A spatial multi-objective decision-making under uncertainty for water resources management
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ABSTRACT

Water resources decision-making is a spatial problem. Topographical features of the region, location of water resources management infrastructure, interaction between the water resources system and other social and ecological systems and impact of different water resources regulation measures are all variables with considerable spatial variability. In this paper a new technique called Spatial Fuzzy Compromise Programming (SFCP) is developed to enhance our ability to address different uncertainties in spatial water resources decision-making. A general fuzzy compromise programming technique, when made spatially distributed, proved to be a powerful and flexible addition to the list of techniques available for decision-making where multiple criteria are used to evaluate multiple alternatives. All uncertain variables (subjective and objective) are modeled by way of fuzzy sets. Through a case study of the Red River floodplain near the City of St. Adolphe in Manitoba, Canada, it has been illustrated that the new technique provides measurable improvement in the management of floods.

Key words | flood management, multi-objective decision-making, natural disasters, spatial compromise programming, spatial fuzzy multi-objective analysis, water resources management

INTRODUCTION

“The nature of floods and their impact depend on both natural and human-made conditions in the floodplain. Economic development and the installation of flood protection measures have political, economic, and social dimensions as well as engineering aspects. Hydrologic and hydraulic analysis of floods provides a sound technical basis for management decision making that must weigh numerous other factors” (Hoggan 1996; Simonovic 2002; Kundzewicz 2002).

Applications of multi-objective techniques to water resources management have come a long way since the early work of Maass et al. (1962), Cohon & Marks (1973) and David & Duckstein (1976), where the decision problems were formulated as linear programming vector optimization problems. There are also applications based on the multi-attribute utility theory (Raiffa 1968), where explicit trade-offs between attributes are utilized. Other popular techniques used for discrete alternative evaluation include the Surrogate Worth Trade Off (Haines 1974), ELECTRE (Roy 1971), Analytical Hierarchy Process (Saaty 1980) and Compromise Programming (Zeleny 1973, 1982).

A typical flood management problem requires selection and implementation of the best structural and/or non-structural solution from the set of potential alternatives. Flood management problems include conflicting quantitative and qualitative evaluation objectives and multiple decision-makers. Multi-objective techniques help in evaluation and ranking of alternatives based on the objective values associated with each of the alternatives, and preferences of the various decision-makers. However, the flood management alternatives exhibit spatial variability. The Geographic Information System (GIS) is a useful computer-based tool to assist in water resources management with spatially distributed variables. “Specific planning
and management tasks for which GIS may be of assistance include comparative analysis, monitoring of dynamic processes, evaluation of current conditions, detection of changes, forecast of future developments, problem assessment, planning of action (e.g., mitigation), identification of regions that meet multiple criteria (e.g., site selection), identification and allocation of resources, analysis of policy options and the determination of cumulative effects based on spatial location” (Kaden 1993). Many GIS applications in water resources management include the work of different research groups (Carver 1991; Banai 1993; McKinney & Maidment 1993; Pereira & Duckstein 1993; Tim 1997; Wolfe 1997). GIS technology facilitates the decision-making process based on its analytical capabilities with spatial information and usually offers a graphical user interface which increases the decision-maker’s comprehension of the spatial information. Based on these two additions to the water resources decision-making process, a GIS is often included as a major component in the development of water resources Decision Support Systems (DSS) (see, among others, Loucks & DaCosta 1991; Simonovic 1995, 1996, 1999; Walsh 1995; Fürst et al. 1993; Leipnik et al. 1993; Watkins et al. 1996).

Conventional multi-objective analysis techniques do not consider the spatial variability of the criteria values, which are used to evaluate potential alternatives. The criteria values, which they use, represent average or total impacts incurred across the entire region being considered. Thus, in identifying the best solution from a set of potential flood mitigation alternatives using conventional multi-objective techniques, only the region as a whole is considered. By doing so the localized, and potentially negative, impacts resulting from the implementation of different flood protection alternatives are ignored. Consequently, the alternative identified as the best for an entire region may not be the best for all locations within that region. Tkach & Simonovic (1997) addressed this spatial variability in the criteria values associated with the various flood management alternatives by combining the Compromise Programming with the GIS technology and called it Spatial Compromise Programming (SCP). SCP can be efficiently used to generate, evaluate and rank a set of potential flood management alternatives. Through the application of, for example, the traditional Compromise Programming technique the best alternative can be determined for the entire region. However, with the SCP the best alternative for each location within the region is determined. Though SCP is capable of accounting for the spatial variability of decision variables, it is unable to address various uncertainties associated with a complex system of multiple structural and non-structural alternatives, multiple objectives and multiple decision-makers.

