerations and the present application is a static one. It would seem more reasonable to use Reissner's static value (5/6) or better yet to use a value obtained for a flat laminate from equilibrium considerations [4, 5].

[1] By J. M. Whitney and C.-T. Sun, published in the June, 1974, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 41, No. 2, TRANS. ASME, Vol. 96, Series E, pp. 471-476.

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[3] Numbers in brackets designate References at end of Discussion.