Frequency analysis of seasonal extreme precipitation in southern Quebec (Canada): an evaluation of regional climate model simulation with respect to two gridded datasets

Loubna Benyahya, Philippe Gachon, André St-Hilaire and René Laprise

ABSTRACT

This study proposes an assessment procedure to compare two gridded (Cubic Spline, CS, and ANUSPLIN) datasets and one regional climate model simulation series (CRCM 4.1.1) of seasonal maximum precipitation (SMP) over southern Quebec (Canada). This study consists of: (1) identifying the appropriate models that could provide the most accurate SMP estimates at a particular grid point; (2) delineating the climatic homogeneous regions; and (3) providing sub-regional intensity–duration–frequency (IDF) estimates. More specifically, five popular probability distributions (Generalized Extreme Value, Generalized Logistic, Weibull, Gamma, Log-Normal) are compared; cluster analysis was employed to delineate a set of homogeneous sub-regions and one empirical model (Montana) was used to represent IDF relationships. From the results, it was found that: (1) CS product is more compatible with mean and maximum observed SMP time series than that of ANUSPLIN and CRCM 4.1.1 datasets, especially in summer; (2) Generalized Extreme Value represents the primary distribution pattern for the study area; (3) southern Quebec can be delineated into two distinct homogeneous sub-regions, especially in winter; and (4) Montana equation provides an accurate IDF model. This study can be viewed as an initial step towards the development of IDF curves under non-stationary conditions within the context of seasonal features in the regional precipitation regime.

Key words | duration and occurrence, frequency analysis, precipitation extremes, seasonal maximum series

INTRODUCTION

Under the global warming scenario, climate models generally project an increase in large-scale precipitation events (Houghton et al. 2001). Indeed, alterations in patterns of global atmospheric circulation are projected to modify mean annual precipitation and to increase inter- and intra-annual variability of precipitation (Easterling et al. 2000; Seneviratne et al. 2006; IPCC 2007). However, at the regional and local scales, impacts of climate changes are felt most strongly through changes in extreme events. Recent studies have shown that there has been an increasing trend of observed extreme precipitation events during recent decades in the USA and Australia (Easterling et al. 2000; Groisman et al. 2001; Kunkel 2003), in the UK in winter (Osborn & Hulme 2000), and in South Africa (Fauchereau et al. 2003). In Canada, extreme rainfall events show no significant and/or consistent trends (Zhang et al. 2001; Kunkel 2003; Vincent & Mekis 2006). However, Vincent & Mekis (2006) noted that the inconsistency in these trends could be due to the high variability of the extreme precipitation events or to the sparse station network, especially in
northern Canada. The lack of consistent patterns in extreme precipitation highlights the need to explore local characteristics of precipitation extremes for regional studies.

In engineering projects, when it comes to making operational water resources planning and designing infrastructure, it is important for water managers to be informed adequately concerning the frequency of extreme precipitations in a region. According to Brian et al. (2006, unpublished paper), rainfall frequency analyses are used extensively in the design of systems to handle storm runoff, including roads, culverts, and drainage systems. The objective of the frequency analysis is to relate the magnitude of an extreme event to its frequency of occurrence through the use of probability distributions (Bobée 1999). This method has been the subject of several studies dealing with floods (e.g., Bobée & Ashkar 1991) and low streamflow (e.g., Kroll & Vogel 2002; Ouarda et al. 2008; Benyahya et al. 2009). In order to better understand extreme events, the focus needs to be not only on their intensity but also on their duration. This is why the evaluation of extreme events, as embodied in the intensity–duration–frequency (IDF) relationship, has been a major focus of both theoretical and applied hydrology (Langousis & Veneziano 2007). IDF curve development and characterization of regional rainfall extremes are areas of continuing research, as evidenced by many studies (Madsen et al. 2002). Prodanovic & Simonovic (2007) developed IDF curves for the current and future climate for the city of London (Ontario, Canada) using a K-Nearest Neighbor (K-NN)-based weather generator. Artificial Neural Networks (ANN) have been successfully used for the rainfall IDF process (Garcia-Bartual & Schneider 2001; Senocak & Acar 2007). Kim et al. (2008) improved the accuracy of IDF curves by using long and short duration separation techniques. They derived the IDF curve by using cumulative distribution function of the site of interest and a multi-objective genetic algorithm (MOGA). Ben-Zvi (2009) estimated IDF relationships using a partial duration series and pointed out improvements in the prediction of maximum rain intensity values, compared to intensity values estimated from the annual series. Huard et al. (2010) applied a Bayesian analysis to the estimation of IDF curves. Mailhot et al. (2007) used outputs from Canadian Regional Climate Models (CRCM A2) to develop IDF over southern Quebec, for the grid scale of 45 km. The results indicated the limitation of using grid scale and acknowledged that they may be improved by using point estimates. Other detailed descriptions of the future changes of extreme precipitations from regional climate model simulation in intensity and duration can be found in Mladjic et al. (2011), as well as from statistical downscaling results over southern Quebec in Jeong et al. (2013).

In Canada, except for the Maritime and Western provinces (where large-scale influences play an important role in the annual maximum precipitation during the winter season), the extremes are prominent during the summer period for most of the country. That is why a seasonally and regionally based analysis needs to be done to explore local characteristics of precipitation extremes. As noted by Mladjic et al. (2011), it should be recognized that there is currently no comprehensive high-resolution observed dataset of precipitation that would allow a satisfactory analysis of precipitation extremes. In view of these issues, the main goal of the present study is to compare two gridded product data interpolated using Cubic Spline (CS) and ANUSPLIN (Australian National University SPline INterpolator; Hutchinson 1995; Hutchinson et al. 2009) methods and one series of CRCM 4.1.1 simulation throughout southern Quebec (Canada). For each product and for winter and summer periods, the specific objectives are: (1) to determine the ‘best’ probability distribution for multi-day seasonal maximum precipitation (SMP) (i.e., 1, 2, and 3 consecutive days); (2) to delineate the climatic homogeneous regions for SMP; and (3) to provide sub-regional IDF estimates, in order to provide those sub-regions with relevant risk assessment and adaptation strategies. To the best of our knowledge, this study is the first attempt to conduct an evaluation (in terms of regional frequency analysis) of various regional climate products in southern Quebec. This investigation is expected to contribute to exploring the complex feature of extreme precipitation in southern Quebec, which is beneficial to engineers and water resource managers, and future climate change studies.