Multi-objective decision-making is moving from optimization methods to more interactive decision tools (Bender & Simonovic 2000). Some of the areas of current and future development are identified by Dyer et al. (1992). One of them is: “sensitivity analysis and the incorporation of vague or imprecise judgments of preferences and/or probabilities in multi-attribute situations and decisions under uncertainty in which states are multidimensional.” Traditional techniques for evaluating discrete alternatives such as ELECTRE (Benayoun et al. 1966), AHP (Saaty 1980), Compromise Programming (Zeleny 1973, 1982) and other do not normally consider uncertainties involved in procuring criteria values. Sensitivity analysis can be used to express decision-maker uncertainty (such as uncertain preferences and ignorance), but this form of sensitivity analysis can be inadequate at expressing decision complexity. There have been efforts to extend traditional techniques, such as PROTRADE (Goicoechea et al. 1982), which could be described as the stochastic compromise programming technique. The problem, though, is that not all uncertainties fit the probabilistic classification. The theory of fuzzy sets, which is a theory of possibility, is not dissimilar to probability theory. In fact, they can be considered complementary. Fuzzy membership functions have a similar appearance to probability distribution functions. However, there are some inherent differences. A probability distribution function provides the probability of specific values occurring. A fuzzy membership function acknowledges that we may not be completely sure what values are being talked about. Statistical precision can be independent of our classification of an event. In many cases, there may not be enough data to make probabilistic predictions with confidence. The dependence of stochastic applications on distribution functions can be restricting and misleading because of the intensity of data requirements. The difference between fuzzy and probabilistic functions is not always so clear. In general,
Fuzzy sets provide an intuitive and flexible framework for interactively exploring a problem that is either ill-defined or has limited available data.

Fuzzy decision-making techniques have addressed some uncertainties, such as the vagueness and conflict of preferences common in group decision making (Blin 1974; Siskos 1982; Seo & Sakawa 1985; Felix 1994; and others), and at least one effort has been made to combine decision problems with both stochastic and fuzzy components (Munda et al. 1995). Application, however, demands some level of intuitiveness for the decision-makers and encourages interaction or experimentation such as that found in Nishizaki & Seo (1994). Authors such as Leung (1982) and many others have explored fuzzy decision-making environments. The fuzzy decision-making process is not always intuitive to all people involved in practical decisions because the decision space may be some abstract measure of fuzziness, instead of a tangible measure of alternative performance. The alternatives to be evaluated are rarely fuzzy. Their performance is fuzzy. In other words, a fuzzy decision-making environment may not be as generically relevant as a fuzzy evaluation of a decision-making problem. The Fuzzy Compromise Programming (FCP) technique developed by Bender & Simonovic (2000) transforms a Compromise Programming distance metric to a fuzzy set by changing all inputs from crisp to fuzzy through the application of the fuzzy extension principle. This approach can address various uncertainties that are associated with the natural hydrological processes occurring in flood management, data monitoring systems, equipment accuracy and lack of knowledge. FCP approach ranks alternatives using fuzzy ranking measures designed to capture the effect of risk tolerance differences among decision-makers.

Time and space play an important role in flood management. Therefore, there are uncertainties involved in flood prediction, in the evaluation of the inundated area and in the estimation of various physical, ecologic, economic and social impacts. The analysis of the current state of the art in multi-objective decision-making and the needs of water resources management pointed out that a new technique is required that will be able to provide: (i) accounting for spatial variability in the water resources decision-making and (ii) accounting for uncertainties involved in the water resources decision-making. A new technique combining these two objectives is developed in this study. It is called herein Spatial Fuzzy Compromise Programming (SFCP). Through a case study of the Red River Basin, Manitoba, Canada it has been successfully demonstrated that SFCP can assist a decision-maker in selecting “the best” flood management alternative, taking into account the spatial variability (using SCP) for each location (5 X 5 m grid) in the entire study region as well as accounting for the uncertainties (using Fuzzy Compromise Programming) involved in the process.

The following section of the paper presents the analytical and numerical aspects of the Spatial Fuzzy Compromise Programming technique. Then, the application of the technique to flood management of the Red River Basin, Manitoba, Canada is discussed. The paper ends with a set of conclusions and suggestions for future research.

**SPATIAL FUZZY COMPROMISE PROGRAMMING**

The general formulation of a multi-objective multi-participant decision problem is based on: (a) a set of potential alternatives, (b) a set of objectives or criteria, (c) a number of decision-makers, (d) a preference structure or weights and (e) a set of performance evaluations of alternatives for each objective. In a mathematical form a multi-objective decision problem can be presented as:

\[
\text{max} - \text{dominant}[Z(x) = [Z_1(x), Z_2(x), \cdots, Z_p(x)]] \\
\text{subject to} \\
x \in X
\]

where \(X\) is a feasible region defined as:

\[
X = \{x : x \in R^n, g_i(x) = 0, x_j \geq 0 \forall i, j\}
\]

where \(R\) is a set of real numbers, \(g_i(x)\) is a set of constraints and \(x\) is a set of decision variables.

Every feasible solution to the problem (1), i.e. all \(x \in X\), implies a value for each objective, i.e. \(Z_k(x), k = 1, \cdots, p\). The \(p\)-dimensional objective function maps the feasible region in decision space \(X\) into the feasible region in objective space \(Z(x)\), defined on the \(p\)-dimensional vector space.
In general, one cannot optimize a vector of objective functions (Haimes & Hall 1974). In order to find an optimal solution, it is required that information about preferences is available. Without this information the objectives are incommensurable and therefore incomparable, implying that the optimum solution could not be achieved since all feasible solutions are not ordered (comparable). A complete ordering can be obtained in this case only by introducing value judgments into the decision-making process.

Most of the water resources multi-objective decision-making problems are discrete. A problem is called discrete if the feasible set $X$ contains only a finite number of points. For example, if the decision-maker can only choose from a finite number of alternatives, then $X$ is necessarily finite and the problem is discrete. The classical outcome of the discrete multi-objective decision problem is the ranking of the alternatives. To obtain that, a number of steps are necessary such as establishing the preference structure, the weights and also the performance evaluations. Among the multi-objective methods, some perform the ranking, some establish the preference structure and some methods come up with the criteria values for each alternative. Some methods have the ability to incorporate qualitative data into the analysis while other methods are capable of including multiple decision-makers in the decision-making process.

