

oscillations of order  $1/3$  and  $1/2$ . Use of previously studied Routh-Hurwitz criterion is applied to stability of periodic solutions. Chapter 10 focuses upon initial conditions leading to different types of periodic oscillations. Initial considerations are given to symmetrical systems and location of fixed points and stability investigations. In turn, one applies this to subharmonic response ( $1/3$  order), harmonic response (no subharmonics) and unsymmetric systems. Chapter 11 dwells upon periodic oscillations. The study includes almost periodic oscillations in a resonant circuit with superimposed direct current, periodic and almost periodic solution. It ends with almost periodic oscillations in a parametric excitation circuit where subharmonic oscillations have order of  $1/2$ . In this part, illustrative examples, numerical solution of the resulting derived equations and experimental investigations comprise this section.

Part IV points out the various aspects concerned with self-oscillations subjected to external forces. Chapter 12 explains entrainment of frequency, i.e., periodic force is applied to a system where free oscillation is of self-excited type. An illustrative example is the Van Der Pol equation containing an additional term for periodic excitation. This follows with harmonic entrainment where the driving frequency is in the neighborhood of unity. We proceed ahead into higher harmonic entrainment and subharmonic entrainment. In all cases, stability investigations and periodic solutions explain in detail the different aspects of the entrainment condition. Chapter 13 considers the subject of almost periodic oscillations. These occur in self-oscillatory systems under periodic excitation. We initiate the topic with Van Der Pol's equation with a forcing term. After completing the analysis, we encounter limit cycles correlated with almost periodic oscillation, transition between external oscillations and almost periodic oscillations, coalescence of singular points at the boundary of harmonic entrainment plus the derivation of a singularity which is a stable focus. We progress to limit cycles correlated with almost periodic oscillations. The authors develops it from higher harmonic oscillation, transition between entrained oscillations and almost periodic oscillations. The chapter concludes with self oscillatory systems with nonlinear restoring force.

Appendix I considers the expansion of Mathieu's function. Appendix II and III delve into the subject of unstable oscillations of Hill's equation and its extended forms. Appendix IV relates the stability criterion by employing the perturbation method. Appendix V focuses upon integral curves, singular points and uses the theorem of Bendixson plus theory of contours. Appendix VI speaks about electronic synchronous switches which proscribe the initial conditions.

In summary, this is a good book. Since almost all nonlinear systems are basically nonlinear, the approximation device (linearization) may range from good to poor. This helps the reader in acquiring a better understanding of nonlinear systems. The reviewer would have preferred seeing a more elaborate section on bifurcation, Nyquist diagrams for nonlinear systems, and employment of Bogoliubov-Metropolisky asymptotic methods. Furthermore, the book should be "updated" to incorporate the digital computer system and associated computer codes. This would definitely enhance the book on this subject.

**Random Vibration of Structures,**  
C. Y. Yang,  
John Wiley & Sons, New York,  
1986, 295 pages.

In the modern day world, most structures and mechanisms are subjected to random forces. The excitation due to dynamic forces of wind pressure, earthquake motion, and ocean waves are usually dealt with in a rather crude fashion. One selects a maximum design load versus time function or the average of several ominous design load time functions. In due consideration both appeals are deficient. At times, they may furnish rational signs of forthcoming load conditions. They do not provide the mechanical or structural engineer with the precise answer to the question of structural safety. The structural response can be highly random in intensity and could contain a wide frequency band. This results in making the problem of structural safety very complex, unwieldy, and hard to solve. The present assumption of mass, stiffness, damping, and failure rules (yield and fatigue) falls within the category of random vibration analysis. It describes the safety of the structure in a probabilistic sense and is most valuable for structural design and planning. As stated by the author, "For practicing structural engineers, the text provides an understanding of the basic concepts and some application of random vibration of structures to enable them to tackle practical problems and to follow the research literature." The author follows his intentions with great vigor.

The book consists of 10 chapters and 3 appendices.

Chapter 1 introduces the idea of random vibration and how it originates. This follows with an interesting discussion of random processes, probability, and statistics. This encompasses frequency definition of probability, probability density, joint probability density, and conditional probability. The statistics entail variance, standard deviation, correlation and its coefficients plus covariance.

Chapter 2 continues with stationary random processes, autocorrelation, and spectral density. The autocorrelation function is defined which then leads to Fourier series and Fourier integral representation. This includes Parseval's relationship. We next meet power spectral density and its kinship with autocorrelation. A slight mention is made of cross correlation but none of cross spectral density. Chapter 3 computes the introduction to random vibration. Beginning with ergodic processes (temporal statistics) and its mathematical relationships, we proceed to autocorrelation and spectral density of one sample function plus an alternative definition of spectral density function directly from the sample function.

Chapter 4 dedicates itself to 3 models of random excitation experienced by a structural system. We superimpose simple fundamental modes with various deterministic parameters and random variables. Finally, we determine the statistical and probabilistic characteristics of the final model. The initial model is a stationary random roadway with mathematical derivations. It starts out with the discrete model and proceeds along to the continuous model. The second is random earthquake motion and is nonstationary and contains both discrete and continuous models. The third is random ocean waves (multivariable stationary model) containing both discrete and continuous models. The spectral density of waves forces conclude the chapter. A most interesting chapter!

Chapter 5 begins the study of the relationship of the transfer or transmission relationship of the random vibration of structures. We initiate this concept by studying the mathematical relationships of the single degree of freedom (SDOF) system. The two solutions are (a) frequency domain (FD) and, (b) time domain (TD). The frequency response function is determined in FD and employed in a number of illustrative examples plus integration in the complex plane. TD considers an excitation as a superposition of impulses using Duhamel's in-

tegral. A number of examples illustrate the TD solution. The chapter concludes with a thoughtful mathematical description and derivation of random excitation. The input excitation is a stationary random process. This is applied to both TD and FD.

