Discussion: "Friction Numbers and Viscous Dissipation Heating for Laminar Flows of Water in Microtubes" (El-Genk, M., and Yang, I., 2008, ASME J. Heat Transfer, 130, p. 082405)

Yutaka Asako
Department of Mechanical Engineering,
Tokyo Metropolitan University,
1-1 Minami-Osawa, Hachioji,
Tokyo 192-0397, Japan
e-mail: asako@tmu.ac.jp

In the above paper [1], the authors determine the friction number and the slip length for laminar water flow in microtubes from the reported measurements in experiments of the temperature rise due to the viscous dissipation heating assuming a velocity slip. However, there is a significant error in the above paper, which is presented below.

Integrating the following steady-state energy balance equation for fluid flow over the total volume of the liquid in the microtube (Eq. (4) in their paper)

$$\rho\, C_p\, v_{z,\ell}(r)\left(\frac{\partial T}{\partial z}\right) = \bar{\mu}\left(\frac{\partial\, v_{z,\ell}}{\partial\, r}\right)^2 \tag{1}$$

the following equation for the temperature rise is derived (Eq. (5) in their paper):

$$\frac{8\pi\bar{\mu}L}{(1+8(\beta/D))^2}\,\bar{v}_{z,\ell}^2 = \left(\frac{\pi}{4}D^2\right)\rho\,\bar{v}_{z,\ell}\,C_p\,\Delta T \tag{2}$$

However, in Eq. (1), the heat conduction term is omitted. The exact expression for the energy equation is

$$\rho\, C_p\, v_{z,\ell}(r)\left(\frac{\partial T}{\partial z}\right) = \lambda\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \bar{\mu}\left(\frac{\partial\, v_{z,\ell}}{\partial\, r}\right)^2 \tag{3}$$

Integrating Eq. (3) with the boundary condition of $(\partial T/\partial r)_{r=D/2} = 0$ and $(\partial T/\partial r)_{r=0} = 0$, the heat conduction term vanishes and Eq. (2) is obtained. However, Eq. (2) is not the equation for the temperature rise for the adiabatic slip flow. This is the equation for the temperature rise for flow in a tube whose wall is "moving" with velocity, $v_{z,\mathrm{slip}}$ [2]. To derive the equation for the temperature rise for the adiabatic slip flow, Eq. (3) should be integrated with the following boundary condition [2]:.

$$\lambda\left(\frac{\partial T}{\partial r}\right)_{r=\frac{D}{2}} = -\mu\, v_{z,\mathrm{slip}}\left(\frac{\partial v_{z,\ell}}{\partial r}\right)_{r=\frac{D}{2}} \tag{4}$$

where $v_{z,\mathrm{slip}}$ and λ are the slip velocity and the thermal conductivity of the fluid. In the case of the slip flow, the energy is transported out from the fluid to the wall even though the fluid temperature and the wall temperature are identical. The difference between the kinetic energy of molecules which arrive on the wall and that of molecules which leave from the wall results in the transported energy. Therefore, the same amount of energy, which is transported out, has to come back to the fluid when the wall is adiabatic. The right hand side of Eq. (4) expresses the energy which is transported out from the fluid to the wall, as the form of the boundary work due to shear. However, note that no work is done by the wall since wall is stationary. The correct equation for the temperature rise for the adiabatic slip flow is as follows:

$$\frac{8\pi\bar{\mu}L}{(1+8(\beta/D))}\,\bar{v}_{z,\ell}^2 = \left(\frac{\pi}{4}D^2\right)\rho\,\bar{v}_{z,\ell}\,C_p\,\Delta T \tag{5}$$

Since no work is done by the wall, the enthalpy of the fluid of the adiabatic slip flow keeps constant along the microtube. This correlation is valid even though the viscous dissipation occurs. Equation (5) can be also obtained from the correlation of constant enthalpy.

References

[1] El-Genk, M., and Yang, I., 2008, "Friction Numbers and Viscous Dissipation Heating for Laminar Flows of Water in Microtubes," ASME J. Heat Transfer, **130**, p. 082405.
[2] Hong, C., and Asako, Y., 2010, "Some Considerations on Thermal Boundary Condition of Slip Flow," Int. J. Heat Mass Transfer, **53**, pp. 3075–3079.

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