Flood management is a typical example of a multi-objective problem, where the objectives, for example, could be to minimize the damage to human lives and property, to minimize the depth of floodwater in the flood inundated region, to maximize the effectiveness of flood protection measures, to minimize the time to provide help to the flood victims or others. Many flood protection alternatives, such as floodwater diversion or a dike construction around the region, include spatially varying variables. Some of the criteria values, such as floodwater depth and/or damage, are also spatially variable. So the general flood management can be addressed by multi-objective analysis. However, spatial variability calls for a modified approach. Tkach & Simonovic (1997) introduced Spatial Compromise Programming (SCP) to account for the spatial variability in multi-objective problems. Uncertainties can be addressed using probability theory but in the case of flood management due to the various kinds of uncertainties a new paradigm is necessary. Bender & Simonovic (2000) explained how uncertainties can be addressed using their Fuzzy Compromise Programming (FCP) method.

### Mathematical formulation of the Spatial Fuzzy Compromise Programming

In this study a discrete set of potential flood management alternatives and a set of criteria/objectives is considered. The main objective is to carry out multi-objective analysis to arrive at the best compromise alternative for each location by accounting for uncertainties and spatial variability in the various elements of the flood management process.

The foundation of the proposed new technique is the discrete modification of Compromise Programming technique introduced by Zeleny (1973). The method of Compromise Programming identifies solutions that are closest to the ideal solution as determined by some measure of distance. The solutions identified as being closest to the ideal solution are called compromise solutions and constitute the compromise set.

The distance from the ideal solution for each alternative is measured by what is referred to as the distance metric. This value, which is calculated for each alternative, is a function of the criteria values themselves, the relative importance of the various criteria to the decision-makers and the importance of the maximum deviation from the ideal solution (Simonovic 1989). All alternatives are ranked according to their respective distance metric values. The alternative with the smallest distance metric is typically selected as the “best compromise solution”. The following mathematical formulation is used to compute the distance metric values ($L_j$) for a set of $n$ criteria and $m$ alternatives:

\[
L_j = \left( \sum_{i=1}^{n} w_i \left| \frac{f_i - f_{ij}}{f_i - f_{i,w}} \right|^p \right)^{1/p}
\]  

(4)

where $L_j$ is the distance metric, $\hat{f}_i$ is the optimal value of the $i$th criteria, $f_{ij}$ is the value of the $i$th criteria for alternative $j$, $f_{i,w}$ is the worst value of the $i$th criteria, $w_i$ are weights indicating decision-maker preferences with respect to different criteria and $p$ is a parameter ($1 \leq p \leq \infty$).

In Equation (4), each criterion is to be given a level of importance, or weight $w_i$, provided by the decision-makers. The parameter $p$ is used to represent the importance of the maximal deviation from the ideal point. If $p = 1$, all
deviations are weighted equally, while, if \( p = 2 \), the deviations are weighted in proportion to their magnitude. Typically, as \( p \) increases, so does the weighting of the deviations. As Tecle et al. (1998) put it, “varying the parameter \( p \) from 1 to infinity, allows one to move from minimizing the sum of individual regrets (i.e., having a perfect compensation among the objectives) to minimizing the maximum regret (i.e., having no compensation among the objectives) in the decision-making process. The choice of a particular value of this compensation parameter \( p \) depends on the type of problem and desired solution. In general, the greater the conflict between players, the smaller the possible compensation becomes.”

Spatial Compromise Programming (SCP) (Tkach & Simonovic 1997) was introduced to include the spatial variability in the criteria, which is often the case in water resources management. For example, in flood control, the impacts of flooding are not the same for all locations within the flood-affected region. Implementation of a particular flood protection measure may reduce flood impacts at one location, while providing no protection at all for another. Using the principles of GIS, spatial considerations can be included into multi-criteria decision-making. The determination of the best spatial location for an alternative according to a predetermined set of criteria has been demonstrated in the literature (Carver 1991; Pereira & Duckstein 1993). In the application of Spatial Compromise Programming a distance metric is calculated for each impacted location, for each alternative. In this approach the region is represented by a raster feature image of the area of interest. Thus an individual raster cell within the feature image represents each location within the region of interest, for which a distance metric is calculated. Criteria values associated with each of the alternatives are contained within sets of criteria images, which are georeferenced with the feature images of buildings, roads and agricultural fields.

Mathematical formulation of the cell-by-cell calculation process requires modification of Equation (4) into

\[
L_{i,j} = \sum_{x=1}^{n} w_i f_{i,j,x,y} / f_{i,x,y} 1^{p} \]

\[(5)\]

where \( L_i \) is the distance metric, \( f_i \) is the optimal value of the \( i \)th criteria, \( f_{ij} \) is the value of the \( i \)th criteria for alternative \( j \), \( f_{ij,w} \) is the worst value of the \( i \)th criteria, \( w_i \) are weights indicating decision-maker preferences; \( p \) is a parameter \( (1 \leq p \leq \infty) \); \( i = 1, n \) criteria; \( j = 1, m \) alternatives; \( x = 1, a \) rows in the image; \( y = 1, b \) columns in the image; \( a \) is the number of rows in the image and \( b \) is the number of columns in the image.

