Application of a modified Griffith criterion to the evolution of fractal damage during compressional rock failure

Ian G. Main,1 Peter R. Sammonds2 and Philip G. Meredith2
1Department of Geology and Geophysics, Grant Institute, West Mains Road, Edinburgh EH9 3JW, UK
2Department of Geological Sciences, University College London, Gower Street, London WC1E 6BT, UK

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SUMMARY
A modified Griffith criterion for a fractal ensemble of cracks is applied to the interpretation of Acoustic Emission (AE) statistics during the compressional deformation of intact and artificially pre-cut rock specimens in the laboratory. A mean energy release rate per unit crack surface area $\langle G \rangle$ is recovered from the observed AE event rate $N$ and the seismic $b$ value, by calculating an inferred mean crack length $\langle c \rangle$ and measuring the differential stress $\sigma$ for a range of experimental conditions. Temporal variations in $\langle G \rangle$ under compressive deformation show very similar trends to those predicted by a synoptic model determined by direct extrapolation from observations of subcritical crack growth under tension. (In the tensile case, deformation is centred on a dominant macrocrack and the stress intensity $K$, which scales as the square root of $G$, is the relevant measured variable.)

The three independent variables measured during the tests ($\sigma, N, b$) are reduced to points that map out a path through a 3-D phase space $\{(G), N, b\}$, which depends on the material type and the experimental conditions. 2-D slices through this phase space $\{(G), N\}$ and $\{(G), b\}$ are compared with results from the tensile tests $\{K, N\}$ and $\{K, b\}$. The event rate $N$ is found to scale with the square root of $G$ according to a power law, with an exponent $n'$ which is smaller than that for tensile fracture, reflecting the greater stability of compressional rock fracture in its early stages. The effective subcritical crack growth index $n'$ is correlated with the material type and degree of apparent ‘ductility’ on a macroscopic scale, with more brittle behaviour corresponding to higher $n'$. The value of $n'$ is similar on unloading of the stress after dynamic faulting as on the loading portion, though the curve is systematically offset, most probably due to the material weakening associated with faulting. The model does not apply near the period of dynamic failure, where strong local interactions are dominant. The seismic $b$ value is also found to scale negatively with the square root of $G$, in a manner similar to experiments where $K$ can be measured independently.

The acceleration of the mean seismogenic crack length $\langle c \rangle = f(t)$ has a similar power-law form to that predicted from Charles' law for a single tensile macrocrack, with an implied subcritical crack growth index $n$ smaller than that for fracture in compression. The extra dimension introduced by the time dependence of $\langle c \rangle$ allows an independent check on the validity of the theory used to calculate $\langle G \rangle$. In particular $n'$ from the diagram $\{(c), t\}$ is found to be similar in magnitude to the exponent obtained from the event rate dependence $\{(G), N\}$, a phenomenon first discovered by empirical observation of tensile subcritical crack growth.

Key words: damage mechanics, fractals, Griffith cracking, seismicity, stress corrosion.
INTRODUCTION

Rocks fail under compressional loading ultimately by the organized, unstable coalescence of microcracks which initially grow in tensile mode, nucleating first on the weakest elements in the polycrystalline matrix. Initially crack growth is stable and distributed throughout the sample. This implies that the sites of incremental crack growth initially tend to avoid each other, testament to a kind of negative feedback operating at a microscopic level. Thus crack growth is initially energetically favourable, but only for a small increment until the energy balance favours arrest of rupture. In this sense the crack growth is quasi-static and subcritical. The stability may be due to the inherent stability of the tensile relief of differential stress in an overall compressional stress field (Ashby & Sammis 1990); a reduction of the rate of subcritical crack growth as the local stress intensity increases due to crack growth itself, controlled by a characteristic ‘domain’ size (Costin 1983) which might represent a local aureole of stress relaxation; the development of an anelastic process zone of smaller scale damage ahead of the crack tip (Rundle & Klein 1989); the mechanism of dilatant hardening in the presence of a pore fluid (e.g. Scholz 1990), or simply to the material heterogeneity itself as the crack runs into a stronger grain.

Although macroscopically stable and quasi-static, crack growth occurs by increments that may be dynamic enough to be recorded by acoustic emission (AE) monitoring. However, once a critical crack density is attained neighbouring cracks begin to interact and nucleate a larger scale shear or tensile crack through positive reinforcement of the stress field at the crack tip. This linkage is often suggested as a possible mechanism for strain softening, though crack–crack interactions may begin to occur before the peak stress has been attained. Such stable crack linkage ultimately leads to the development of a runaway instability as avalanches of such cracks are produced under more dynamic, critical, conditions where positive feedback between an increment of crack growth and further growth dominates. Fig. 1 schematically illustrates the general transition from distributed damage to concentrated fault development under compression, due to the transition from local rules involving negative and positive feedback in crack growth (e.g. Henderson & Main 1992).

