

An optimization approach to wastewater systems planning at regional level

Joaquim Sousa, Alexandra Ribeiro, Maria da Conceição Cunha and António Antunes

ABSTRACT

A regional wastewater system planning problem consists of finding the minimum-cost configuration for the system needed to drain the wastewater produced at the communities located within a region, while meeting the quality standards defined for the receiving water bodies and complying with all (other) relevant regulatory aspects. There are several possible solutions to this kind of problem. They range from solutions where each community treats its wastewater separately, to solutions where all the wastewater produced in the region is sent to a single treatment plant. It is likely that the most effective solution both in terms of public expenditure, equipment reliability, and environmental impact will be found somewhere between these two extremes. This paper presents an optimization approach to wastewater systems planning at regional level. The approach applies only to sanitary sewer networks. A simulated annealing algorithm is used to solve the optimization model upon which the approach is based. For the application of this approach a user-friendly computing tool was developed. Within this tool, both the acquisition of data and the output of results are made through a flexible GIS interface.

Key words | computer-aided design, geographic information systems, optimization, simulated annealing, wastewater systems

Joaquim Sousa
Alexandra Ribeiro
Coimbra Polytechnic Institute,
Quinta da Nora,
3030 Coimbra,
Portugal

Maria da Conceição Cunha
(corresponding author)
António Antunes
Civil Engineering Department,
University of Coimbra,
Pinhal de Marrocos,
3030 Coimbra,
Portugal

INTRODUCTION

The discharge of wastewater improperly treated or untreated into lakes and rivers has been growing during recent decades at an almost exponential rate. As a result, despite a considerable amount of intervention by national and municipal authorities, serious water quality problems can now be found in many regions of the world.

A regional wastewater system planning problem consists, generally and succinctly, of finding the minimum-cost configuration for the system needed to drain the wastewater produced in the communities (wastewater sources) located within a region, while meeting the quality standards defined for the receiving water bodies and complying with all (other) relevant regulatory aspects. The essential components of the system are sewer networks and treatment plants.

The problem stated above includes a location component and a sizing component. These two aspects are not independent of each other, and should be handled simultaneously. There are several possible solutions. They range from solutions where each community treats its wastewater separately without paying heed to the important scale economies and reliability gains that would be permitted if larger treatment plants were chosen, to solutions where all the wastewater produced in the region is sent to a single treatment plant. It is likely that the most effective solution both in terms of public expenditure, equipment reliability and environmental impact will be found somewhere between these two extremes.

Finding the best solutions to this kind of problem is a delicate task, considering the vast number of alternatives for the location/layout and other characteristics of sewer

networks and treatment plans. Normally, the task cannot be performed without resorting to optimization techniques. The mathematical nature of the optimization models that represent the problem is quite complex. In fact, both the objective function and some constraints are non-linear, and some variables are discrete (e.g. diameters have to be chosen from what is commercially available).

Formerly, traditional optimization methods were often used to solve regional wastewater systems planning problems. Melo & Câmara (1994) present a wide-ranging survey of the literature in this domain (41 papers are quoted). The methods most commonly used are as follows: linear programming (Deininger 1965; Loucks *et al.* 1967); dynamic programming (Converse 1972; Klemetson & Grenney 1985); non-linear programming (Graves *et al.* 1972); convex programming (Deininger & Su 1973); linear mixed-integer programming (Wanielista & Bauer 1972; Joeres *et al.* 1974; Brill & Nakamura 1978); and different types of heuristic techniques (McConagha & Converse 1973; Weeter & Belarti 1976; Lauria 1979; Smeers & Tyteca 1982; Câmara 1985; Melo 1992; Voutchkov & Boulos 1993). Regardless of the potential interest of these models, in practice they have been utilized only to a very small extent. In some cases they were excessively difficult to apply in real-world situations; in other cases they did not represent real-world problems but rather simplified versions of them.

It is only relatively recently that advances in computing technologies and operations research made it possible to deal efficiently with the non-linear combinatorial optimization models applicable to regional wastewater systems planning. The role played by several global optimization methods is, in this context, particularly important. These methods, which include genetic algorithms, simulated annealing, tabu search and neural networks, have been used successfully to solve difficult problems in several areas of science and technology. A recent application of genetic algorithms is reported in Wang & Jamieson (1998). The authors have been working on the application of simulated annealing to a variety of civil engineering problems (Cunha & Sousa 1999, 2001; Cunha 1999; Antunes & Peeters 2000, 2001) and they have confirmed the good performance of this method when compared to other methods.