The remainder of the paper is structured as follows. The next section presents the study area and the data. This is followed by the methodology used for local frequency analysis, delineation of homogeneous sub-regions and IDF analysis of SMP and application of an empirical regional prediction model. The results and discussion follow and finally, conclusions are drawn.
METHODS

Data and study area

The study region encompasses the southern Quebec (Canada) area (south of 50°N) (Figure 1). In order to compare with CRCM simulations, two gridded datasets obtained by interpolation technique are used. The first product corresponds to CS data in which we have used a Cubic Spline method for interpolating daily precipitation from Environment Canada meteorological stations (69) in southern Quebec onto a 45-km polar stereographic grid over the 1961–1999 period. The second product corresponds to ANUSPLIN data developed by Hutchinson et al. (2003), who have developed a Canada-wide spatial interpolation tool (≈10-km gridded climate dataset) for daily precipitation from Environment Canada stations. The 10-km gridded dataset of daily precipitation has been downgraded to 45-km grid spacing by Eum et al. (2012) over the same area. ANUSPLIN data were generated using thin-plate smoothing splines fitted to observations in three dimensions: latitude, longitude, and elevation. These are analogous to cubic splines in one dimension. For the validation of the above two methods, the CRCM simulations driven by NCEP/NCAR (National Centers for Environmental Protection, National Centre for Atmospheric Research) version 4.1.1 data (e.g. Brochu & Laprise 2007; Music & Caya 2007) were selected. This selection was based on the availability of continuous daily series for the period 1961–2001 at a spatial resolution of 45 km.

We first reasoned that, as smoothing effect is inherent to all interpolation methods, there is still no interpolation method which will guarantee the best results for all datasets. Second, smoothing always increases as the number of neighbors increases (Herzfeld et al. 1993, Figure 3). However, to assess the smoothing effect, the cross-validation technique (i.e., Jackknife resampling) can be used. It should be kept in mind that even if interpolation methods tend to smooth data in space and time, they remain widely used. Indeed, as part of the European Union Framework 6 ENSEMBLES project, Haylock et al. (2008) used a two-stage process in their methodology: station data are first interpolated as...
point estimates to a fine grid, after which the point estimates are averaged to obtain area averages for the 25 and 50-km grids used by various regional modeling centers to validate regional-scale simulations over Europe (see the recent study of Kjellström et al. (2010)).

Local frequency analysis of seasonal maximum precipitation (SMP)

SMP values for the winter (January, February, and March) and the summer (June, July, and August) were extracted from the original time series of daily precipitation. In traditional design frequency analysis, the following steps are used:

- verify hypotheses of independence, stationarity, and homogeneity;
- select a group of ‘candidate distributions’;
- fit each statistical distribution to the observed maxima by estimating the distribution parameters;
- select the most adequate frequency distribution based on goodness-of-fit;
- estimate quantiles for specific return periods.

The time series should be composed of independent (no autocorrelation), homogeneous (i.e., all data come from a single population), and stationary (absence of trend) data. Independence was tested using the autocorrelation coefficient of lag-1, homogeneity was verified using the Wilcoxon test (Wilcoxon 1945), and stationarity was assessed using the Kendall test (Kendall 1975). For all tests, a significance level of 1% was used to accept or reject the null hypotheses. If all hypotheses are verified, statistical distributions can be fitted to the data. In engineering practice, the choice of a suitable probability model is still a problem since there is no general agreement as to which distribution(s) should be used for extreme events. Hence, five candidate distributions were considered at each grid point: Generalized Extreme Value (GEV), Weibull (WEI3), Generalized Logistic (GLO), Gamma (GAM), and Log-Normal (LN2). Table 1 presents the probability density function and the associated parameters for each of these models. In general, more parameters lead to greater flexibility and hence, to a better fit to the data. However, in some cases, difficulties in the parameter estimation arise. Testing two- and three-parameter distributions allows for selecting the best trade-off between goodness-of-fit and parsimony. To estimate the parameters for each of the aforementioned distributions, different procedures exist, including the method of L-moments, the maximum likelihood method, and the method of probability-weighted moments (e.g., Kite 1977; Hosking & Wallis 1997). However, in the present study, the method of maximum likelihood was used since it gives asymptotically

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability density function</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalized Extreme Value (GEV)</td>
<td>$f(x) = \frac{1}{\alpha} \exp\left(\left(1 + \frac{x - \xi}{\alpha}\right)^{-1/k}\right) \left(1 + \frac{x - \xi}{\alpha}\right)^{-1-1/k}$</td>
<td>$\alpha$: scale parameter; $k$: shape parameter; $\xi$: location parameter</td>
</tr>
<tr>
<td>Weibull (WEI3)</td>
<td>$f(x) = \frac{\alpha}{\beta} \left(\frac{x - \gamma}{\beta}\right)^{a-1} \exp\left(-\left(\frac{x - \gamma}{\beta}\right)^a\right)$</td>
<td>$a$: shape parameter; $\beta$: scale parameter; $\gamma$: location parameter</td>
</tr>
<tr>
<td>Generalized Logistic (GLO)</td>
<td>$f(x) = \frac{x - \mu}{\sigma} e^{-\frac{x - \mu}{\sigma}} \left(1 + \frac{x - \mu}{\sigma}\right)^{-1/k}$</td>
<td>$\sigma$: scale parameter; $\mu$: location parameter</td>
</tr>
<tr>
<td>Gamma (GAM)</td>
<td>$f(x) = \frac{1}{b \Gamma(a)} x^{a-1} e^{-x/b}$</td>
<td>$a$: shape parameter; $b$: scale parameter; $\Gamma(.)$: gamma function</td>
</tr>
<tr>
<td>Log-Normal (LN2)</td>
<td>$f(x) = \frac{1}{\sqrt{2\pi}} x e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$</td>
<td>$\sigma$: scale parameter; $\mu$: location parameter</td>
</tr>
</tbody>
</table>
optimal estimators of the parameters (i.e., unbiased with minimum variance; Ashkar et al. 1994) and it is one of the most commonly used methods for estimating parameters of a statistical distribution. Moreover, this method can produce good estimators for three parameter distributions (Bobée 1979).