In order to address uncertainties in the water resources multi-objective decision-making Bender & Simonovic (2000) introduced the Fuzzy Compromise Programming (FCP) approach. Techniques involve the transformation of a distance metric into a fuzzy set by changing all inputs from crisp to fuzzy and applying the fuzzy extension principle. However, it should be noted that some of the inputs could remain in deterministic form provided the level of confidence about their accuracy is satisfactorily high. In this way a combination of fuzzy and deterministic inputs can also be handled by the FCP approach. Measurement of the distance between an ideal solution and the perceived performance of an alternative can no longer be given as a single value, because many distances are at least somewhat valid. Choosing the shortest distance to the ideal solution is no longer a straightforward ordering of distance metrics, because of overlaps and varying degrees of possibilities. The resulting fuzzy distance metric has the following form:

\[
\tilde{L}_i = \left[ \sum_{i=1}^{n} \tilde{w}_i \left| \tilde{f}_i - \tilde{f}_{i,w} \right|^{p} \right]^{1/p} \]

\[(6)\]

where \( \tilde{L}_i \) is the fuzzy distance metric, \( \tilde{f}_{i,w} \) is the fuzzy worst value of the \( i \)th criteria, \( \tilde{f}_i \) is the fuzzy value of the \( i \)th criteria for alternative \( j \), \( \tilde{f}_{ij} \) is the fuzzy optimal value of the \( i \)th criteria, \( \tilde{p} \) is a fuzzified parameter \( (1 \leq \tilde{p} \leq \infty) \), \( \tilde{w}_i \) are fuzzified weights indicating decision-maker preferences, \( i = 1, n \) criteria and \( j = 1, m \) alternatives.

Equation (6) contains a great amount of additional information about the consequences of a decision and the effect of subjectivity. Non-fuzzy distance-based techniques measure the distance from an ideal point, where the ideal alternative would result in a distance metric, \( L : X \rightarrow \{0\} \). In the Fuzzy Compromise Programming approach, the distance is fuzzy, such that it represents all of the possible valid evaluations, indicated by the degree of possibility or the membership value. Alternatives, which tend to be closest to the ideal solution, may be selected.
Literature is available on the techniques for encoding information in a fuzzy set in order to generate input fuzzy sets. Articles on demonstrating decision problems with qualitative or subjective criteria are many. Fuzzy sets are able to capture many qualities of relative differences in perceived value of criteria among alternatives. Placement of modal values, along with curvature and skew of membership functions can allow decision-makers to retain what they consider the degree of possibility for subjective criteria values. As a subjective value, criteria weights may be more accurately represented by fuzzy sets as shown by Despic & Simonovic (2000) for the flood management problem domain.

In Equation (6), \( p \) is likely the most uncertain element of the distance metric computation. There is no single acceptable value of \( p \) for every problem and also it is not related to problem information in any way except by providing parametric control over the interpretation of distance. Fuzzification of the distance metric exponent, \( p \), can take many forms but in a practical way it might be defined by a triangular fuzzy set with a mode of 2. Similarly, weights \( w_i \) can be fuzzified to account for indecisiveness in their boundary values, for example, a value of 0.5 could be defined as approximately 0.5. This means that fuzzy boundaries of weight values will take care of the uncertainties associated with crispness. Expressing possibility values with fuzzy inputs allows experience to play a significant role in the expression of input information. The shape of a fuzzy membership function expresses the experience or the interpretation of a decision-maker.

In the application of the Spatial Compromise Programming technique the best alternative for each location is determined by comparing the values in the distance metric images for each individual raster cell. As stated earlier, in Compromise Programming the alternative with the smallest distance metric is typically selected as best. However, for convenience, Equation (6) has been rewritten in such a way that the better the alternative, the larger the distance metric value, following Tkach & Simonovic (1997):

\[
L_j = \left[ \sum_{i=1}^{n} w_i \left( \frac{\tilde{h}_{i,j} - \tilde{f}_{i,j}}{\tilde{f}_{i,j} - \tilde{f}_{i,w}} \right)^{\frac{1}{p}} \right]^{p}
\]

(7)

where \( L_j \) is the fuzzy distance metric, \( \tilde{f}_{i,w} \) is the fuzzy worst value of the \( i \)th criteria, \( \tilde{f}_{i,j} \) is the fuzzy value of the \( i \)th criteria for alternative \( j \), \( \tilde{f}_{i,j} \) is the fuzzy optimal value of the \( i \)th criteria, \( \tilde{p} \) is a fuzzified parameter (\( 1 \leq p \leq \infty \)), \( w_i \) are fuzzified weights indicating decision-maker preferences, \( i = 1, n \) criteria and \( j = 1, m \) alternatives.

Spatial Fuzzy Compromise Programming (SFCP) works on the same principle as that of Compromise and Spatial Compromise Programming. The additional information that is required as input for the computation of the distance metric is in the form of fuzzified criteria images, a fuzzified parameter \( p \) and fuzzified weights \( \tilde{w}_i \). This fuzzification has been proposed to account for the vagueness or uncertainty in the decision-making process. The process of cell by cell fuzzification of each input image can be carried out using an appropriate membership function, such as Gaussian, triangularly shaped, sigmoidally shaped or Z-shaped.