The aim of this paper is to compare the phenomenon of subcritical crack growth under tensile and compressive loading, by using a modified Griffith criterion for a fractal ensemble of cracks (Main 1991) to quantify a mean seismic energy release rate (per unit surface area) during stable crack growth under compression. First we review the available literature on subcritical crack growth under tension, and how constitutive relations derived from such observations have been used to develop a synoptic model for acoustic emission statistics during compressional failure. We then compare these semi-quantitative predictions with those inferred by applying the modified Griffith criterion under a range of experimental conditions for compressional loading. The major difference between the tensile and compressional results is that the constitutive relations, although they have the same form, are found to be much less non-linear for crack growth under compression. Thus subcritical damage under compression is inherently more stable than in tension.

![Figure 1. Idealized synoptic model of damage development in a heterogeneous brittle solid under compression. (a) The first crack grows on the weakest element (left-hand diagram) and is arrested by one of the mechanisms discussed in the text. The local probability of fracture (right-hand diagram) is reduced in a domain of diameter $d$ around this crack, but enhanced slightly elsewhere. Damage progresses with similar negative feedback until the whole sample is pervaded by microcracks (b). At this stage the deformation is optimally distributed and the local probability of fracture has returned to a uniform level. (c) The first two cracks coalesce as an incipient shear fault. This leads to stress concentration and an increase in probability of fracture around the nucleating faultlet, which grows rapidly to form a throughgoing fault. In reality the transition from (b) to (c) will be gradual, and have a statistical element that depends on the crack density, but the general principle of a transition from negative to positive feedback persists. (d) Slip on the fault closes the dilatant microcracks and further deformation is concentrated on and around the new fault.](https://academic.oup.com/gji/article/115/2/367/583200)
Subcritical crack growth in tension

Subcritical crack growth in polycrystalline materials, such as rock, is inherently non-linear. For example the quasi-static rupture velocity \( V \) is found experimentally to be related to the stress intensity \( K = Y\sigma\sqrt{c} \) by Charles' law:

\[
V = V_0(K/K_0)^n.
\]

(1)

\( K \) is a measure of the intensity of the stress field at the crack tip, \( \sigma \) is the stress applied to the remote boundary of the material, \( c \) is the crack semi-length and \( Y \) is a dimensionless constant depending on the loading configuration. The constants \( V_0 \) and \( K_0 \) depend on the temperature, pressure, rock type and chemical environment (Atkinson & Meredith 1987). \( K_0 \) is a threshold stress intensity for subcritical crack growth, below which no crack growth occurs, and is an upper bound for the occurrence of crack healing. The exponent \( n \) is known as the subcritical crack growth index.

In monocristalline materials and ceramics the velocity often reaches a temporary plateau with increasing stress intensity (region 2 in fig. 4.1 of Atkinson & Meredith 1987), before accelerating even faster towards dynamic rupture (region 3) at even higher \( n \) as the stress intensity approaches a critical value known as the fracture toughness. The plateau has rarely if ever been seen in experiments on rocks, most probably due to their very heterogeneous nature (Atkinson & Meredith 1987). The exponent \( n \) is respectively greater at low stress intensities for single crystal silicates, monominerallic rocks and polyminallic rocks (Atkinson & Meredith 1987). This implies a fundamental relationship between material heterogeneity and non-linearity expressed by \( n \). \( n \) is also very sensitive to the chemical activity of the environmental species, when subcritical crack growth is assisted by chemical weakening of atomic bonds at the crack tip due to the activity of a pore fluid. This mechanism is known as stress corrosion, and is an important process inferred to operate in the Earth at depth on the basis of structural and geochemical observations of exhumed rocks (Kerrich, La Tour & Barnet 1981; Etheridge 1983). For this reason \( n \) is sometimes referred to as the stress corrosion index. In general, for monocristalline rocks under tension \( 20 < n < 60 \), so the process of accelerating crack growth is in fact extremely non-linear. The acceleration of crack length predicted from eq. (1) and the definition of stress intensity has the form

\[
c = c_0(1 - t/t_f)^{2(2-n)},
\]

where \( c_0 \) is the initial crack length at time \( t = 0 \), and \( t_f \) is the failure time (Das & Scholz 1981). For earthquakes \( t_f \) depends to second order on the loading history (Main 1988). In the case of a constant stress \( n > 2 \) is required for the development of instability.