This paper presents an optimization approach to wastewater systems planning at regional level. The approach applies only to sanitary sewer networks (i.e. networks which only carry domestic and industrial wastewater). Surface runoff and other uncontaminated waters are assumed to be collected in storm sewer networks. Separate sewer networks are the dominant solution in the USA and Mediterranean countries (Hammer 1991). This paper consists of three parts. The first part describes the model upon which the optimization approach is based. The second part presents the simulated annealing algorithm developed to solve the model. The third part introduces a user-friendly computing tool designed to facilitate the application of the model to real-world problems, within which both the acquisition of data and the output of results are made through a flexible Geographical Information System (GIS) interface. Tools of this kind are essential to enhance the application of advanced optimization techniques to wastewater systems planning problems. In their absence, the gap between research and practice, which is already wide in this and many other civil engineering fields, will continue to enlarge.

OPTIMIZATION MODEL

As stated before, a regional wastewater system planning problem consists of finding the minimum-cost configuration for the system needed to drain the wastewater produced at the communities (wastewater sources) located within a region, while meeting the quality standards defined for the receiving water bodies and complying with all (other) relevant regulatory aspects.

The optimization model presented below encompasses all the aspects involved in this kind of problem. Once solved, the model determines the layout of sewer networks, the diameter of the sewers, the location and capacity of treatment plants, and the location and size of pumps (if needed). This is made in accordance with regulations regarding minimum and maximum slopes for sewers, minimum and maximum flowing velocity, minimum diameters for sewers, and receiving water quality parameters. Since the model is designed for planning

purposes, it assumes static reference values for wastewater production and wastewater pollution loads (unlike combined or storm sewer networks, sanitary sewer networks can be planned without taking account of time variations in loadings). Though being designed for planning purposes, operation (manpower, energy, etc.) and maintenance costs are considered in addition to investment costs.

Within the model, sewer networks connect the wastewater sources to treatment plants, either directly or indirectly through possible intermediate nodes. These nodes must be included to allow both the adequate representation of topography or the early regrouping of wastewater.

In mathematical notation, the optimization model can be formulated as follows:

$$\text{Min} \sum_{i=1}^N \sum_{j=1}^N C_{ij}(Q_{ij}, L_{ij}, E_i, E_j, x_{ij}) + \sum_{k=m+1}^N C_k(QT_k, y_k) \quad (1)$$

subject to:

$$\sum_{j=1}^N Q_{ji} - \sum_{j=1}^N Q_{ij} = -QR_i, \quad \forall i = 1, \dots, n \quad (2)$$

$$\sum_{j=1}^N Q_{jl} - \sum_{j=1}^N Q_{lj} = 0, \quad \forall l = n+1, \dots, m \quad (3)$$

$$\sum_{j=1}^N Q_{jk} - \sum_{j=1}^N Q_{kj} = QT_k, \quad \forall k = m+1, \dots, N \quad (4)$$

$$\sum_{i=1}^N QR_i = \sum_{k=m+1}^N QT_k \quad (5)$$

$$Q_{\min,ij} \cdot x_{ij} \leq Q_{ij} \leq Q_{\max,ij} \cdot x_{ij}, \quad \forall i = 1, \dots, N; \forall j = 1, \dots, N \quad (6)$$

$$QT_k \leq QT_{\max,k} \cdot y_k, \quad \forall k = m+1, \dots, N \quad (7)$$

$$x_{ij} \in \{0,1\}, \quad \forall i = 1, \dots, N; \forall j = 1, \dots, N \quad (8)$$

$$y_k \in \{0,1\}, \quad \forall k = m+1, \dots, N \quad (9)$$

$$QT_k \geq 0, \quad \forall k = m+1, \dots, N \quad (10)$$

$$Q_{ij} \geq 0, \quad \forall i = 1, \dots, N; \forall j = 1, \dots, N \quad (11)$$

where:

N : total number of nodes (wastewater sources plus possible intermediate nodes plus possible treatment plants);

$1, \dots, n$: wastewater sources;

$n+1, \dots, m$: locations for possible intermediate nodes;

$m+1, \dots, N$: locations for possible treatment plants;

C_{ij} : discounted costs for installing, operating, and maintaining a sewer linking node i to node j ;

Q_{ij} : flow carried from node i to node j ;

L_{ij} : length of the sewer linking node i to node j ;

E_i and E_j : hydraulic heads at nodes i and j respectively;

C_k : discounted costs for installing, operating and maintaining a treatment plant k ;

QT_k : amount of wastewater treated at treatment plant k ;

QR_i : amount of wastewater produced at node i ;

$Q_{\min,ij}$ and $Q_{\max,ij}$: minimum and maximum flow allowed in the sewer linking node i to node j respectively;

$QT_{\max,k}$: maximum amount of wastewater that may be treated in treatment plant k ;

x_{ij} : binary variable that will take value 1 if there exists a sewer linking node i to node j , otherwise will take value 0;

y_k : binary variable that will take value 1 if there exists a treatment plant in node k , otherwise will take value 0.