Local estimates of SMP quantiles were obtained using the selected statistical distribution for each season. The goodness-of-fit between the frequency of occurrence of the gridded/simulated data and the expected frequencies obtained from the hypothesized distribution was assessed using statistical criteria. They include the Anderson–Darling (AD) (D’Agostino & Stephens 1986) and Kolmogorov–Smirnov (KS) statistics. The best-fitted distribution is the one associated with the smallest value of each criterion. Finally, the grid-point SMP quantiles corresponding to the return periods \( T = 5, 10, 20, 50, \) and 80 years were produced. All frequency analyses and statistical tests were performed using the Matlab (version 7.8.0 R2009a) software. It is important to note that for the remainder of the document, the term ‘gridded’ is related to CS and ANUSPLIN datasets, the term ‘simulated’ is related to CRCM, and the slash ‘/’ character means ‘or’ depending on the type of data used.

Delineation of homogeneous sub-regions

In the present study, the task is to delineate the study area into sufficiently small numbers of homogeneous sub-regions by grouping grid points or area according to their similarities in precipitation regime. Several attempts have been made by different authors to identify homogeneous regions based on geographical, hydrological, climatic, and physiographic variables. These techniques include the regression analysis, L-moment method (Onibon et al. 2004), multivariate methods (Chokmani & Ouarda 2004), and non-parametric approaches (Ouarda & Shu 2009). Chen & Hong (2012) used the principal component analysis, self-organizing maps, and the L-moment method, for improving estimation of desired rainfall quantiles of ungauged sites in Taiwan. As there is no unique approach for identifying homogeneous regions, in the present study, the hierarchical cluster analysis was employed based on quantiles of daily SMP estimated from the ‘best’ distribution. For each data product and over the 83 grid points, the technique synthesized the SMP data by splitting them into small and homogeneous clusters such that the data inside the same cluster are more similar to each other than to the data inside the other clusters. The resulting clusters can be represented in the form of the joining diagram, e.g., dendrogram with similar groups appearing on the same ‘branches’. As there is no formal measure of an ‘optimal’ number of clusters, the choice of a suitable threshold is subjective. In the present study, as the study area is relatively small, the threshold is chosen to define two sub-regions. Then, in order to combine the multiple partitions obtained by each data product into a single consensus cluster, the Cluster-based Similarity Partitioning Algorithm (CSPA; Strehl & Ghosh 2002) was used. The CSPA is a novel, robust, and efficient combination method that uses the combined similarity matrix to recluster the data using a similarity-based clustering algorithm (METIS; Karypis & Kumar 1998).

Intensity–duration–frequency analysis of SMP and development of regional prediction models

The third part of the study was devoted to the SMP IDF analysis, in which the objective is to estimate the maximum intensity \( i \) of seasonal precipitation (in mm/d) for any duration \( d \) (in days) and return period \( T \) (in years). The IDF relation is expressed mathematically as follows:

\[
i = f(T, d) \tag{1}
\]

In the first step, samples of seasonal maximum 1, 2, and 3-day precipitation amounts were drawn from each grid-point record using a moving window. In the second step, the quantiles of a set of selected return periods (e.g., 5, 10, 20, 50, and 80 years) were estimated for each duration. This is done by using the ‘best’ probability distribution functions obtained from 1-SMP analysis. In the third step, the empirical formulas are used to construct the SMP IDF curves. The least-square method is applied to determine the parameters of the empirical IDF equation that is used to represent intensity–duration relationships. As the selected durations are less or equal to 3 days, a simple empirical formula with fewer parameters is selected to avoid the problem of over-parameterization. In the present study, one IDF model was tested, as it is one of the most commonly used
functions in water resources engineering (Montana type; i.e., Mohymont & Demarée 2006).

\[ i(d, T) = \frac{a(T)}{d(nT)} \]  

(2)

where \( a \) and \( \eta \) are coefficients to be estimated.

RESULTS AND DISCUSSION

The SMP data over the 83 grid points obtained from each dataset (CS, ANUSPLIN, and CRCM4.1.1) are first compared with the observed meteorological stations (69) over the whole study area and the entire time window (1961–1999). The summary of various statistics (i.e., minimum, maximum, mean, skewness, and coefficient of variation) for 1-day SMP is presented in Figure 2. For both summer and winter seasons, the observed variability of all the statistics is greater than that of the gridded/simulated data products, which means that these products systematically underestimate the observed variance. This result can be the consequence of the so-called ‘smoothing effect’ often associated with interpolation (or the search neighborhood). Moreover, with respect to the mean, it is clear that, for both seasons, CS shows values close to the observations, while ANUSPLIN and CRCM tend to show negative (underestimation) bias. For the minimum SMP, in winter, the two gridded products (CS and ANUSPLIN) overestimate the observed values, while the CRCM underestimates them. In summer months, the gridded/simulated SMP values reproduced the observed data quite well; while there is a slight overestimation from the CS values. For the maximum SMP, in winter, the values are also quite well reproduced by all gridded/simulated data, except for the ANUSPLIN which underestimates the observed values. In summer, this location parameter is systematically underestimated by all gridded/simulated values, especially with the CRCM.
datasets (i.e., by more than 20 mm/d in median values). It may be noted that the range of coefficient of variation values of all data products are generally less than 0.5 both in winter and summer seasons, which indicates that the data are relatively closely clustered. More differences between CS and ANUSPLIN vs. CRCM values are present in the winter season for this parameter than for summer. Also, it is indicated that the distributions of precipitation are more positively skewed in summer than in winter, and all gridded/simulated products reproduce relatively well these features.