Acoustic emissions may be used to monitor the evolution of damage during subcritical crack growth experiments above a detection threshold magnitude \( m_c \). Meredith & Atkinson (1983) showed that AE from tensile subcritical growth experiments exhibited the same frequency–magnitude distribution as earthquakes, which we will write in the form

\[
\log N_c = a - b(m - m_c),
\]

where \( N_c \) is the number of events of magnitude greater than or equal to \( m \) in a unit time interval, \( N = N_c(m_c) = 10^a \) is the event rate for occurrence above a threshold magnitude \( m_c \), and \( b \) is the seismic \( b \) value. Meredith & Atkinson (1983) showed that the event rate \( N \) was also non-linearly related to the stress intensity, via

\[
N = N_0(K/K_0)^n.
\]

(4)

We will refer to \( n \) as an ‘effective’ stress corrosion index, because the exponents \( n \) and \( n' \) are found to be equal within a few per cent (e.g. Fig. 2(a), where \( n = 29.0 \) and \( n' = 29.1 \)).

In contrast the exponent \( b \) is negatively correlated to the stress intensity (Fig. 2b). Within the resolution of the experiments, all crystalline rocks studied so far fall on the same apparently linear curve for a given chemical environment when normalized by the critical stress intensity (or fracture toughness) \( K_c \)

\[
b = b_0 - (b_0 - b_c)(K - K_0)/(K_c - K_0).
\]

(5)

### Evolution of scaling exponents during damage under compression

Meredith, Main & Jones (1990) used eq. (5) to predict changes in seismic \( b \) value for a variety of stress histories during the compressional failure of intact rock samples in the laboratory, ranging from nearly elastic brittle failure at low confining pressure to cataclastic flow at high confining pressure. The semi-quantitative synoptic model they developed is shown in Fig. 3, with only minor modifications for reference to the work to be described later in this paper. The reader is referred to the original paper for a full description of the model. The good qualitative agreement in the general shape of the predicted and observed anomalies available at that time indicated that laws established for tensile failure apply equally well to compressional failure, although the values of the controlling constants may be different (Meredith et al. 1990). During these experiments AE event rates were measured but their behaviour was not included in the model. This extra degree of freedom is explicitly included in the model used in the present paper.

### A mean field theory for the evolution of fractal damage

Main (1991) developed a modified Griffith criterion for the evolution of damage due to an array of tensile cracks, all of which could grow because of the increase in free energy available to promote crack growth due to the chemical activity of a pore fluid. Ignoring the interaction potential between neighbouring cracks, a mean potential strain energy release rate (per unit crack surface area \( A \)) is defined by

\[
(G) = -(\partial U/\partial A) = B^2 \sigma^2(c),
\]

(6)

where \( U \) is the elastic strain energy, \( \sigma \) is the stress applied to the boundary of each element containing a crack, assumed uniform, and \( c \) is the crack semi-length in a volume element. For different loading configurations the combined geometric and scaling constant may be different from the tensile case, where \( B^2 = \pi/E \), and \( E \) is Young’s modulus. By assuming a uniform stress field, which is everywhere continuous and differentiable, and neglecting the effect of crack–crack interactions, the theory is a mean field approximation to the real case. In the early stages of crack growth there will be little distortion of the stress field by the growth of relatively...
Figure 2. (a) Dependence of crack velocity (dots) and event rate (crosses) on measured (mode I) stress intensity \( K_I \) from the double torsion tensile failure experiments of Meredith & Atkinson (1983), carried out in water at room temperature. (b) Synoptic diagram of the variation in \( b \) value (and inferred crack-length distribution exponent, \( D \)) with stress intensity and crack tip 'humidity' for a variety of crystalline rocks, after Meredith et al. (1990). The data are normalized to the fracture toughness \( K_c \) of the particular rock type.

Small cracks, the effective elastic constants will be relatively unvarying, and there will be relatively little crack-crack interaction. Under these conditions the mean field solution (6) is a reasonable approximation to the mean potential energy release rate in an ensemble of \( N_T \) aligned cracks, which are small compared to the average crack spacing. Such conditions apply most appropriately to the early stages of damage.

By assuming that the damage had the form of a fractal array of cracks within specific upper and lower bounds \((c_{\text{min}}, c_{\text{max}})\) with a probability density distribution \( p(c) \sim c^{-(D+1)} \), Main (1991) showed that

\[
\langle c \rangle = c_{\text{min}} \left( \frac{D}{D-1} \left( 1 - \frac{c_{\text{max}}}{c_{\text{min}}} \right)^{D} \right) ; D \neq 1. \tag{7a}
\]

\[
\langle c \rangle = c_{\text{min}} \left( \frac{\ln \left( \frac{c_{\text{max}}}{c_{\text{min}}} \right)}{1 - \left( \frac{c_{\text{max}}}{c_{\text{min}}} \right)^{D}} \right) ; D = 1. \tag{7b}
\]