The objective function (1) expresses the minimization of the total discounted costs for installing, operating and maintaining sewer networks and treatment plants. The first term corresponds to sewer network costs, which will depend on the wastewater flow (and, thus, on the diameter of sewers), on the length of sewers and on the hydraulic heads at the extreme points of the sewers. The network may require pump stations to carry wastewater from low-energy points to high-energy points. The second term corresponds to treatment plant costs, which, for a given type of treatment plant, will depend on the amount of wastewater treated there.

Constraints (2)–(4) are the continuity equations for three types of network nodes: wastewater sources; possible intermediate nodes; and possible treatment plants. Constraint (5) ensures that all the wastewater produced by the communities located in the region will be treated. Constraints (6) guarantee that the flow carried by sewers will be within given minimum and maximum values. These values depend both on the diameter and slope of sewers, and on flow velocity requirements. The hydraulic calculations needed to determine the diameter and slope of sewers are based on the Manning–Strickler equation. Constraints (7) ensure that the wastewater sent to any treatment plant will not exceed given maximum values. These values depend on the quality standards defined for the receiving water bodies. Constraints (8) and (9) are 0–1 constraints. Constraints (10) and (11) are non-negativity constraints.

MODEL SOLVING

Several alternative methods can be used to solve the complex non-linear combinatorial optimization model presented in the previous section. Given their previous experience, the authors decided to use a simulated annealing algorithm. The first application of a simulated annealing algorithm was made by Metropolis *et al.* (1953), and involved a thermodynamic problem that was not explicitly an optimization problem. Some authors, notably Kirkpatrick *et al.* (1983) and Cerny (1985), had the innovative idea of applying the principles of the Metropolis algorithm to a well-known combinatorial problem (the travelling salesman problem), and they obtained very good results. Since then, simulated annealing algorithms have been successfully applied to a wide variety of problems. The main reason explaining the success of simulated annealing algorithms is their ability to avoid getting trapped in poor local optimums. Furthermore, they offer attractive theoretical properties regarding convergence towards global optimum. Detailed information on simulated annealing algorithms can be found in Aarts *et al.* (1997).

The simulated annealing algorithm implemented for this study consists of the following steps (Figure 1).