The hypotheses of independence, homogeneity, and stationarity were verified using respectively, autocorrelation coefficient of lag-1, Wilcoxon and Kendall tests, on each grid point for winter and summer (Table 2). All time series of seasonal maximum precipitations were tested for stationarity (Kendall test), independence (serial autocorrelation), and homogeneity (Wilcoxon test), as a prerequisite for frequency analysis. Table 2 shows the percentage of grid points that successfully passed these tests at significance levels of 1%. The test results confirmed that the majority of the grid-point series are homogeneous and independent (Table 2). However, an exception has occurred in cases where the stationarity and homogeneity conditions have not been satisfied. For example, during the winter, CRCM4.1.1 product displayed significant trend for only 6 and 8% of grid points for durations of 1 day and 2 days, respectively (Table 2). Also, during the summer, a non-homogeneity in CS was detected for three (3.6%) and two (2.4%) grid points for durations of 2 days and 3 days, respectively (Table 2). In terms of serial autocorrelation, with the exception of some grid points for CRCM4.1.1 (between 1 and 3), the results showed that all the autocorrelation coefficients of lag-1 achieved values lower than 0.4 (in absolute value). This implies, in the present study, that the SMP data are serially independent. In the present study, grid points for which the above-mentioned conditions are not fully satisfied were further removed from the analysis.

As stated earlier, in order to determine the ‘best’ fit model(s) at each grid point that met the requirements for frequency analysis, five probability distribution models were subjected to two (2) statistical criteria: the Anderson–Darling (AD) and Kolmogorov–Smirnov (KS) tests. The assessment of the probability distribution models was based on the total test score obtained from all the criteria. Criteria scores ranging from one to five (1–5) are awarded to each distribution model based on the criteria that the distribution with the highest total score is chosen as the ‘best’ distribution model for the data in the study area. In general, the ‘best’ distribution is awarded a score of five (5), the next best is awarded four (4), and so on, in descending order. Overall ranks of each distribution were obtained by summing the individual ranks of each criterion. Table 3 summarizes the overall ranking results for winter and summer seasons, with ranks given in parentheses. Examination of the goodness-of-fit test results reveals that in many cases there was negligible difference between the various distributions for the whole study area. The overall ranks

<table>
<thead>
<tr>
<th>CS (winter)</th>
<th>ANUSPLIN (winter)</th>
<th>CRCM 4.1.1 (winter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 day</td>
<td>2 days</td>
<td>3 days</td>
</tr>
<tr>
<td>Independence</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Stationarity</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Homogeneity</td>
<td>98.8%</td>
<td>100%</td>
</tr>
<tr>
<td>CS (summer)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Independence</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Stationarity</td>
<td>100%</td>
<td>96.4%</td>
</tr>
<tr>
<td>Homogeneity</td>
<td>100%</td>
<td>96.4%</td>
</tr>
</tbody>
</table>

Table 2 | Results of the Mann–Kendall and Wilcoxon tests as well as autocorrelation coefficients on SMP for various durations (1, 2, and 3 days) and for CS, ANUSPLIN, and CRCM4.1.1. Percentages of grid points that showed p-values values greater to the significance level of 0.01.
combined show that the GEV distribution was the most appropriate probability distribution model to describe the SMP series for all products for summer and winter seasons in southern Quebec (Table 3). It was also found that LN and WEI3 gave quite good performances, depending on the product and the season. For instance, the WEI3 distribution was found to be the second ‘best’ fitted distribution for CS and ANUSPLIN in winter and for CRCM 4.1.1 in summer. The GLO and the GAM distributions ranked consistently poorly compared to the others. However, the fact that a distribution has a low ranking does not necessarily mean that it performed poorly, since the differences in goodness-of-fit between different distributions was relatively small in some cases. Moreover, it can be expected that distributions with three parameters (GEV and WEI3) could provide better fit to the data. This appropriateness of the GEV distribution was also suggested in the literature by Overeem et al. (2008), Veneziano et al. (2009), and Kingumbi & Mailhot (2010). The popularity of the GEV stems, in part, from the fact that it has been shown to be a combination of the three families of extreme value distributions: Weibull, Fréchet, and Gumbel (e.g., Katz & Brown (1992) for details).

With series of SMP modeled by the two ‘best’ distributions (namely GEV and/or LN and WEI3), it becomes possible to estimate the SMP for different return periods. Since the dataset covers 39 years, in practice, the highest return period to be considered for further analysis is typically less or equal to twice the length of record ($T \leq 2 \times 39 \approx 80$ years). Quantiles associated with return periods of 2, 5, 10, 20, and 80 years (corresponding respectively to probabilities of non-exceedance of 0.80, 0.90, 0.95, 0.98, and 0.9875) were estimated. For each data series and each season, the degree of fit of each distribution was visually examined (Figures 3 and 4) with gridded/simulated vs. values produced by the frequency analysis for the whole study area (considered as one population). For low return periods, no noticeable differences between the GEV, LN and between GEV and WEI3 distributions were suggested, whereas for high return periods (e.g., 50 and 80 years), more consistent probabilities were revealed from LN and WEI3 distributions, respectively for CS and CRCM4.1.1 datasets, irrespective of the season (Figures 3 and 4). Moreover, in winter, the overall quantiles for ANUSPLIN and CRCM4.1.1 were more often lower than those for the CS product (Figure 3), confirming the systematic underestimation with respect to observed values suggested in the box plots (i.e., Figure 2). This is also the case in summer, but more clearly for high return periods (e.g., 50 and 80 years,