The minimum condition for subcritical crack growth \( K > K_0 \) implies the inequality \( c_{\text{min}} > (K_0/\gamma \sigma) \), so that \( c_{\text{min}} > 0 \). This is also a necessary condition for defining a finite mean crack length from a fractal distribution. A finite upper bound \( c_{\text{max}} \) is required so that the strain energy stored in the body \( U \propto c^2 \) for Griffith cracks) remains finite. The upper bound may be calculated by assuming that there is only one crack of this size, whence \( c_{\text{max}} = c_{\text{min}} (D N_T)^{\frac{1}{1+(D+1)}} \) (Main 1991). As the fracture system grows, \( c_{\text{max}} \) increases but remains finite. Thus \( \langle G \rangle \) is a function of three independent variables \( (\sigma, N_T, D) \). When any two of these is held constant, \( \langle G \rangle \) is found to be positively correlated with \( \sigma \) and \( N_T \), but negatively correlated with the exponent \( D \) of the crack length distribution. In general, however, the relationships between these parameters will depend on the precise path through this 3-D phase space with three independent variables, which will in turn depend on the experimental conditions.

On their own these formulae are of little use in compressional tests because the crack system is not amenable to direct observation. Accordingly, Main & Meredith (1991) predicted variations of the seismic parameters \( a \) and \( b \) in eq. (2) by assuming respective proportionality between the measured event rate \( N \) and \( b \)-value and the number of cracks \( N_T \) and their distribution exponent \( D \), according to

\[
N = 10^a = \lambda N_T, \tag{8a}
\]

\[
b = cD/3. \tag{8b}
\]

The factor \( c \) depends on the time constants associated with the seismic source and the recording instrument. In this case \( c_{\text{min}} \) is set by the minimum level of complete detection of AE signals above the background noise. Eq. (8a) predicts an exponential increase in event rate \( N = dN_T/dt \) after integration, controlled by the rate constant \( \lambda \), exactly as seen in the early stages of damage (e.g. Meredith et al.)
Figure 3. Synoptic model of the evolution of seismic $b$ values predicted by Fig. 2(b). Column (a) is the differential stress history, running from ideal elastic–brittle behaviour (Model A) through dynamic failure after strain hardening (Model B), to dynamic failure after strain softening (Model C) and finally cataelastic flow (Model D). Column (b) is the accelerating length of the nucleating macrocrack, predicted from eq. (2), and (c) is the stress intensity. Column (d) is the predicted $b$-value anomaly, assumed proportional to the power-law exponent, $D$ (eq. 8b). The model for catalastic flow involves no macrocrack development. Two variants of Model C are possible (I and II).

1990). Eq. (8b) results from a simple dislocation model of the seismic source (Kanamori & Anderson 1975), and has obtained recent experimental confirmation from direct measurement of crack length distributions and $b$ values under open tensile loading (Hatton, Main & Meredith 1993). These two assumptions are therefore reasonable to first order at least.

The stress intensity is a measure of the concentration of stress at the crack tip, but is related to the energy release rate $G$ for a single crack by $G \propto K^2$ (Irwin 1958). This allows a direct comparison of experimental results with theoretical models developed from first principles from a free-energy formulation such as the one used in the present paper. One of the successful predictions from Main (1991) is that more heterogeneous materials (corresponding to higher $D$) have higher stress corrosion indices than more homogeneous materials (low $D$). This is consistent with the observation that $n$ is greater for rocks that are polycrystalline rather than monominerallic (e.g. Atkinson & Meredith 1987, Fig. 4.14), and that both have higher indices than single crystals.
This is further confirmation of the utility of the simple approach of characterizing damage evolution by a mean energy release rate. In this paper this quantitative model will be applied to interpret AE statistics under a wide variety of compressive loading conditions. The major difference from previous work is that seismic data is inverted using the theory, rather than forward predictions being made.

**MEAN FIELD INTERPRETATION OF EXPERIMENTAL RESULTS**

**Time dependence**

Table 1 summarizes the experimental conditions under which AE was monitored for the six selected experiments to be analysed below. The individual experiments are described in detail elsewhere, and the reader is referred to the original papers for a fuller description (Meredith *et al.* 1990; Sammonds, Meredith & Main 1992, 1993). The prime interest in the present paper is to relate the different macroscopic rheologies to the inferred shape of the change in mean seismogenic crack length \(c\) and mean seismic energy release rate \(\langle G \rangle\) per unit crack surface area, and to compare these quantitative inferences with the synoptic model of Fig. 3, where stress intensity on a single dominant crack of length \(c\) is the relevant variable. The experimental results and the mean field interpretation are presented in Figs 4–9. For all of these inversions we have assumed \(\lambda = 1\) and \(D = 2b\) in the absence of direct methods of checking on the value of these constants at this stage. A cut-off event rate of 400 min\(^{-1}\) has been applied to ensure a reliable statistical estimate of the mean crack length and energy release rate. The presentation in all of these figures takes the form: (a) measured differential stress and strain; (b) normalized mean crack length \(\langle c \rangle / \langle c \rangle_{\text{max}}\) inferred from \(N\) and \(b\); (c) normalized mean seismic energy release rate \(\langle G \rangle / \langle G \rangle_{\text{max}}\) calculated from (a) and (b). This last parameter is plotted as its square root for ease of comparison with the stress history, and because \(G \propto K^2\) for a single crack; and (d) measured \(b\)-value and event rate from the AE. These can be compared directly with the columns (a)–(d) on Fig. 3. The models are appropriate for quasi-static crack growth, and hence do not apply in the region approaching dynamic failure, where crack–crack interactions are likely to lead to an underestimate in \(\langle G \rangle\).