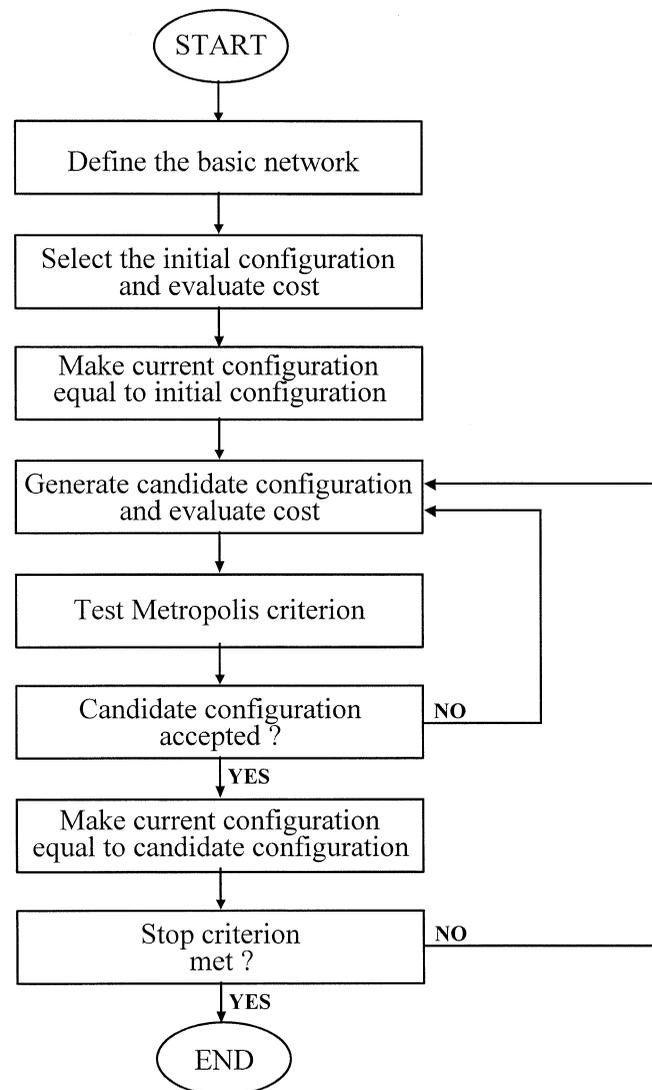


Figure 1 | Flowchart for the simulated annealing algorithm.

First, the basic network of the problem is defined. The nodes of this network are the wastewater sources, the locations for possible treatment plants and the possible intermediate nodes. The arcs of this network are the possible sewers linking the different types of nodes.

Next, an initial configuration of the system is established, and evaluated with regard to (total discounted) costs. A configuration of the system is defined by the layout and the size of sewers, the location and capacity of pump stations (if needed), and the location and capacity

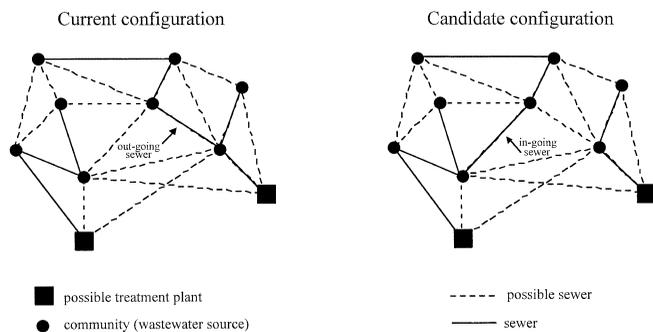


Figure 2 | Generation of candidate configurations.

of treatment plants. Any feasible configuration linking wastewater sources, either directly or indirectly through other source and/or intermediate nodes, to treatment plants can be chosen. Feasible configurations are those that verify the applicable technical, economic and environmental criteria. The initial configuration plays the role of current configuration for the development of the algorithm.

Then, through a perturbation mechanism, a new (feasible) configuration of the system, called the candidate configuration, is generated from the current configuration. The perturbation mechanism consists of replacing a randomly chosen sewer included in the current configuration by a randomly chosen sewer not included in the current configuration but connected to the same wastewater source or intermediate node (Figure 2). The candidate configuration is evaluated with regard to costs. The costs of the current configuration and the candidate configuration are compared and the candidate configuration is accepted, or not, according to some appropriate acceptance criterion. If it is accepted, the candidate configuration becomes the current configuration. In successive iterations, new candidate configurations are generated. Those being accepted become current configurations. The process continues until no further cost improvements are possible.

The acceptance criterion most commonly used is the Metropolis criterion, according to which the probability of replacing the current configuration (S) by a candidate configuration (S') is given by

$$p = \min \left\{ 1, \exp \left(\frac{\Delta C}{\theta} \right) \right\} \quad (12)$$

where $\Delta C = C(S) - C(S')$; θ ; positive parameter.

According to the Metropolis criterion, configurations leading to cost decreases will always be selected ($p = 1$) while configurations leading to cost increases may be selected or not, with probability of selection depending on the value of the parameter θ . This parameter is called temperature, a name that refers to the physical process upon which the simulated annealing algorithm is based.

In most implementations, the temperature decreases as the algorithm proceeds. The function describing the decrease of temperature is called the cooling schedule. The performance and robustness of a simulated annealing algorithm are highly affected by the cooling schedule.

The cooling schedule adopted in this study, which is similar to the one employed by Johnson *et al.* (1989) in their simulated annealing algorithm for the graph partitioning problem, involves four parameters: the initial temperature, θ_1 ; the tread length, λ ; the cooling rate, γ ; and the stop number, σ . The initial temperature defines the rate at which candidate configurations with cost $x\%$ larger than the cost of the initial configuration are accepted. The tread length is the minimum number of candidate configurations to be tried at each temperature. If the heuristic is unable to find at least one better single solution or a better average solution over λ iterations, the temperature is decreased. The cooling rate is the rate at which temperature decreases. The stop number is the maximum number of temperature reductions that may occur without finding any solution improvements, if the rate of acceptance for candidate configurations is inferior to 10%. When the stop number is attained the system becomes frozen, and the annealing process reaches the end. The links between the four parameters and the way they work together are depicted in Figure 3.