---

Table 3 | Summarized results of the fitted distributions (GEV, GLO, GAM, LN, and WEI3) on SMP for 1-day duration of CS, ANUSPLIN, and CRCM 4.1.1 and their corresponding test scores (in parentheses)*. The values are averaged from all grid-point information of the study area.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Distribution</th>
<th>CS Average values (winter)</th>
<th>ANUSPLIN Average values (winter)</th>
<th>CRCM 4.1.1 Average values (winter)</th>
<th>CS Average values (summer)</th>
<th>ANUSPLIN Average values (summer)</th>
<th>CRCM 4.1.1 Average values (summer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>GEV</td>
<td>0.39 (5)</td>
<td>0.29 (5)</td>
<td>0.26 (5)</td>
<td>0.27 (5)</td>
<td>0.34 (5)</td>
<td>0.29 (5)</td>
</tr>
<tr>
<td></td>
<td>GLO</td>
<td>0.57 (1)</td>
<td>0.43 (1)</td>
<td>0.76 (1)</td>
<td>0.78 (1)</td>
<td>1.03 (1)</td>
<td>0.78 (1)</td>
</tr>
<tr>
<td></td>
<td>GAM</td>
<td>0.43 (2)</td>
<td>0.34 (3)</td>
<td>0.50 (2)</td>
<td>0.53 (2)</td>
<td>0.87 (2)</td>
<td>0.61 (2)</td>
</tr>
<tr>
<td></td>
<td>LN</td>
<td>0.43 (3)</td>
<td>0.36 (2)</td>
<td>0.54 (4)</td>
<td>0.37 (4)</td>
<td>0.64 (3)</td>
<td>0.44 (3)</td>
</tr>
<tr>
<td></td>
<td>WEI3</td>
<td>0.40 (4)</td>
<td>0.33 (4)</td>
<td>0.40 (3)</td>
<td>0.41 (3)</td>
<td>0.63 (4)</td>
<td>0.42 (4)</td>
</tr>
<tr>
<td>KS</td>
<td>GEV</td>
<td>0.10 (5)</td>
<td>0.10 (5)</td>
<td>0.08 (5)</td>
<td>0.08 (5)</td>
<td>0.09 (5)</td>
<td>0.09 (5)</td>
</tr>
<tr>
<td></td>
<td>GLO</td>
<td>0.10 (5)</td>
<td>0.10 (5)</td>
<td>0.11 (2)</td>
<td>0.11 (2)</td>
<td>0.10 (4)</td>
<td>0.11 (3)</td>
</tr>
<tr>
<td></td>
<td>GAM</td>
<td>0.10 (5)</td>
<td>0.11 (4)</td>
<td>0.11 (2)</td>
<td>0.11 (2)</td>
<td>0.11 (3)</td>
<td>0.12 (2)</td>
</tr>
<tr>
<td></td>
<td>LN</td>
<td>0.10 (5)</td>
<td>0.10 (5)</td>
<td>0.09 (4)</td>
<td>0.096 (4)</td>
<td>0.10 (4)</td>
<td>0.10 (4)</td>
</tr>
<tr>
<td></td>
<td>WEI3</td>
<td>0.10 (5)</td>
<td>0.10 (5)</td>
<td>0.10 (3)</td>
<td>0.10 (3)</td>
<td>0.11 (3)</td>
<td>0.10 (4)</td>
</tr>
<tr>
<td>Total scores</td>
<td>GEV</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>GLO</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>GAM</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>LN</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>WEI3</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

KS: Kolmogorov-Smirnov test; AD: Anderson Darling statistic; RMSE: root mean square error. *Best distribution is awarded a score of five (5), the next best is awarded four (4), and so on, in descending order.
Figure 3 | Plots of SMP gridded/simulated probabilities of non-exceedence (PNE) vs. values produced by the frequency analysis for different return period for all grid points and for the winter period.
Figure 4  |  Same as Figure 3 but for the summer season.
Climate classification resulting from the hierarchical cluster algorithm and based on winter maximum quantiles (d = 1 day) estimated by GEV for CS (a), ANUSPLIN (c), and CRCM 4.1.1 (e) data and geographical representation of their respective sub-regions (SR) (b), (d), and (f); (g) is the cluster ensembles (black unfilled circles represent the grid points that were removed from the analysis).
Figure 6  Climate classification resulting from the hierarchical cluster algorithm and based on summer maximum quantiles ($d = 1$ day) estimated by GEV for CS (a), ANUSPLIN (c), and CRCM 4.1.1 (e) data and geographical representation of their respective sub-regions (SR) (b), (d), and (f); (g) is the cluster ensembles.
as also revealed in mean and maximum SMP values in Figure 2).

Before performing the IDF analysis, it was necessary to delineate homogeneous regions in southern Quebec for each data product. Hierarchical clustering was performed for the 5, 10, 20, 50 and 80th quantiles of SMP estimated by GEV for each data product and season. The dendrograms are presented in the left panels of Figures 5 and 6, while the geographical representation of each cluster is shown in the right panels. Using the linkage distance of 15 specified in Figure 5(a), two sub-regions (SR) have been identified (SR1 and SR2). The results of the reclustering classification for southern Quebec are shown at the bottom of Figures 5 and 6 for winter and summer, respectively. In winter, the two sub-regions are defined along the axis of the St-Lawrence River (southwest–northeast axis), reflecting some of the climatological differences in the precipitation regime associated with the known synoptic patterns of storm tracks coming from the Great Lakes area toward the Gulf of St-Lawrence. However, in summer, these regional features disappear whereas the spatial variability of precipitation increases, i.e., as those are more driven by regional and local forcing factors as topography or surface conditions with a decrease in influence of large-scale systems during such season.

For each of the analyzed grid points, the frequency analysis described earlier is carried out using the time series of SMP with durations of 1, 2, and 3 days. In the present case, the statistical distributions tested on the SMP with various durations were limited to the GEV. Over the whole area, the average values of the KS and AD tests’ performance criteria revealed that the GEV was accepted for all durations (with a significance level of 1%) and was shown to provide the ‘best’ fit for southern Quebec (see Tables 3 and 4). For instance, locally, for the winter period and for all durations, GEV was selected for 53%, 55%, and more than 65% of the analyzed grid points of ANUSPLIN, CS, and CRCM4.1.1, respectively, followed by LN and WEI distributions that account for 35% and less than 45% of the grid points of CRCM4.1.1 and of both CS and ANUSPLIN, respectively.