First, Fig. 4 shows the results for a sample of Westerley Granite exhibiting a rheology similar to Model B of Fig. 3, involving dynamic failure at peak stress after a period of quasi-static strain hardening. The strain rate is seen to increase slightly during the phase of strain hardening, and the inferred mean crack length accelerates rapidly at the same time. The energy release rate has a similar sharp increase, and is concave upward, as predicted for the stress intensity by Synoptic Model B.

Figures 5 and 6 show the results of two experiments on Darley–Dale sandstone, where the sample fails dynamically after a period of quasi-static strain hardening and strain softening, similar to Model C in Fig. 3. Fig. 5 shows results from an experiment in which the strain rate was maintained constant during deformation of an intact, water-saturated specimen. This resulted in a short strain-softening phase associated with seismic quiescence (Main, Meredith & Sammonds, 1992). Fig. 6 shows results from an experiment in which the phase of strain softening was extended by maintaining a constant pore fluid volume in an external pressure intensifier. This allows the pore pressure to drop as the sample dilates due to opening tensile microcracks, thereby stabilizing the system by local dilatant hardening (Sammonds, Meredith & Main 1992).

Figure 5 shows characteristics more similar to Model C(I), which has a single minimum in the \(b\) value. Here the small decrease in stress in the strain softening phase is more than offset by the increase in mean crack length, to produce a monotonically increasing \(\langle G \rangle\). The acceleration in mean crack length is non-linear over almost an order of magnitude in \(\langle c \rangle\). Dynamic failure is marked by a sudden stress drop and a jump in the strain curve, a peak in \(\langle c \rangle\) and \(\langle G \rangle\), and a sharp drop in event rate and \(b\) value. In the phase of quasi-static strain softening the event rate flattens out or falls slightly, and may be an example of precursory seismic quiescence.

Figure 6 shows behaviour much more like Model C(II), with a double minimum in the seismic \(b\) value. The strain rate shows a smaller, more stable jump on dynamic failure, which is also marked by a peak in mean crack length and event rate, and a sharp minimum in the \(b\) value. The mean energy release rate shows the double maximum predicted by Model C(II). In Model C(II) the first maximum is predicted solely as a result of the reduction in stress during the strain softening phase, so that \(K\) decreases temporarily before increasing again more rapidly due to accelerating crack growth. In contrast the decrease in \(\sqrt{\langle G \rangle}\) in Fig. 6 is dominated by the temporary decrease in mean crack length, which reduces more rapidly than the stress in the phase of strain softening. In this phase we might normally expect microcracks to be linking up with strong positive feedback due to these interactions, as in Fig. 1. The increase in \(\langle c \rangle\) and the extended period of strain softening are testament to the effectiveness of the dilatant hardening mechanism in

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**Table 1. Summary of experimental conditions.**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Rock Type*</th>
<th>Sample Type</th>
<th>Humidity</th>
<th>Servo-control</th>
<th>Reference†</th>
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<td>Intact</td>
<td>Vacuum</td>
<td>None</td>
<td>MMO 90</td>
</tr>
<tr>
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<td>Intact</td>
<td>Saturated</td>
<td>Strain rate</td>
<td>MMS 92</td>
</tr>
<tr>
<td>(111)</td>
<td>DDS</td>
<td>Intact</td>
<td>Saturated</td>
<td>Fluid volume &amp; Strain rate</td>
<td>SMM 92</td>
</tr>
<tr>
<td>(1 v)</td>
<td>DDS</td>
<td>Intact</td>
<td>Saturated</td>
<td>None</td>
<td>MMO 90</td>
</tr>
<tr>
<td>(v)</td>
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<td>Compress-</td>
<td>Saturated</td>
<td>Strain rate</td>
<td>SMM 93</td>
</tr>
<tr>
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<td>Dilational</td>
<td>Saturated</td>
<td>Strain rate</td>
<td>SMM 93</td>
</tr>
</tbody>
</table>

* WG—Westerley Granite
  DDS—Darley–Dale Sandstone
  † MMO 90—Meredith, Main & Jones (1990)
  MMS 92—Main, Meredith & Sammonds (1992)
  SMM 92, 93—Sammonds, Meredith & Main (1992, 1993)
Figure 4. Results for experimental condition (i): (a) measured differential stress (solid line) and strain (dashed line); (b) inferred mean crack length; (c) inferred mean energy release rate; (d) observed seismic b value (solid line) and AE event rate (dashed line). Experimental details are given in Table 1. (To be compared with Model B.)