After a detailed evaluation of different sets of parameters, made for a large number of case studies and different sequences of random numbers, it was found that the best values for the parameters are as follows:

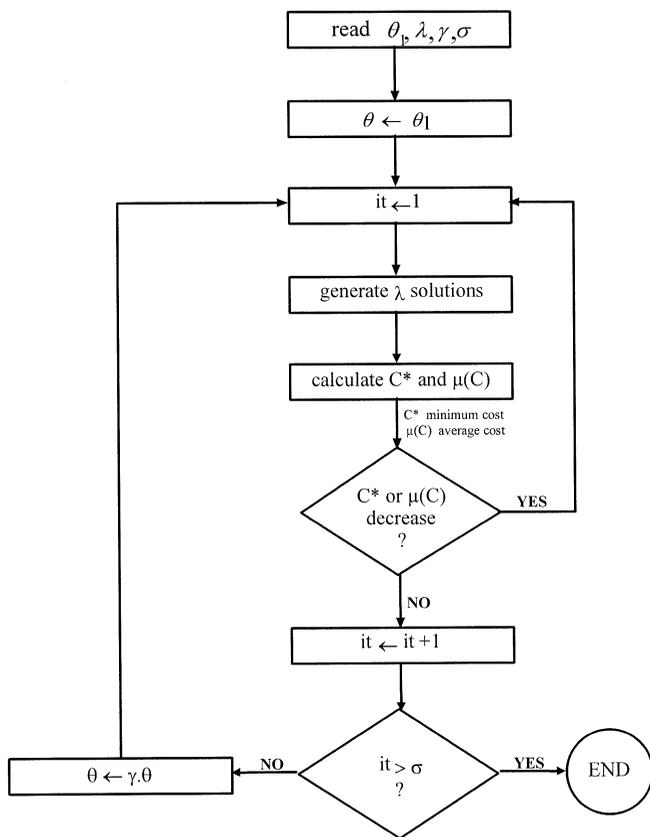


Figure 3 | The cooling schedule.

- where $\theta_1 = 1.95C_0$, where C_0 is the cost of the initial configuration;
- $\lambda = 10Z$, where Z is the number of possible sewers;
- $\gamma = 0.9$;
- $\sigma = 2$.

Detailed information on the process leading to the choice of these parameters can be found in Cunha & Sousa (1999) and Sousa and Cunha (1999).

COMPUTING TOOL

In order to facilitate the application of the optimization model presented above to real-world problems, the authors developed the package RWSP (Regional Wastewater Systems Planning), a user-friendly computing

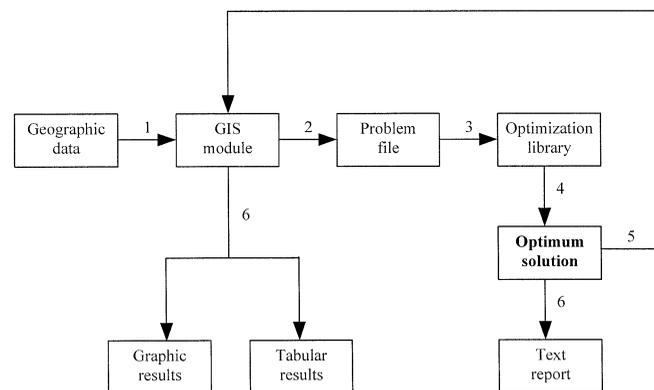


Figure 4 | General structure of RWSP.

tool showing the interface encountered in most applications prepared for the operating systems Windows 95/98/2000. The package makes the connection between a GIS developed with the mapping language *MapObjects* and a library of optimization subroutines stored as Dynamic Link Libraries (DLLs). The main code and the optimization subroutines were respectively written in Microsoft Visual Basic 6.0 and Fortran PowerStation.

The general structure of RWSP is presented in Figure 4. The GIS module manages all problem information. This information is then assembled in a problem file written in the appropriate text format. The optimization library interprets the problem and determines the corresponding optimum solution. This solution is made available to the user both in graphic and tabular format, and through a text report.

The detailed description of the input and output operations required by the utilization of RWSP is done below, with reference to a hypothetical problem involving a region with 13 communities (wastewater sources) and 6 possible sites for the location of wastewater treatment plants. The size of communities ranges between 1,000 and 30,000 inhabitants.