Having defined the homogeneous sub-regions, the next step was to develop IDF estimates. Figure 7 represents the sub-regional IDF curves in which the quantiles were obtained by fitting the GEV distribution to d-SMP (duration-SMP) data. According to this figure, for the entire data products and for the same return period, short SMPs are more intense than long SMPs, especially in summer and for higher return periods. It is also noted that the spreading of the IDF curves is relatively similar among the various datasets for high return period values, but is much larger in the case of the SR2, especially in summer and for CS values (Figure 7). For example, for 1-day SMP based on CS data, the ratio between the intensity corresponding to \( T = 80 \) years and the intensity corresponding to \( T = 5 \) years is equal to 1.89 for SR2 but to 1.69 for SR1.

### Table 4

<table>
<thead>
<tr>
<th>Duration</th>
<th>Criterion</th>
<th>Distribution</th>
<th>Average values (winter)</th>
<th>Average values (summer)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>CS</td>
<td>ANUSPLIN</td>
</tr>
<tr>
<td>2 days</td>
<td>AD</td>
<td>GEV</td>
<td>0.32 (2)</td>
<td>0.26 (2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WEI3/LN</td>
<td>0.37 (1)</td>
<td>0.28 (1)</td>
</tr>
<tr>
<td></td>
<td>KS</td>
<td>GEV</td>
<td>0.09 (2)</td>
<td>0.09 (2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WEI3/LN</td>
<td>0.10 (1)</td>
<td>0.10 (1)</td>
</tr>
<tr>
<td></td>
<td>Total scores</td>
<td>GEV</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WEI3/LN</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3 days</td>
<td>AD</td>
<td>GEV</td>
<td>0.34 (2)</td>
<td>0.33 (2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LN/WEI</td>
<td>0.39 (1)</td>
<td>0.41 (1)</td>
</tr>
<tr>
<td></td>
<td>KS</td>
<td>GEV</td>
<td>0.09 (2)</td>
<td>0.09 (2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LN/WEI</td>
<td>0.10 (1)</td>
<td>0.10 (1)</td>
</tr>
<tr>
<td></td>
<td>Total scores</td>
<td>GEV</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LN/WEI</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

KS: Kolmogorov-Smirnov test; AD: Anderson Darling statistic; RMSE: root mean square error.

*As we have two distributions the best distribution is awarded a score two (2) and the next best is awarded one (1).
Figure 7 | Sub-regional IDF curves (gridded/simulated): intensity SMP vs. duration d (1, 2, and 3 days) for different return periods T (5, 10, 20, 50, and 80 years) from CS, ANUSPLIN, and CRCM 4.1.1 during the winter (four top panels) and summer (four bottom panels) seasons obtained from the GEV distribution, and for the two sub-regions (SR1 and SR2) defined in Figure 5(g).
The present study compared extreme seasonal precipitation from two gridded product data (CS and ANUSPLIN) with those estimated by the CRCM4.1.1. The ANUSPLIN and CRCM4.1.1 values were calculated for winter and summer periods and for the two sub-regions (SR1 and SR2). The parameters of the IDF equations and the root mean square errors (RMSEs) for different return periods (5, 10, 20, 50, and 80 years) were calculated for winter and summer (Table 5) and Figure 8 show that the Montana equation may fit well in southern Quebec, with RMSE values less than 1 mm/d. Indeed, in the case of CS/SR2, the 5-year return period was calculated for winter and the 3-year return period for summer. The parameters of the IDF equations and the root mean square errors (RMSEs) were calculated for winter and summer periods. The ratios between the 1-day extreme quantity and the 3-day extreme quantity corresponding to all return periods were between 2.21 and 2.27 for the gridded values and between 2.11 and 2.27 for those estimated by Montana and between 2.09 and 2.24 for those estimated by CRCM4.1.1, respectively. SR2 is a region that is known for convective summer precipitations in the vicinity of the Appalachian Mountains that are more intense than in the northwestern area. For example, the Lennoxville station, located in the southeastern corner of the study area, receives on average more than 119 mm during the August month, by comparison with only around 93 mm during the same period at the Val-d’Or station located in the northwest of the domain (i.e., values corresponding to 1971–2000 climate normal; see http://climate.weather.gc.ca/climate_normals/index_f.html). These features are not reproduced by the ANUSPLIN and the CRCM-simulated values. Again, caution should be required when using ANUSPLIN and CRCM4.1.1 products. Indeed, Mladjic et al. (2014) arrived at similar conclusions. Their validation of precipitation quantiles with 20-, 50-, and 100-yr return periods for single- and multi-day events against the observed values suggests an underestimation of extreme events by the CRCM over most of Canada.

In order to provide the regional intensity-duration relationships for engineers and water resources managers without necessarily carrying out complex calculations, one empirical IDF equation (Equation (2)) was investigated for each data product in southern Quebec. The parameters of the IDF equations and the root mean square errors (RMSEs) for different return periods (5, 10, 20, 50, and 80 years) were calculated for winter and summer (Table 5). Table 5 and Figure 8 show that the Montana equation may fit well in southern Quebec, with RMSE values less than 1 mm/d. Indeed, in the case of CS/SR2, the 5-year return period was calculated for winter and the 3-year return period for summer. The ratios between the 1-day extreme quantity and the 3-day extreme quantity corresponding to all return periods were between 2.21 and 2.27 for the gridded values and between 2.11 and 2.27 for those estimated by Montana and between 2.09 and 2.24 for those estimated by CRCM4.1.1, respectively. SR2 is a region that is known for convective summer precipitations in the vicinity of the Appalachian Mountains that are more intense than in the northwestern area. For example, the Lennoxville station, located in the southeastern corner of the study area, receives on average more than 119 mm during the August month, by comparison with only around 93 mm during the same period at the Val-d’Or station located in the northwest of the domain (i.e., values corresponding to 1971–2000 climate normal; see http://climate.weather.gc.ca/climate_normals/index_f.html). These features are not reproduced by the ANUSPLIN and the CRCM-simulated values. Again, caution should be required when using ANUSPLIN and CRCM4.1.1 products. Indeed, Mladjic et al. (2014) arrived at similar conclusions. Their validation of precipitation quantiles with 20-, 50-, and 100-yr return periods for single- and multi-day events against the observed values suggests an underestimation of extreme events by the CRCM over most of Canada.