Figure 5. Results for experimental condition (ii): (a) measured differential stress (solid line) and strain (dashed line); (b) inferred mean crack length; (c) mean energy release rate; (d) observed seismic b value (solid line) and AE event rate (dashed line). Experimental details are given in Table 1. (To be compared with Model C(I).)
Figure 6. Results for experimental condition (iii): (a) measured differential stress (solid line) and strain (dashed line); (b) inferred mean crack length; (c) inferred mean energy release rate; (d) observed seismic $b$ value (solid line) and AE event rate (dashed line). Experimental details are given in Table 1. (To be compared with Model C(II)).

Figure 7. Results for experimental condition (iv): (a) measured differential stress (solid line) and strain (dashed line); (b) inferred mean crack length; (c) inferred mean energy release rate; (d) observed seismic $b$ value (solid line) and AE event rate (dashed line). Experimental details are given in Table 1. (To be compared with Model D.)
Figure 8. Results for experimental condition (v)—compressional jog specimen: (a) measured differential stress (solid line) and strain (dashed line); (b) inferred mean crack length; (c) inferred mean energy release rate; (d) observed seismic b value (solid line) and AE event rate (dashed line). Experimental details are given in Table 1.

Figure 9. Results for experimental condition (vi)—tensional jog specimen: (a) measured differential stress (solid line) and strain (dashed line); (b) inferred mean crack length; (c) inferred mean energy release rate; and (d) observed seismic b value (solid line) and AE event rate (dashed line). Experimental details are given in Table 1.
delaying the final instability. Thus the positive reinforce-
ment of crack linkage is temporarily overcome by the
negative feedback of a reduction in pore pressure in
growing, dilatant microcracks. The first peak in the mean
crack length is not predicted by the synoptic model (Fig. 3),
which did not explicitly include the mechanical effect of pore
fluids.

Figure 7 shows the results for a sample of Darley–Dale
Sandstone deformed at a much higher confining pressure,
and where the sample fails by cataclastic (microscopically
brittle) flow (Meredith et al. 1990). Thus the macroscopic
rheology is ductile even though the microscopic rheology is
brittle, with a prolonged period of essentially constant stress
after a phase of strain hardening representing an extended
yielding. This closely resembles Model D behaviour,
where the stress intensity is undefined and the $b$-value
anomaly was predicted by using Scholz’s (1968) observation
of $b$ being negatively correlated to the stress. The
quantitative model of Main (1991) shows that this is not true
in general, and allows us to fill in the gaps in Fig. 3 using a
mean energy release rate and crack length. In this
experiment there was no servo-control on the strain rate,
which accelerates throughout the deformation as the
material weakens. The mean crack length reaches a peak
some few minutes after the flattening of the differential
stress curve, as does the mean energy release rate. The $b$
value decreases monotonically whereas the event rate peaks
before the mean crack length. The decrease in $c$ in the
latter stages is consistent with the comminution in grain size
that often accompanies cataclastic flow, where the whole
material fails due to widely distributed microcracking and
clast rotation. This is accompanied by a reduction in $c$ (G)
consistent with the weakening of the material by these
processes.

All of the above experiments involve the deformation of
initially intact rock specimens. In contrast Figs 8 and 9 show
results from samples of Darley–Dale Sandstone which were
artificially pre-cut to simulate the effect of compressional
(anti-dilational) and tensional (dilational) jogs or offsets in
the fault trace. The importance of these geometries was
highlighted by Sibson (1985), and the experimental
conditions are fully described in Sammonds et al. (1993).

Figure 8 shows results from an experiment designed to
investigate the properties of a compressional jog. The
sample was loaded at a strain rate maintained at an accurate
constant level by servo-control. The stress history shows a
quasi-elastic phase followed by a sharper yield point than in
Fig. 7, and then an extended phase of strain hardening and
strain softening with no macroscopic instability. The mean
crack length peaks at the first yield point, and declines
monotonically thereafter. The mean energy release rate
reaches a plateau during the strain hardening phase, where
the effect of an increase in differential stress is exactly
balanced by the decrease in $c$, and then declines in the
phase of strain softening. The event rate and the $b$ value are
anticorrelated prior to the yield point, which is marked by a
stable minimum in the $b$ value, and positively correlated
thereafter. The significance of the latter is that large changes
in the distribution of microcracks can be effected without
changing $c$ significantly, whereas the former is consistent
with the stable increase in $G$ in the loading phase
predicted by the model of Main (1991). Here the implied
reduction in the mean fracture length occurs even during the
phase of strain hardening, in contrast to Fig. 6, where this
type of behaviour is associated only with strain softening.