The information themes for the hypothetical problem are displayed in Figure 5. WASTEWATER_SOURCES is a point theme representing the communities located in the region (circles 1–13). POSSIBLE_TREAT_PLANTS is also a point theme representing the possible treatment

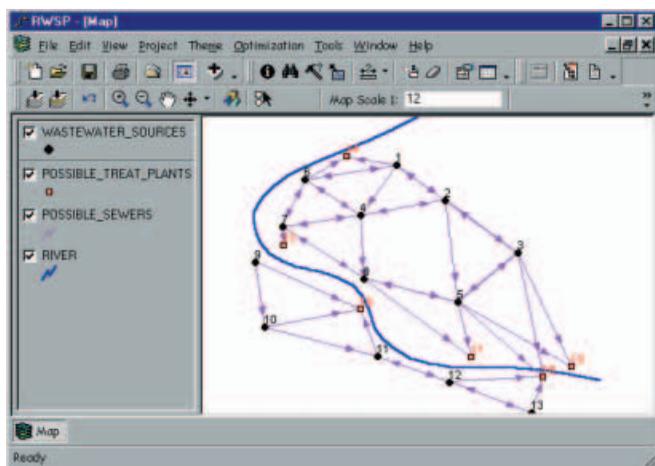


Figure 5 | Geographic setting for the hypothetical problem.

plants (squares 14–19). POSSIBLE_SEWERS is a polyline theme representing the possible sewers. The figure also contains the image theme RIVER (the receiving water body).

Each theme must be associated with an attribute table consisting of as many lines as the number of features (communities, possible treatment plants, possible sewers) in the theme, and of as many columns (fields) as the number of attributes characterizing the features. The attribute table corresponding to the theme WASTEWATER_SOURCES must contain numerical information on the attributes ELEVATION, POPULATION and (wastewater) PER_CAPITA_FLOW for each community. The attribute table corresponding to the theme POSSIBLE_TREAT_PLANTS must contain numerical information on the attributes ELEVATION and MAX_CAPACITY of the possible treatment plants. If some treatment plants already exist, this information must be supplied in a 0–1 column corresponding to the attribute EXISTING_TREAT_PLANTS. Finally, the attribute table corresponding to the theme POSSIBLE_SEWERS must contain numerical information on the attribute LENGTH. In addition to these essential (obligatory) attributes, optional attributes may be included to further describe the problem features.

The attribute table for the theme WASTEWATER_SOURCES is displayed in Figure 6 (rear window). The

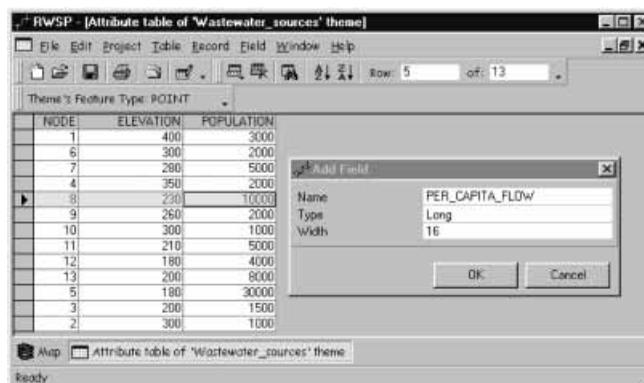


Figure 6 | Attribute table of wastewater sources.

attributes visible there (ELEVATION and POPULATION) were created through the command *Add Field*, included in the *Field* menu. The utilization of this command to create the field PER_CAPITA_FLOW is illustrated in the same figure (front window).

Information relating to the type (materials) and size (diameters) of sewers, physical characteristics of treatment plants and pump stations, and investment, operating and maintenance costs are supplied through dialog boxes. The parameters required by the simulated annealing algorithm are also supplied through a dialog box. The user may apply the default values suggested by RWSP, or change them if appropriate.

All data may be displayed, altered, or printed whenever necessary.

Output operations

Problem solutions are presented, and saved onto disk, in three formats:

- A text report containing a brief description of the problem and numerical information on the five best solutions identified through the application of simulated annealing;
- Three information themes, TREATMENT_PLANTS, SEWERS and PUMP_STATIONS, representing the location of treatment plants, the location of pump stations and the layout of sewer networks;