Mathematical formulation of the distance metric for Spatial Fuzzy Compromise Programming is given by

\[
L_{i,x,y} = \left[ \sum_{i=1}^{n} \tilde{w}_i \left( \frac{\tilde{f}_{i,w,x,y} - \tilde{f}_{i,j,x,y}}{\tilde{f}_{i,j,x,y} - \tilde{f}_{i,w,x,y}} \right)^{\frac{1}{p}} \right]^{p}
\]

(8)

where \( L_{i,x,y} \) is the fuzzy distance metric, \( \tilde{f}_{i,w,x,y} \) is the fuzzy worst value of the \( i \)th criteria, \( \tilde{f}_{i,j,x,y} \) is the fuzzy value of the \( i \)th criteria for alternative \( j \), \( \tilde{p} \) is a fuzzified parameter (\( 1 \leq p \leq \infty \)), \( \tilde{w}_i \) are fuzzified weights indicating decision-maker preferences, \( i = 1, n \) criteria, \( j = 1, m \) alternatives, \( x = 1, a \) rows in the image, \( y = 1, b \) columns in the image, where \( a \) is the number of rows in the image and \( b \) is the number of columns in the image.

Using the values in the fuzzified distance metric images the best alternative is determined for each location. The fuzzified distance metric values for each location in the region of interest, as described by the feature image, are compared between the alternatives. The alternative having the largest fuzzified distance metric value for each raster cell is selected as the best. This cell-by-cell comparison between the alternatives is undertaken for each location in the region of interest. This process is illustrated in mathematical form as

\[
\text{max} - \text{dominant} \left\{ L_{i,x,y} = \left[ \sum_{i=1}^{n} \tilde{w}_i \left( \frac{\tilde{f}_{i,w,x,y} - \tilde{f}_{i,j,x,y}}{\tilde{f}_{i,j,x,y} - \tilde{f}_{i,w,x,y}} \right)^{\frac{1}{p}} \right]^{p} \right\}
\]

(9)
where $L_{i,x,y}$ is the fuzzy distance metric, $f_{i,\text{worst},x,y}$ is the fuzzy worst value of the $i$th criteria, $f_{i,x,y}$ is the fuzzy value of the $i$th criteria for alternative $j$, $f_{i,\text{optimal},x,y}$ is the fuzzy optimal value of the $i$th criteria; $\hat{w}_i$ are fuzzified weights indicating decision maker preferences; $\hat{p}$ is a fuzzified parameter ($1 \leq \hat{p} \leq \infty$), $i = 1, n$ criteria, $j = 1, m$ alternatives, $x = 1, a$ rows in the image, $y = 1, b$ columns in the image, where $a$ is the number of rows in the image and $b$ is the number of columns in the image.

Based on this comparison, an image identifying the best alternative for each location is produced. By inspecting this image, decision-makers are able to identify the alternative providing the greatest benefit for each location contained in the feature image.

### Implementation of Spatial Fuzzy Compromise Programming technique

In this study flood protection alternatives are evaluated and ranked using the proposed new technique of SFCP in order to illustrate a step-by-step implementation process. A schematic presentation of the process is shown in Figure 1. Initial data requirements include: (a) a Digital Elevation Model (DEM) of the region of interest, (b) separate feature images of buildings, roads, agricultural fields and any other features which might suffer damage in the region of interest, (c) hydraulic data, including river reach cross section profiles, expansion and contraction coefficients and Manning’s roughness coefficient and (d) a flood event data set, which forms the

![Figure 1 | Schematic presentation of the Spatial Fuzzy Compromise Programming.](http://iwaponline.com/jh/article-pdf/7/2/117/392719/117.pdf)
basis of the simulation process of flood protection alternatives.

The next step is to consider a set of potential flood protection alternatives that are feasible in the region of interest. Further, a set of relevant criteria/objectives needs to be decided upon. For example, in the case of flood protection planning one of the criteria could be to minimize the depth of floodwaters. Minimum damage to property and people is another potential criterion.

Having decided upon the criteria, a raster image is prepared for each of the criteria in which each raster cell contains the criteria values for all distinct geographic locations. This is accomplished using a combination of the flooded feature images, the water surface elevations as contained in the image and the DEM of the region of interest. Raster cells in locations which were unaffected by floodwaters retain a value of zero. In this way an image containing the criteria values for all flooded locations in the study region can be produced for each alternative.

Criteria values associated with each of the alternatives are contained within sets of criteria images, which are georeferenced with the feature images. Therefore the total number of criteria images equals the product of the number of criteria and the number of alternatives. Each raster cell in a criteria image contains the criteria value for that geographic location associated with a particular alternative. If the criteria is spatially variable then each affected cell, or location, within the image has a different value. If the alternative impacts all locations within the region of interest equally, all impacted cells contain the same criteria value. Using GIS the spatial distribution of the criteria values is captured.

The best and the worst criteria values are also required for computation of the distance metrics. Once again, rather than having just a single value for each criteria, the best and worst criteria values are determined for each location, or raster cell, in the feature image. This way each criterion has the best and the worst value image. The criteria values contained in the images, to be used for computation of the distance metrics, may be the actual or absolute minimum or absolute maximum. The choice of this is dependent on the criteria themselves and the opinion of the decision-makers. If it is the actual extreme values that are desired, these may be determined by comparing the values of the individual criteria for each location between the alternatives. The best and the worst value for each location can be extracted and placed into separate images using GIS commands. By using actual values, if the criteria values are spatially variable so too will be the best and the worst criteria value images. If the absolute maximum and minimum criteria values are required, new images georeferenced to the feature image are produced, whose initial value is that of the best or the worst criteria value.