In contrast Fig. 9 shows the effect of a pre-cut tensional
jog on the stress history and AE statistics. Here the
rheology looks very ductile, with no clear yield point, and
strain hardening behaviour is observed throughout the
experiment, up to a final strain of 10 per cent. The
maximum stress is 200 MPa in contrast to 150 MPa in Fig. 8.
On both counts the tensional jog is stronger, testament
again to the operation of the dilatant hardening mechanism.
This is consistent with Sibson’s (1985) observation that
earthquake ruptures often arrest at the site of dilatational jogs
in the fault trace. Here the mean crack length shows a sharp
initial peak, followed by two broad peaks not associated
with any strong features on the stress history. The mean
energy release rate shows a monotonic increase which
appears to be flattening out with time. The event rate
increases very non-linearly at first to a sharp peak, and then
stays approximately constant, declining significantly only in
the latter stages. The $b$ value shows one sharp minimum
associated with the local sharp peaks in event rate and mean
crack length, and then two broad minima. This complexity
of behaviour implies that large changes in crack-density and
crack-length scaling can occur even when the energy release
rate is changing monotonically.

Scaling of AE parameters with $\langle G \rangle$

Figures 4–7 show that the mean energy release rate and
mean crack length for seismic damage in compressional
failure have a very similar qualitative form to the stress
intensity and macroscopic crack length for a model based
on observation of tensile cracks. In this section we explore
this comparison further by directly investigating the scaling of
event rate and $b$ value with $\langle G \rangle$. The rationale for this is
that the experimental data map out paths through a 4-D
phase space ($\langle G \rangle$, $\sigma$, $N$, $b$), with three degrees of freedom
representing the independent experimental observations
($\sigma$, $N$, $b$), or ($\langle G \rangle$, $N$, $b$) after applying eqs (6)–(8). These
data may then be compared with similar slices taken by
direct measurement in tensile tests ($K$, $N$, $b$), where each
parameter can be measured independently.

Figure 10 shows this inferred scaling for the six
experimental conditions described above. The experiments
are presented in the same order as above, numbered (i)–(vi)
as designated in Table 1. The data are presented as line
curves with marker arrows to allow the reader to follow the
history of the experiments. The results are plotted against
the square root of the normalized energy release rate to
enable direct comparison with Fig. 2.

First we note that the event rate scales with $\sqrt{\langle G \rangle}$
according to the power-law scaling of Charles’ law, but with
power-law indices $n'$ (marked by the dashed lines in the
figure) in the range 0.7–3.5 rather than 20–60 for Fig. 1.
This less severe non-linearity is consistent with the greater
stability of crack growth in the early phase of compressional
damage. This scaling breaks down near dynamic rupture for
equations (ii) and (iii), where the mean field theory is not
applicable, and where there is an obvious transition between
separate curves for loading and stress relaxation after
dynamic failure. An unexpected result is that the slope $n'$ is
similar on stress relaxation to that on the stable part of the loading curves, as evidenced by the parallel nature of the two curves in each of the two diagrams 10(IIa) and 10(IIIa). The offset of the relaxation curve to lower \( \langle G \rangle \) is evidence of the weakening effect of fault formation in these cases. The log-log curve of event rate against \( \langle G \rangle \) for experiment (i) is similarly straight up to very near failure, and that for experiment (iv) is remarkably linear throughout. In contrast the jog specimens (v) and (vi) see a flattening of the event rate curve at high \( \langle G \rangle \), reminiscent of region 2 behaviour in the velocity curves of Atkinson & Meredith (1987). Perhaps the most interesting result is that the more macroscopically

**Figure 10.** Functional dependence of (a) AE event rate \( N \) and (b) seismic b value on \( \sqrt{\langle G \rangle} = f(\sigma, N, b) \) for experiments (i)–(vi) inclusive.
brittle rheologies have systematically higher indices $n'$, ranging from 0.7 for the tensional jog (vi) to 3.5 for brittle failure after strain hardening (i). Thus the greater the constitutive non-linearity in event rate dependence, the greater the degree of brittle behaviour. No instability is seen at all (iv–vi) when $n = 2$, consistent with the prediction from eq. (2) by Das & Scholz (1981).

A second property of the event rate results is that the material involved in the intact specimens has a first-order effect on $n'$, with the sandstone samples (ii–iv) having values of $n'$ that cluster around 2.6 ± 0.2 compared to a value of 3.5 for Westerley granite (i). The relative ratio between rock types of 3.5:2.4 in compression is similar to the ratio in tension of 30–50 for granites to 15–25 for sandstones.
(Atkinson & Meredith 1987). Thus it is difficult in practice to distinguish between the effects of material heterogeneity and a macroscopic rheology, which may in any case be strongly dependent on material type.