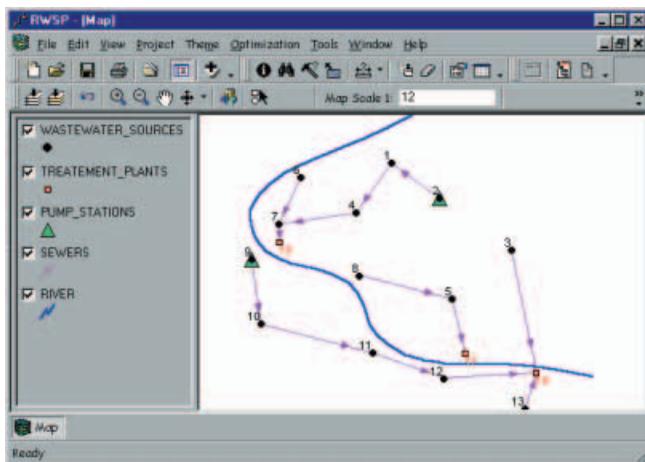


Figure 7 | Graphic solution for the hypothetical problem.

START	END	LENGTH	MS_COEF	DIAMETER	UNIT_COST	TOTAL_COST
1	4	2000	100	200	8937	17874000
2	1	2000	100	125	6684	13368000
6	7	1000	100	200	8937	8937000
7	15	200	100	200	8937	1787400
13	18	1000	100	200	8937	8937000
3	18	4000	100	200	8937	35748000
5	17	1000	100	400	15933	15933000
4	7	2000	100	200	8937	17874000
9	10	2000	100	140	7082	14164000
10	11	5000	100	200	8937	44685000
11	12	2000	100	250	10394	20788000
12	18	3000	100	250	10394	31182000
8	5	5000	100	315	12258	61290000

Figure 8 | Attribute table of sewers.

- (c) The attribute tables for each theme, containing numerical information on the characteristics of treatment plants (capacity, investment costs, annual operation and maintenance costs), pump stations (capacity, head, power, investment costs, annual operation and maintenance costs), and sewers (length, Manning–Strickler coefficient, diameter, unit costs, total costs).

The output obtained through the application of RWSP to the hypothetical problem is exemplified in Figures 7 and 8. Figure 7 displays the themes WASTEWATER_SOURCES, TREATMENT_PLANTS, PUMP_STATIONS and SEWERS for the optimum solution. This solution involves the construction of three treatment plants (at locations 15, 17 and 18) and two pump stations (next to communities 2 and 9). Figure 8 shows the attribute table associated with the theme SEWERS. It contains the fields LENGTH, MS_COEFS, DIAMETER, UNIT_COST and TOTAL_COST. The time needed to find the optimum solution on a Pentium III PC running at 500 MHz was 1.5 s.

CONCLUSION

Regional wastewater systems planning will be made more efficient both in terms of public expenditure, equipment

reliability and environmental protection, if the corresponding problems are handled through an optimization approach. This paper presents a non-linear combinatorial optimization model aimed at being the base for that type of approach. The model permits us to find optimum decisions for both the layout and design of sewer networks and the location and capacity of treatment plants, taking into account the quality standards defined for the receiving water bodies. The method used to solve the model is simulated annealing, a relatively recent global optimization method that is being used on complex engineering problems with remarkable results. In order to facilitate the application of the approach to real-world problems, a user-friendly computing tool was developed, within which both the acquisition of data and the output of results are made through a flexible GIS interface. Tools of this kind are essential to enhance the application of optimization techniques to wastewater systems planning problems. In their absence, the gap between research and practice, which is already wide in this and other civil engineering fields, will continue to enlarge.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge support received from Fundação para a Ciência e a Tecnologia through grant PRAXIS/PE/ECM/12266/98.