Based on the criteria images, and the decision-maker’s preferences, a distance metric is calculated for each alternative. Contained in the distance metric images are distance metric values for each impacted raster cell in the region of interest. As illustrated in Figure 1, the fuzzified distance metric values within the images are calculated by comparing impacts for each location on a cell-by-cell basis between all alternatives and applying the decision-makers’ preferences, which are in the fuzzy form as well. All necessary computations are performed using GIS commands and Equation (8). Locations, or raster cells, in the study area for which there is no criteria value, or in other words, no impacts, are assigned a distance metric value of zero. Fuzzified distance metrics are then defuzzified for ranking purpose. Spatially variable ranking of flood protection alternatives is carried out to come up with the final picture of preferences of each alternative for each location in the region of interest.

RED RIVER BASIN (MANITOBA, CANADA) CASE STUDY

A floodplain analysis of the Red River Valley has been selected to demonstrate the capabilities of the Spatial Fuzzy Compromise Programming (SFCP) technique for multi-objective decision-making. This area is located in the south-central portion of the province of Manitoba, Canada. It consists of low-lying flat prairies predominantly used for agricultural purposes. The main population center in this area is the city of Winnipeg, which is located in the downstream portion of the valley, at the confluence of the Red River and Assiniboine River. Other communities of significant size further upstream in the Red River Valley include the towns of St. Adolphe, St. Agathe, Morris and Emerson.
The Red River Valley, which borders North Dakota and Minnesota in the US and expands north toward Lake Winnipeg in Manitoba, Canada, is very prone to flooding and has historically (1826, 1950, 1979 and 1997) incurred extensive damage to both urban and agricultural areas from floodwaters. The major floods are typically seasonal in nature and are the result of combined spring snowmelt and rainfall runoff along both the Red and Assiniboine Rivers (Krenz & Leitch 1993). For this study, the community of St. Adolphe located 20 km south of Winnipeg has been taken into consideration.

**Flood protection alternatives**

To alleviate the damage produced by flooding in the Red River Valley a number of structural and non-structural flood protection measures were implemented. A schematic presentation of the complex flood protection system for the city of Winnipeg is shown in Figure 2. The main infrastructure includes: (i) dikes along both the Red and Assiniboine Rivers, (ii) flood pumping stations within the city of Winnipeg, (iii) the Shellmouth Reservoir, (iv) Portage diversion and (v) the Red River Floodway. For the purpose of illustrating the methodology for the proposed Spatial Fuzzy Compromise Programming (SFCP), two flood protection measures for the community of St. Adolphe are considered: (a) a dike along the river bank and (b) modified operations of the Red River Floodway located immediately downstream from the town.

The floodplain analysis of the Red River Valley is selected as a case study in order to demonstrate the benefits of the new SFCP technique for addressing the conflict between upstream and downstream communities. The study focus is a 2.015 × 1.720 km region encompassing the community of St. Adolphe along the Red River. As St. Adolphe is the closest community upstream from the floodway inlet and gate structure, it is the one which is most heavily influenced by the floodway operation. In normal operations of the floodway, the backwater that it produces extends many kilometers upstream beyond St. Adolphe. As a result its operation is frequently responsible for heavy damage to the community and surrounding areas. For this reason, the largest conflict in the region is between St. Adolphe and the city of Winnipeg.

Three illustrative flood protection alternatives are developed for the application of SFCP:

1. A dike around the community. This dike has been simulated only on the right bank of the river to protect the community of St. Adolphe.
2. Alteration of the controlled floodway operation so as to let more floodwater flow through the floodway in order to protect the larger city downstream. This is achieved by raising the floodway gate height in such a way that the water surface elevation at the floodway entrance is increased by 1 meter above the normal level. This alternative will be referred to herein as Floodway 1.
3. Alteration of the controlled floodway operation so as to let less floodwater flow through the floodway in order to protect a community upstream. This is achieved by lowering the floodway gate height in such a way that the water surface elevation at the floodway entrance is decreased by 1 meter below the normal level. This alternative will be referred to herein as Floodway 2.

The basic spatial data set includes the digital elevation model (DEM) for the region under consideration. Figure 3 shows the DEM of the study region. This 5-meter resolution DEM was acquired from LIDAR (LIght Detection And Ranging) remote sensing data. Feature image data sets were acquired for the purpose of damage assessment due to flooding. In Figure 4, buildings in St. Adolphe are visible as small square-shaped grey elements, roads can be seen as the straight lines around the buildings and also across the river and the agricultural fields are illustrated as polynomials in shades of grey. Red River is shown on the left side of the image.
The next set of data required for the implementation of the SFCP technique includes hydraulic data of the Red River in the region of interest, which is needed for HEC-RAS hydraulic simulations (river cross section profiles, river flows and coefficients, such as Manning’s $n$, contraction and expansion coefficients). Using the HEC-RAS hydraulic model (Hydrologic Engineering Center 2001) the simulation of all three flood protection alternatives is performed. The results of these simulations are listed in Table 1.

Criteria for evaluation of flood protection alternatives

Two criteria that exhibit a spatial variability are selected for evaluating the alternatives: (a) water depth and (b) flood damage. The computational procedures necessary to produce the raster criteria images involve the use of GIS software and data on damage curves for buildings, agriculture and roads.

