almost identical with those advanced in this paper) yields three real modes of propagation. These are shown in Fig. 1 of this Discussion for the case of a Thornel-Carbon phenolic laminate. Material properties used for this calculation are given in Table 1.

References

1 Hegemier, G. A., "On a Theory of Interacting Continua for Wave Propagation in Composites," *Dynamics of Composite Materials*, ASME, New York, 1972, pp. 70–121.

2 Hegemier, G. A., and Nayfeh, A. H., "A Continuum Theory for Wave Propagation in Laminated Composites, Case 1: Propagation Normal to the Laminates," ASME Paper No. 72-WA/APM-20.

3 Hegemier, G. A., and Bache, T. C., "A Continuum Theory for Wave Propagation in Laminated Composites, Case 2: Propagation Parallel to the Laminates," Air Force Office of Scientific Research Report, Grant AF-AFOSR-70-1957, Technical Report No. 6, July 1972.

Authors' Closure

We wish to thank Drs. Bache and Gurtman for their comments. In reply to their specific points, we would like to add the following: First, the effects that errors in the stress and displacement fields have on transient pulses is a topic we wish to address in the future; however, there is reason to suspect that the integrated nature of the transient pulse would tend to reduce the net effects of these discrepancies. Second, our version of the "effective stiffness theory" also admits cutoff frequencies and additional optical modes of wave propagation which are discussed in detail in "Wave Propagation in Elastic Laminates Using a Second-Order Microstructure Theory" by Drumheller, D. S., and Bedford, A., *Journal of Solids and Structures* (in press).

On the Buckling of Cylinders in Axial Compression[1]

J. G. Simmonds[2] and D. A. Danielson.[3] We wish to point out that in [1][4] we derived a reduced set of buckling equations for circular cylindrical shells that were only slightly more complicated than the Donnell equations but yet as accurate as the unreduced Koiter-Budiansky-Sanders equations [2]. In particular, an explicit solution for the very problem considered by the author was given in the last section of [1]. Our proof of accuracy in [2], based on energy arguments, was mathematically *rigorous* and thus avoided the need for detailed numerical computation and comparison. Finally, we note that the Euler buckling of a long tube was also treated by us, as a limiting case, in a paper in which we applied our equations to the study of the buckling of a circular cylindrical shell under axial load and subject to certain relaxed boundary conditions [3].

References

1 Danielson, D. A., and Simmonds, J. G., "Accurate Buckling Equations for Arbitrary and Cylinderical Elastic Shells," *International Journal of Engineering Science*, Vol. 7, 1969, pp. 459–468.

2 Danielson, D. A., and Simmonds, J. G.; "A Proof of the Accuracy of a Set of Simplified Buckling Equations for Circular Cylindrical Shells," Developments in Theoretical and Applied Mechanics (*Proceedings 5th Southeastern Conference Theoretical and Applied Mechanics*, 1970), University of North Carolina Press, 1971, pp. 1015–1028.

3 Simmonds, J. G., and Danielson, D. A., "New Results for the Buckling Loads of Axially Compressed Cylindrical Shells Subject to Relaxed Boundary Conditions," Journal of Applied Mechanics, Vol. 37, Trans. ASME, Vol. 92, Series E, 1970, pp. 93–100.

Author's Closure

The author is grateful to Professors Simmonds and Danielson for pointing out these additional references. However, in spite of the rigor of the paper cited by the discussers, the author is not convinced that the need for "numerical experiments" has been obviated. Further, a forthcoming paper by the author ["On Approximations of the Buckling Stresses of Axially Compressed Cylinders," to appear in the Journal of Applied Mechanics] will leave clear the idea that there exists more than one legitimate approximation to the eigenvalues under discussion, from a single shell theory! Finally, the author did not wish to infer that he was the first to discuss the limiting case of the Euler buckling of a tubular column. That limiting case has been discussed at least as far back as Flügge's classical monograph on shell behavior [Flügge, W., "Statik und Dynamik der Schalen," Springer-Verlag OHG, Berlin, 1934].