In order to provide the regional intensity-duration relationships for engineers and water resources managers without necessarily carrying out complex calculations, one empirical IDF equation (Equation (2)) was investigated for each data product in southern Quebec. The parameters of the IDF equations and the root mean square errors (RMSEs) for different return periods (5, 10, 20, 50, and 80 years) were calculated for winter and summer (Table 5). Table 5 and Figure 8 show that the Montana equation may fit well in southern Quebec, with RMSE values less than 1 mm/d. Indeed, in the case of CS/SR2, the 5-year return period was calculated for winter and the 3-year return period for summer. The ratios between the 1-day extreme quantity and the 3-day extreme quantity corresponding to all return periods were between 2.21 and 2.27 for the gridded values and between 2.11 and 2.27 for those estimated by Montana and between 2.09 and 2.24 for those estimated by CRCM4.1.1, respectively. SR2 is a region that is known for convective summer precipitations in the vicinity of the Appalachian Mountains that are more intense than in the northwestern area. For example, the Lennoxville station, located in the southeastern corner of the study area, receives on average more than 119 mm during the August month, by comparison with only around 93 mm during the same period at the Val-d’Or station located in the northwest of the domain (i.e., values corresponding to 1971–2000 climate normal; see http://climate.weather.gc.ca/climate_normals/index_f.html). These features are not reproduced by the ANUSPLIN and the CRCM-simulated values. Again, caution should be required when using ANUSPLIN and CRCM4.1.1 products. Indeed, Mladjic et al. (2014) arrived at similar conclusions. Their validation of precipitation quantiles with 20-, 50-, and 100-yr return periods for single- and multi-day events against the observed values suggests an underestimation of extreme events by the CRCM over most of Canada.

In order to provide the regional intensity-duration relationships for engineers and water resources managers without necessarily carrying out complex calculations, one empirical IDF equation (Equation (2)) was investigated for each data product in southern Quebec. The parameters of the IDF equations and the root mean square errors (RMSEs) for different return periods (5, 10, 20, 50, and 80 years) were calculated for winter and summer (Table 5). Table 5 and Figure 8 show that the Montana equation may fit well in southern Quebec, with RMSE values less than 1 mm/d. Indeed, in the case of CS/SR2, the 5-year return period was calculated for winter and the 3-year return period for summer. The ratios between the 1-day extreme quantity and the 3-day extreme quantity corresponding to all return periods were between 2.21 and 2.27 for the gridded values and between 2.11 and 2.27 for those estimated by Montana and between 2.09 and 2.24 for those estimated by CRCM4.1.1, respectively. SR2 is a region that is known for convective summer precipitations in the vicinity of the Appalachian Mountains that are more intense than in the northwestern area. For example, the Lennoxville station, located in the southeastern corner of the study area, receives on average more than 119 mm during the August month, by comparison with only around 93 mm during the same period at the Val-d’Or station located in the northwest of the domain (i.e., values corresponding to 1971–2000 climate normal; see http://climate.weather.gc.ca/climate_normals/index_f.html). These features are not reproduced by the ANUSPLIN and the CRCM-simulated values. Again, caution should be required when using ANUSPLIN and CRCM4.1.1 products. Indeed, Mladjic et al. (2014) arrived at similar conclusions. Their validation of precipitation quantiles with 20-, 50-, and 100-yr return periods for single- and multi-day events against the observed values suggests an underestimation of extreme events by the CRCM over most of Canada.

In order to provide the regional intensity-duration relationships for engineers and water resources managers without necessarily carrying out complex calculations, one empirical IDF equation (Equation (2)) was investigated for each data product in southern Quebec. The parameters of the IDF equations and the root mean square errors (RMSEs) for different return periods (5, 10, 20, 50, and 80 years) were calculated for winter and summer (Table 5). Table 5 and Figure 8 show that the Montana equation may fit well in southern Quebec, with RMSE values less than 1 mm/d. Indeed, in the case of CS/SR2, the 5-year return period was calculated for winter and the 3-year return period for summer. The ratios between the 1-day extreme quantity and the 3-day extreme quantity corresponding to all return periods were between 2.21 and 2.27 for the gridded values and between 2.11 and 2.27 for those estimated by Montana and between 2.09 and 2.24 for those estimated by CRCM4.1.1, respectively. SR2 is a region that is known for convective summer precipitations in the vicinity of the Appalachian Mountains that are more intense than in the northwestern area. For example, the Lennoxville station, located in the southeastern corner of the study area, receives on average more than 119 mm during the August month, by comparison with only around 93 mm during the same period at the Val-d’Or station located in the northwest of the domain (i.e., values corresponding to 1971–2000 climate normal; see http://climate.weather.gc.ca/climate_normals/index_f.html). These features are not reproduced by the ANUSPLIN and the CRCM-simulated values. Again, caution should be required when using ANUSPLIN and CRCM4.1.1 products. Indeed, Mladjic et al. (2014) arrived at similar conclusions. Their validation of precipitation quantiles with 20-, 50-, and 100-yr return periods for single- and multi-day events against the observed values suggests an underestimation of extreme events by the CRCM over most of Canada.