The right-hand column of Fig. 10 shows that the $b$-value is negatively correlated to $\sqrt{\langle G \rangle}$, similar to the approximately linear, negative correlation of $b$ with $K$ in Fig. 2. This is most strikingly illustrated in experiment (iv), involving cataclastic flow. However, it should be borne in mind that an exact comparison is inappropriate because of the inability of the mean field model to determine $\langle G \rangle$ accurately, in contrast to the well-documented measurement of the fracture toughness $K_C$ in Fig. 2(b). There is some evidence of a systematic offset in the curves before and after failure in experiments (ii) and (iii). The unloading curve for experiment (v), where $\langle G \rangle$ is decreasing, also has a linear form but with a different slope than the loading portion. Nevertheless the scaling of $b$ value with $\langle G \rangle$ either on loading or unloading has a remarkable consistency with Fig. 2(b) within the experimental scatter.

In summary, the scaling of event rate and $b$ value has a remarkably similar form to that of Fig. 2(b), though quantitatively the event rate has a much less non-linear dependence on $\sqrt{\langle G \rangle}$ for these compressional experiments than on $K$ for the tensile tests of Fig. 2(a). This is consistent with the greater stability of compressional loading under subcritical crack growth. We might expect this result to hold from the general rules of fracture mechanics for critical, dynamic failure, but this is the first time this general phenomenon has also been demonstrated for subcritical crack growth.

### Acceleration of mean crack length

If the event rate dependence with $\sqrt{\langle G \rangle}$ has a power-law scaling with exponent $n'$, then it also seems appropriate to investigate the scaling of the inferred mean crack length with time, in order to determine an equivalent index $n$ for comparison, as $n = n'$ for the tensile tests (Fig. 2a). The extra dimension provided by the time dependence is an important independent check on the validity of the theory used to calculate $\langle G \rangle$, and map out its variation with the seismic parameters. On Fig. 11 the mean crack length for

![Figure 11. Acceleration of mean crack length $\langle c \rangle = f(t, N, b)$ as a function of time $t$ relative to the failure time $t_o$, for experiment (ii), for comparison with eq. (2).](image-url)

experiment (ii), which had the largest range of inferred values for $\langle c \rangle$ is plotted against $t_f - t$ in order to allow such a comparison. The negative slope on the log–log plot is 1, apart from a deviation near failure, marked by the peak value of $\langle c \rangle$. This deviation may be due either to a sampling rate (30 s in this case) insufficient to determine the nominal failure time accurately enough, or more likely to the poor performance of the theory when cracks begin to interact. The slope reduces to an index $n$ for the crack acceleration of 4. This compares reasonably well with the value of 2.7 or so for $n'$ determined from the event rate dependence in Fig. 10 for the same experiment, given the uncertainties involved. Thus the inferred indices $n$ and $n'$ are similar to each other for these compressional experiments, but are both systematically lower than their counterparts in Fig. 2. This confirms the validity of the theory used to calculate $\langle G \rangle$, and reinforces the implication of greater stability of subcritical crack growth in compression.

### CONCLUSION

The results of six compression rock mechanical experiments have been interpreted using a mean field theory for subcritical crack growth based on a modified Griffith criterion for a fractal ensemble of cracks. Despite the limitations of the theory near dynamic failure, where crack–crack interactions are important, the theory produces results that are very similar to an earlier but less quantitative model based on observations of tensile cracking. The inferred non-linear constitutive relations are similar to those determined for subcritical crack growth due to stress corrosion in tension, except that the stress intensity is replaced by the square root of a mean energy release rate, and the macrocrack length by a mean crack length, both inferred from the seismic parameters in the fractal range. The stress corrosion indices $n$ and $n'$ determined from these constitutive laws are lower than those for tensile loading, testament to the greater stability of subcritical crack growth under compressive loading. These indices are also positively correlated with the degree of macroscopically brittle rheology, implying a positive correlation between the degree of constitutive non-linearity and instability. The indices are also strongly dependent on rock type, with a similar relative variation to that found in tensile tests. They have a similar magnitude on unloading as well as loading for experiments involving the formation of a fault after a dynamic stress drop, although the curves are systematically offset due to the associated material weakening.

The low constitutive indices are testament to the effectiveness of mechanisms of negative feedback during subcritical crack growth, which in turn justifies the utility of the theory well into the phase of stable crack growth. The implication is that neighbouring cracks are effectively shielded from the short-range effects of each others' deformation until very late in the cycle, where positive reinforcement due to crack coalescence dominates and a mean field theory is inappropriate.

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