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[4] Numbers in brackets designate References at end of Discussion.

The Origin of Instability in Airlift Pumps[1]

G. B. Wallis.[2] It is interesting to note that the solution presented in this paper is analogous to one of the cases treated by Wallis and Heasley.[3] Wallis and Heasley considered the stability of a natural-circulation loop with one leg consisting of a boiling section followed by a riser in which liquid and vapor flowed together.

Equations (79) and (80) of Hjalmars' paper have exactly the same form as equations (18) and (19) of the paper by Wallis and Heasley. The method of attack is essentially the same, if one replaces the rate of gas addition by the rate of vapor generation due to boiling. Wallis and Heasley, however, did not neglect the difference in velocity between the gas and the liquid but assumed that the gas was in the form of "slug flow bubbles" in order to calculate the "slip."

Author's Closure

The author would like to thank Professor Wallis for drawing attention to the interesting case, treated by him and Heasley. It illuminates the fact that the same sort of control instability occurs in different sorts of devices, where changes in the motion have a delayed effect on the driving force itself, e.g., by changing the density and thus after a while the gravitational action on the driven mass. Another example, quite close to that mentioned by Wallis, is the case of the heated one-phase fluid loop, studied

[1] By S. Hjalmars and published in the June, 1973, issue of the Journal of Applied Mechanics, Vol. 40, No. 2, Trans. ASME, Vol. 95, Series E, pp. 399–404.

[2] Professor, Thayer School of Engineering, Dartmouth College, Hanover, N. H. Mem. ASME.

[3] Wallis, G. B., and Heasley, J. H., "Oscillations in Two-Phase Flow Systems," *Journal of Heat Transfer*, Trans. ASME, Series C, Vol. 83, 1961, pp. 363–369.

by Welander,[4] which seems to be of interest, e.g., in oceanography.

In conclusion I should like to give the following qualitative description of the mechanism for the generation of the instability in the airlift pump, a description which can be easily translated to other similar systems. Consider a harmonic small oscillation of the velocity around its stationary value at the injection zone. Under the half period, when this oscillation is negative, it causes a decrease in the density below its stationary value, which decrease is propagated with the flow and builds up an accelerating force. This accelerating force must reach its maximum on the elements in the injection zone after a lapse of time, which is of the same order of magnitude as the time of travel of the fluid through the pipe. If now this maximum is in phase with the maximum of the acceleration of the original oscillation, an overshooting occurs, producing growing amplitude of the oscillation, i.e., instability. If the condition (86) of my paper is interpreted according to this view, it is seen to give a time of travel as calculated from the velocity (42) of about a third ($1/\sqrt{8}$) of the time of oscillation as calculated from the coefficient of κ in (70).

[4] Welander, P., "On the Oscillatory Instability of a Differentially Heated Fluid Loop," *Journal of Fluid Mechanics*, Vol. 29, Part 1, 1967, pp. 17–30.

Behavior of Dilute Polymer Solutions in the Inlet Region of a Pipe[1]

W. MICHAEL LAI.[2] One of the conclusions, stated in the concluding remarks of the paper is that for dilute polymer solutions, characterized by the constitutive equations (24) and (31), the theory predicts shorter inlet length and greater pressure drop, in agreement with the results of Tomita and Yamane [10]. On examining equations (48), (51), and Table 1, it is seen that the inlet length is shorter and the pressure drop greater for $1.0 \leq m \leq 1.30$, whereas for $m = 1.50$, exactly the opposite is true and it appears that at some value of m between 1.30 and 1.50, the presence of elasticity has no effect at all on either the inlet length or the pressure drop. What significance, if any, is to be given to this value of m? Can one extrapolate the result of Table 1 to conclude that for say, $m = 2$, C_2 is positive, and $(\Delta p)_{\text{elastic}}$ is negative?