REFERENCES

- Aarts, E. M., Korst, J. H. & Van Laarhoven, P. J. 1997 Simulated annealing. In *Local Search in Combinatorial Optimization* (ed. E. M. Aarts & J. K. Lenstra), pp. 91–120. John Wiley and Sons, New York.
- Antunes, A. & Peeters, D. 2000 A dynamic optimization model for school network planning. *Socio-Econ.-Plann. Sci.* **34**(2), 101–120.
- Antunes, A. & Peeters, D. 2001 On solving complex multi-period location models using simulated annealing. *Eur. J. Opl. Res.* **130**, 190–201.
- Brill, E. D. & Nakamura, M. 1978 A branch and bound method for use in planning regional wastewater treatment systems. *Wat. Res. Res.* **14**(1), 109–118.
- Câmara, A. S. 1985 Optimization of regionalized wastewater treatment systems—an algorithm based on network theory. *Workshop on Water Quality Modelling, LNEC, Lisbon.*
- Cerny, V. 1985 A thermodynamical approach to the traveling salesman problem. *J. Optim. Theor. Appl.* **45**, 41–51.
- Converse, A. O. 1972 Optimum number and location of treatment plants. *J. Wat. Pollut. Control Fed.* **44**(8), 1629–1636.
- Cunha, M. C. 1999 On solving aquifer management problems with simulated annealing algorithms. *Wat. Res. Mngnt.* **13**(3), 153–169.
- Cunha, M. C. & Sousa, J. 1999 Water distribution network design optimization: a simulated annealing approach. *J. Wat. Res. Plann. Mngnt. ASCE* **125**(4), 215–221.
- Cunha, M. C. & Sousa, J. 2001 Hydraulic infrastructures design using simulated annealing. *J. Infrastruc. Syst. ASCE* **7**(1), 32–39.
- Deininger, R. A. 1965 Water quality management: the planning of economically optimal pollution control systems. *PhD thesis*, Northwestern University, Evanston, IL.
- Deininger, R. A. & Su, S. Y. 1973 Modeling regional wastewater treatment systems. *Wat. Res.* **7**(4), 663–646.
- Graves, G. W., Hatfield, G.B. & Whinston, A. B. 1972 Mathematical programming for regional water-quality management. *Wat. Res. Res.* **8**(2), facing p. 273.
- Hammer, M. J. 1991 *Water and Wastewater Technology*. Prentice-Hall, Englewood Cliffs, NJ.
- Joers, E. F., Dressler, J., Choand, C. C. & Falkner, C. H. 1974 Planning methodology for the designing of regional wastewater treatment systems. *Wat. Res. Res.* **10**(4), 643–649.
- Johnson, D., Aragon, C., McGeoch, L. & Schevon, C. 1989 Optimization by simulated annealing: an experimental evaluation—part I, graph partitioning. *Opns. Res.* **37**, 865–892.
- Kirkpatrick, S., Gellatt, C. & Vecchi, M. 1983 Optimization by simulated annealing. *Science* **220**, 671–680.
- Klemetson, S. L. & Grenney, W. J. 1985 Dynamic optimization of regional wastewater treatment systems. *J. Wat. Pollut. Control Fed.* **57**(2), 128–134.
- Lauria, D. T. 1979 Desk calculator model for wastewater planning. *J. Environ. Engng, ASCE* **105**(1), 113–120.
- Loucks, D. P., ReVelle, C. S. & Lynn, W. R. 1967 Linear programming models for water pollution control. *Mngnt Sci.* **14B**, facing p. 166.
- McConagha, D. L. & Converse, A. D. 1973 Design and cost allocation algorithm for waste treatment systems. *J. Wat. Pollut. Control Fed.* **45**(12), 2558–2566.
- Melo, J. J. 1992 Optimization of regional wastewater treatment systems – the OPTTAR model. *PhD thesis*, New University of Lisbon, Lisbon.
- Melo, J. J. & Câmara, A. S. 1994 Models for the optimization of regional wastewater treatment systems. *Eur. J. Opl. Res.* **13**, 1–16.
- Metropolis, N., Rosenbluth, M., Rosenbluth, A., Teller, A. & Teller, E. 1953 Equations of state calculations by fast computing machines. *J. Chem. Phys.* **21**, 1087–1092.
- Smeers, Y. & Tyteca, D. 1982 Optimal location and design of wastewater treatment plants under river quality constraints. In *Environmental Systems Analysis and Management* (ed. S. Rinaldi), pp. 289–310. North-Holland, Amsterdam.
- Sousa, J. & Cunha, M. C. 1999 Soluções optimizadas para sistemas regionais de águas residuais: a utilização do método de recozimento simulado. *Actas do IV SILUSBA*, CD edn. Coimbra, Portugal.
- Voutchkov, N. S. & Boulos, P. F. 1993 Heuristic screening methodology for regional wastewater treatment planning. *J. Environ. Engng., ASCE* **199**(4), 603–613.
- Wang, C. G. & Jamieson, D. G. 1998 Use of a genetic algorithm in regional wastewater treatment planning. *Proceedings of the First International Conference on New Information Technologies for Decision Making in Civil Engineering, Montreal, Canada*, pp 791–802. Institut National des Sciences Appliquées de Lyon, Lyon, France.
- Wanielista, M. P. & Bauer, C. S. 1972 Centralization of waste treatment facilities. *J. Wat. Pollut. Control Fed.* **44**(12), 2229–2238.
- Weeter, D. W. & Belarti, J. G. 1976 Analysis of regional water treatment system. *J. Environ. Engng., ASCE* **102**(1), 233–237.