The first criterion used in evaluation of the alternatives is the floodwater depth for the study region. An image is prepared by combining the flooded feature images, the water surface elevations as contained in the image and the DEM of the region of interest. For all flooded areas the ground surface elevations in the DEM are subtracted from the simulated water surface elevation. Raster cells in locations which were unaffected by floodwaters retained a value of zero. In this way an image containing the water depths for all flooded locations in the study region is produced for each alternative.

For the SFCP technique, separate images showing the best and the worst criteria values for each location in the study region are also required. For the floodwater depth criteria, the absolute minimum water depth has been considered the best criteria value. The actual maximum floodwater depths are used to represent the worst criteria value. The second criterion used in evaluation of the alternatives is the monetary value of damage to buildings.

### Table 1 | HEC-RAS simulation results for three flood protection alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Total discharge at floodway entry point (m$^3$/s)</th>
<th>Water surface elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dike</td>
<td>3650</td>
<td>232.89</td>
</tr>
<tr>
<td>Floodway 1</td>
<td>4730</td>
<td>233.83</td>
</tr>
<tr>
<td>Floodway 2</td>
<td>2900</td>
<td>231.71</td>
</tr>
</tbody>
</table>
roads and agricultural land within the region of interest. KGS Group (2000) recommendations, which are based on the 1997 flood event, are implemented to arrive at the dollar value damages associated with each of the three categories. KGS Group (2000) data are used to arrive at the depth–damage relationship for buildings:

$$y = 76879x^3 - 344873x^2 + 470283x + 538659$$  \(10\)

where $y$ is the dollar value of damage to buildings; and $x$ is the floodwater depth.

Damage to roads is expressed (KGS Group 2000) as the relationship between the monetary value of damage and the total length of submerged roads:

$$rd = 18.889L^2 + 261.25L + 300000$$  \(11\)

where $rd$ is the dollar value of damage to roads and $L$ is the total length of flooded roads.

Agricultural damage assessment depends on the time of year and the type of crop in the region of interest. Though spatial variability in crop type would be there in the study region, an accurate account of such raster data was not available. Therefore, only one crop, namely, R.S. Wheat, is assumed to be in the agricultural fields at the time of flooding. The following relationship is used to assess the agricultural damage in the region (KGS Group 2000):

$$ad = \sum [(1 - \text{yield}) \times (cp) \times A \times \text{price}]$$  \(12\)

where $ad$ is the dollar value of agricultural damage, $\text{yield}$ is the expected yield (fraction of optimum) as a function of seed date, $cp$ is crop percentage of a typical distribution ($cp = 1$ in this case), $A$ is the area of cropland (acres) and $\text{price}$ is the three-year average price of the crop ($/bushel$).

**Numerical analysis**

The utility of Spatial Fuzzy Compromise Programming is demonstrated through the comparison between the deterministic and fuzzy analysis. Three flood protection alternatives are evaluated spatially according to the two criteria. Further insights into the SFCP technique are obtained by comparing the results obtained by implementing various shapes of the fuzzy membership functions. The following set of experiments is performed:

1. Deterministic spatial multi-objective analysis of flood management options for the case study region with three different weight sets.
2. Fuzzy spatial multi-objective analysis of flood management options for the case study region with the triangular membership function and three weight sets.
3. Fuzzy spatial multi-objective analysis of flood management options for the case study region with the Z-shaped membership function and three weight sets.

Weights indicating the relative decision-maker preferences towards the two criteria are symbolized as $w_i$ in Equation (5). In order to represent the potential different opinions of the various stakeholder groups of interested decision-makers in the case study, three different sets of weights are selected and shown in Table 2. The first weight set is selected to give an equal level of importance to both of the criteria. The other two weight sets were chosen to represent the difference (to the order of extreme nature) in opinions and interests between various decision-makers. The importance of the maximum deviation from the ideal solution, accounted for by variable $p$ in Equation (5), is also the necessary input for the deterministic analysis. In this case study, a single value of $p = 2$ is used in the evaluation of all alternatives. Selection of this value is based on the results produced by Simonovic (1989), wherein it is determined that a selection of $p = 2$ can be used as a reasonable approximation of the best compromise alternative from a set of potential compromise solutions.

Fuzzy spatial multi-objective analysis is performed by fuzzifying the criteria image inputs. Through the application of fuzzy set theory the vagueness or uncertainties associated with stakeholder preferences, the parameter $p$ and criteria values can be addressed in an efficient and accurate manner. Fuzzification of criteria images is performed using the environment of MathWorks’ Fuzzy Logic Toolbox of

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight set 1</th>
<th>Weight set 2</th>
<th>Weight set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floodwater depth</td>
<td>0.5</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Damages</td>
<td>0.5</td>
<td>0.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>
MATLAB (MATLAB 2000). Selection of suitable membership function is based on the nature of the criteria values (Despic & Simonovic 2000). In this case study two membership functions are found to be appropriate and fitting, namely the triangular membership function (T-MF), which is illustrated in Figure 5(a), and the Z-shaped membership function (Z-MF) as shown in Figure 5(b).