On comparing equation (32) and (33) of the paper with equations (4) and (5) of reference [10], it appears that for the case $n = 1$ and $m = 2$ $\left(\text{in the notation of the present paper, i.e., } t_{zr} = -\eta\left(-\frac{\partial u}{\partial r}\right) \text{ and } t_{zz} - t_{rr} = 2\tau\left(-\frac{\partial u}{\partial r}\right)^2\right)$, the conclusions from the present paper and reference [10] are opposite (if extrapolation of the numerical values are valid).

Is it really necessary to invoke the Weissenberg's conjecture that $t_{rr} - t_{\theta\theta} = 0$? It appears that with α_i's assumed to be $0(\delta^2)$ it follows $t_{rr} - t_{\theta\theta} = 0(1)$ so that its contribution to the pressure change across the boundary layer is $0(\delta)$.

It is noted that the tensors $A_n(n = 2, 3, \ldots)$ used in the paper, equations (22) and (23), are related to but not the same as the Rivlin-Ericksen tensors as defined in say, *Encyclopedia of Physics*, Vol. III/3, p. 54.

[1] By E. Bilgen, published in the June, 1973, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 40, TRANS. ASME, Vol. 95, Series E, pp. 381–387.

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Author's Closure

The analysis presented is strictly on the behavior of dilute polymer solutions and $m > 1.30$ does not have any physical significance. From the mathematical point of view, it should be noted however that the sign change in Table 1 for $m > 1.30$ and the impossibility to extrapolate the results for m near 2 can be attributed to the failure of the integral method and in that sense, a refinement using other methods discussed in the Introduction of the paper may be of help.

$\alpha_i = 0\ (\delta^2)$ is the necessary condition obtained from the stress equation of motion; hence if it is assumed that $\alpha_i = 0(\delta^2)$, obviously the Weissenberg's conjecture will prevail and vice versa.

The Response of an Elastic Disk With a Moving Mass System[1]

C. D. Mote, JR.[2] The discusser finds this paper by Prof. W. D. Iwan and Dr. K. J. Stahl very interesting and a noteworthy contribution to the now voluminous circular plate literature. The discusser has also been interested in these problems for sometime and he would like to remark that one of his papers [1][3] may be an important companion for the present research. In his paper the Greens' function for a centrally clamped, peripherally free, circular plate is formulated as a eigenfunction expansion, similar to that in the authors' paper, and the plate response is investigated for two circumnavigating, peripheral prescribed loads. One is the rotating harmonic load, which includes as a special case the critical speed phenomenon mentioned by the authors. The second as a load whose speed contains a harmonic component which is similar to what occurs in some industrial processes. The paper includes detailed, exact eigensolutions including 50 eigenvalues and corresponding eigenfunctions in a tabular form for these plates with $b/a = 0.5$. The discusser believes this is not unlike clamping radii used in computer disk file memory units. It appears that these eigensolutions can be directly applied to the formation of D in [28] and thereby extend the authors' results to the other plates with little effort.

The question of disk operation above its critical speed is a very interesting one, and the discusser understands from the authors' introduction that this is now common. The discusser has held the idea that stable operation above the lowest critical speed was unlikely. Tobias and Arnold [17] showed in the laboratory that instability extended over a wide rotation range, even for a concentrated moving load, because of nonlinear effects not contained in the linear, critical speed analysis. Dugdale [2] has independently commented that stable operation above the critical speed is unlikely. In some of the discussers' studies [1, 3] operation above the critical speed was possible only in the absence of loading. The results in Fig. 4 of the present paper indicate a "broadened" instability region above the critical speed. One can assume that nonlinear effects would broaden it somewhat further. In many critical speed problems the modes (number of nodal diameters and nodal circles) of potential instability are quite close. That is, in the authors Fig. $4j/\bar{P}_{jn}$ does not differ greatly for values of $j = 2 - 6$. It appears then that overlapping of instability regions will occur. Besides the interesting implications of this overlapping, the present paper seems to indicate that the potential for instability can only be increased by the moving

[1] By W. D. Iwan and K. J. Stahl, published in the June, 1973, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 40, TRANS. ASME, Vol. 95, Series E, pp. 445–451.

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[3] Numbers in brackets designate References at end of Discussion.