In order to provide the regional intensity-duration relationships for engineers and water resources managers without necessarily carrying out complex calculations, one empirical IDF equation (Equation (2)) was investigated for each data product in southern Quebec. The parameters of the IDF equations and the root mean square errors (RMSEs) for different return periods (5, 10, 20, 50, and 80 years) were calculated for winter and summer (Table 5). Table 5 and Figure 8 show that the Montana equation may fit well in southern Quebec, with RMSE values less than 1 mm/d. Indeed, in the case of CS/SR2, the 5-year return period was calculated for winter and the 3-year return period for summer. The ratios between the 1-day extreme quantity and the 3-day extreme quantity corresponding to all return periods were between 2.21 and 2.27 for the gridded values and between 2.11 and 2.27 for those estimated by Montana and between 2.09 and 2.24 for those estimated by CRCM4.1.1, respectively. SR2 is a region that is known for convective summer precipitations in the vicinity of the Appalachian Mountains that are more intense than in the northwestern area. For example, the Lennoxville station, located in the southeastern corner of the study area, receives on average more than 119 mm during the August month, by comparison with only around 93 mm during the same period at the Val-d’Or station located in the northwest of the domain (i.e., values corresponding to 1971–2000 climate normal; see http://climate.weather.gc.ca/climate_normals/index_f.html). These features are not reproduced by the ANUSPLIN and the CRCM-simulated values. Again, caution should be required when using ANUSPLIN and CRCM4.1.1 products. Indeed, Mladjic et al. (2014) arrived at similar conclusions. Their validation of precipitation quantiles with 20-, 50-, and 100-yr return periods for single- and multi-day events against the observed values suggests an underestimation of extreme events by the CRCM over most of Canada.
Figure 8  |  Subregional IDF curves (produced by the frequency analysis): intensity SMP vs. return period $T$ and duration $d$ for CS estimated using Montana equation for winter and summer seasons obtained from the GEV distribution.
interpolated products) and one series of Regional Climate Model simulation (CRCM4.1.1) using frequency analysis. This analysis provides a better overall understanding of the extreme precipitation regime and will contribute to improve climatological and engineering studies. The major conclusions of this study are as follows:

1. CS gridded data are more compatible with the mean and maximum observed SMP, as the ANUSPLIN and CRCM4.1.1 datasets underestimate these observed values, especially in summer. Indeed, over most of Canada, Mladjic et al. (2011) have shown an underestimation of extreme events by the CRCM. Therefore, ANUSPLIN and CRCM4.1.1 products should be used with care for winter and summer SMPs. In particular, ANUSPLIN shows important biases for daily precipitation extreme.

2. Among the five distributions considered, the GEV, WEI3, and LN distributions provided the best performance for daily SMP in southern Quebec, depending on the season and the data product. However, the highest scores of Anderson–Darling and Kolmogorov–Smirnov tests were found for the GEV (three-parameter distribution), as it best fitted the SMP data series for all data products, as well as for various durations and seasons (winter and summer). For the winter period and for all durations, GEV was the best choice for 53%, 55%, and more than 65% of the analyzed grid points of ANUSPLIN, CS, and CRCM4.1.1, respectively, followed by WEI3 distribution that accounted for less than 45% of the grid points of CS and ANUSPLIN LN, and by LN that accounted for 35% of the grid points of CRCM4.1.1. Generally, if one wishes to use a two-parameter distribution to model the current datasets, the best choice is LN.

3. There is a regionally varying change in seasonal extreme precipitation event occurrence across southern Quebec, and two sub-regions have been identified. In winter, they are defined along the axis of the St-Lawrence River (southwest–northeast axis), reflecting some of the climatological differences in the precipitation regime associated with the known synoptic patterns of storm tracks coming from the Great Lakes area toward the Gulf of St-Lawrence. However in summer, because of site specificity, the regional features disappear whereas the local variability of precipitation increases, as it is more driven by some regional-scale forcing factors, such as topography or surface conditions.

4. One empirical function with two parameters (Montana equation) was tested to represent IDF relationships. The IDF empirical model of Montana was appropriate to estimate SMP intensity values for short time duration in the study area.

This study is a first step towards providing an accurate prediction of seasonal maximum precipitation in southern Quebec. In fact, it is anticipated that the research presented in this paper will be built upon to examine the further possibilities of:

1. quantifying the uncertainties by using multi-model ensemble systems for future changes in IDF curves (as suggested by Mailhot et al. 2007) for seasonal maximum rainfall depth;
2. comparing seasonal maximum to peak-over-threshold (POT) approach to combine magnitude and duration;
3. combining multiple RCMs simulations from NARCCAP and CORDEX runs (i.e., http://www.narccap.ucar.edu/ and http://wcrp-cordex.ipsl.jussieu.fr/, respectively) along with multisite statistical downscaling simulations to evaluate the potential changes in seasonal precipitation extremes in the study region (as shown recently in Jeong et al. 2013).

ACKNOWLEDGEMENTS

The authors gratefully acknowledge financial support from the National Sciences and Engineering Research Council of Canada (NSERC). We would like to also acknowledge the Data Access Integration (DAI, see http://loki.qc.ec.gc.ca/DAI/) Team for providing the data and technical support, in particular the help of Ms Milka Radojevic in preparing the data. The DAI data download gateway is made possible through collaboration among the FQRNT-funded Global Environmental and Climate Change Centre (GEC3), the Adaptation and Impacts Research Section (AIRS) of Environment Canada, and the Drought Research Initiative (DRI). The Ouranos Consortium also provides IT support to the DAI team. The CRCM time
series data have been generated and supplied by Ouranos’ Climate Simulations Team.

REFERENCES


IPCC 2007 Climate change 2007: the physical science basis, summary for policymakers. International Panel on Climate Change Secretariat, Geneva.


Kim, T., Shin, J., Kim, K. & Heo, J.-H. 2008 Improving Accuracy of IDF Curves Using Long- and Short- duration Separation and
Multi-Objective Genetic Algorithm. 2008 EWRI Congress, 12–16 May, Honolulu, Hawaii.


Music, B. & Caya, D. 2007 Evaluation of the hydrological cycle over the Mississippi River basin as simulated by the Canadian Regional Climate Model (CRCM). J. Hydrometeorol. 8, 969–988.


First received 13 April 2013; accepted in revised form 4 June 2013. Available online 5 July 2013