Chapter 6 considers multidegree of freedom (MDOF) systems. Beginning with the two degree of freedom (TDOF) system, the deterministic vibration is developed. This leads to random vibration of TDOF and MDOF systems with derivation and application of auto and cross correlation and response spectral density for both damped and undamped conditions. The alternative approach generalizes the complex frequency response for the equivalent response functions to matrix of functions. The complex frequency response, impulse response and stationary random function conclude the chapter.

Chapter 7 focuses upon response of continuous systems. The shear beam is used as the prime example. As a previous chapters, deterministic vibration via modal solutions initiates the subject. Stationary and concentrated random excitation follow in order. The author touches upon random vibration of a thin plate. Our next topic is the alternative solution of shear beams, where again, the uncoupled SDOF systems are applied in either time or frequency domain. The random vibration of a dam reservoir system subjected to vertical ground acceleration is exploited. The alternative impulse response function solves the dam problem where the hydrodynamic pressure corresponds to the shear force. The frequency response function and power spectral density response conclude this section. In a similar manner, the dam-reservoir is excited by a horizontal acceleration simulating an earthquake event. A very good chapter!

The next chapter covers the design of structures for random excitations. The stationary Gaussian process (random variable) is defined. The probability of upcrossings over a specified level and probability density of the peaks introduce this phase of the subject. The author applies this to the concept of structural design against yield failure using an SDOF and a dam-reservoir subjected to vertical ground acceleration. The chapter concludes with structural damage against fatigue and includes the Palmgren-Miner (PM) rule and stationary narrow band random loading. The reviewer would have preferred seeing actual random test data since PM is limited as to excitations and may be unconservative at times.

Chapter 9 treats nonstationary response which is a time dependent response of structures. Beginning with an SDOF system having stationary excitation, we again utilize the dam-reservoir system (vertical acceleration excitation). This continues into systems with nonstationary excitation containing Priestley's complex integral model. Zero damped and lightly damped SDOF systems inaugurate this topic. An alternative approach is Bendat and Piersol's model which employs the autocorrelation function for zero mean excitation and nonstationary spectrum. The dam-reservoir with nonstationary excitation concludes the chapter.

The last chapter develops random vibration of structures in the plastic range. The random walk model is the eye-opener. It encompasses basic probability definitions (mass probability, mass function, conditional probability) and then derives the Chapman-Kalmagarov-Smolchoewski equation applied to structural response. Application of random walk model to a nonlinear structure and solution of the Fokker-Plunck equation complete the chapter.

The appendices include Fast Fourier Transform (FFT) in random vibration and Monte Carlo simulation plus references.

In summary, this is a good book. The author spares no effort in making the reader understand random vibration. The

mathematics which may seem to be a roadblock are easily mastered. The reviewer would have preferred seeing sections on Hilbert Transform and cepstrum analysis. This would be a great help to those interested in the application of random vibrations to present-day problems. The reviewer recommends this book to those interested in learning about random vibrations and their applications.

#### **Finite Elements—Special Problems in Solid Mechanics—Vol. V,**

J. Tinsley Oden and G. F. Carey,  
Prentice-Hall Inc., Englewood Cliffs, N.J.,  
1984, 273 pages, \$37.95.

Finite element (FE) has tripped the light fantastic toe from the bare elements in the late 1950's to the more sophisticated state of today. This book resides in the latter realm and covers topics and procedures that are in the realm of modern research. This little volume is meant for the scholar and the analytical engineer interested in applying the latest information on FE. As stated by the editors, "A more useful objective was to identify a small collection of special problems in solid mechanics, that shares the following characteristics (1) they are important in a practical sense (i.e., the solutions are of significant interest in application), (2) they are difficult and involve intricate mathematical features that require special attention in order to obtain reasonable results, (3) an analysis of the problem based on sound mathematical and physical arguments is possible and methods for their solution can be described which have a reasonable degree of reliability, (4) their analysis and numerical results obtained have a degree of permanence (i.e., the accounts of the subject given should be of interest to readers working in these subjects for some time to come), and (5) the nature of the problems are such that they generally cannot be solved by brute force application of the FE methods—special FE algorithms must be tailored to deal with the special features of each problem." To this, the reviewer says "amen."

There are 5 chapters, each written by author or authors from both here and abroad. Chapter 1 describes numerical analysis of thin shell problems. The linear equations are derived at the mid-surface. This follows with classes of nonlinear shallow shell problems. The middle surface strain tensor and Koiter's modified change of curvature tensor are employed. This follows with stress measures which are derived from strain measures, variational formulations, exceedance, and uniqueness theories. This leads to conforming FE methods with numerical integration, effect of numerical integration (abstract error estimate), asymptotic error estimates with examples of the following  $c'$  triangular FE (a) Bell, (b) Hermite, (c) reduced Hermite, (d) Lagrange type, and (e), Argyris. This continues with  $c'$  composite triangular FE, i.e., (a) Hsieh-Clough-Tocher (HCT) and reduced HCT, (b) Hermite and reduced Hermite, and (c) Lagrange of type 2 and 3. The  $c'$  rectangular FE is the Bogner-Fox-Schmidt. This is then applied to the computation of an arch dam problem using the Argyris triangular scheme.

Chapter 2 reports on FE in nonlinear incompressible elasticity. The chapter begins with basic mechanic preliminaries, i.e., (a) stress-free response configuration, (b) equilibrium equation, and (c) minimization formulations. With this under our belt, the mathematical formulation, including equilibrium equations, are considered. This follows with the approximate problem, i.e., finite domain second discrete approximation spaces, compatibility conditions plus