The triangular membership function is a function of a vector, \( x \), and depends on three scalar parameters \( a \), \( b \) and \( c \), as given by

\[
f(x; a, b, c) = \begin{cases} 
0, & x \leq a \\
\frac{x - a}{b - a}, & a \leq x \leq b \\
\frac{c - x}{c - b}, & b \leq x \leq c \\
0, & c \leq x 
\end{cases}
\] (13)

The parameters \( a \) and \( b \) locate the “feet” of the triangle and the parameter \( c \) locates the peak as shown in Figure 5(a). The choice of triangular membership has been made due to its characteristic that this function expands a crisp value on both sides to convert a value into a range format. For example, a crisp value of “4” can be converted to a range of “3.5 to 4.5” while keeping the value “4” as the peak value. This is a fairly convenient way of fuzzifying any number.

Among the other membership functions there are the Gaussian curve membership function, the bell-shaped membership function, the \( P \)-shaped membership function, the product of two sigmoidally shaped membership functions and the trapezoidally shaped membership function. However, they are not very different from the triangular membership function in terms of impacts produced by the application of these membership function shapes.

The Z-shaped function is basically a spline-based function of \( x \). The parameters \( a \) and \( b \) \((a < b)\) locate the extremes of the sloped sections of the curve (Figure 5(b)). The Z-shaped membership function is defined by

\[
Z(x; a, b) = \begin{cases} 
0, & x \leq a \\
1 - 2\left(\frac{x - a}{b - a}\right)^2, & a < x \leq \frac{a + b}{2} \\
2\left(\frac{b - x}{b - a}\right)^2, & \frac{a + b}{2} < x \leq b \\
1, & b < x 
\end{cases}
\] (14)

Z-MF takes any crisp value \( x \) and expands it according to the shape of the membership function, which is defined by the parameters \( a \) and \( b \). The fuzzified value is always in the form of a decreasing function (maintaining the Z-shape) between one and zero. For application in the case study, the Z-shaped MF is appropriate because of its shape, which varies from the highest value of MF (one) to the lowest value of MF (zero). This shape is suitable for both of the criteria considered, namely flood depth and flood damage, because when flood depth is minimum (zero on the x axis) then the degree of membership is highest (one on the y axis) and vice versa. Similarly, minimum damage provides the highest degree of membership, which suits the particular objective of minimizing the flood damages.

**Results of the analysis**

Application of spatial multi-objective analysis ends with the map that shows which alternative is the best compromise solution for each location in the region. All the final results are shown in Figures 6, 7 and 8. Each figure contains three
Comparison of deterministic and fuzzy analyses

Looking at all three sets of experiments for weight set 1 (equal weights assigned to both criteria), it is observed in Figures 6(a), 7(a) and 8(a) that alternative “Dike” provides the highest protection for most of the case study region except for the left bank of the Red River, where alternative “Floodway 2” is found to be providing better protection, and some scattered spots where alternative “Floodway 1” offers better protection. The spatial fuzzy approach using T-MF (Figure 7(a)) illustrates that alternative “Floodway 1” provides the highest protection for most of the study region and alternative “Floodway 2” offers protection to some buildings, roads and the floodplains on both sides of the river. The SFCP approach using Z-MF indicates that alternative “Floodway 2” is the best compromise for most of the region and alternative “Floodway 1” is recommended for some scattered locations. A point to note here is that alternative “Dike” dominates the deterministic analysis, alternative “Floodway 1” provides better protection to most of the region in the SFCP analysis using T-MF and “Floodway 2” dominates the entire region using SFCP analyses using Z-MF. A separate comparison of all the alternatives using three different experiments gives an impression that the selection of one approach over the others is not possible at this point where the comparison of alternatives has been done based on equal weight assignment to both criteria.

Weight set 2 assigns different weights to two criteria considered in this study. Figures 6(b), 7(b) and 8(b) present the ranking of alternatives using the three approaches. The deterministic and SFCP using Z-MF produce similar results for most of the case study region except for some scattered spots that are shown as suitable for alternative “Floodway 2” in Figure 6(b). SFCP using T-MF (Figure 7(b)), however, shows a completely different picture of suitability of the three alternatives.
Figure 7 | Final ranking of the alternatives using spatial fuzzy analysis with triangular membership function: (a) weight set 1, (b) weight set 2 and (c) weight set 3.

Figure 8 | Final ranking of the alternatives using spatial fuzzy analysis with Z-shaped membership function: (a) weight set 1, (b) weight set 2 and (c) weight set 3.
alternatives. In this case, alternative “Floodway 1” is found to be most effective in most of the region and “Floodway 2” is found to be suitable in the floodplains on both sides of the river plus some buildings and roads. The alternative “Floodway 2” is mostly found suitable only in the floodplain on the left side of the river using deterministic and SFCP with Z-MF analyses.

Lastly, weight set 3 rankings are shown in Figures 6(c), 7(c) and 8(c). It can be noted that all three approaches generate different results. Deterministic analysis shows alternative “Floodway 2” as the most preferred, alternative “Floodway 1” is next and alternative “Dike” is shown to be attractive only for some scattered points in the region. SFCP with T-MF analysis ranks alternative “Dike” as the best compromise for most of the region and “Floodway 2” for the left river bank floodplain. SFCP with Z-MF shows a completely different ranking by choosing alternatives “Dike” and “Floodway 2” as the most preferred alternatives.

Comparison of SFCP with T-MF for three different weight sets

The results of spatial fuzzy compromise programming analyses with a triangular membership function obtained for three different weight sets are shown in Figure 7. For the case of equal weights assigned to both criteria, alternative “Floodway 2” is the best compromise for the floodplains and some of the roads and buildings. For the rest of the region alternative “Floodway 1” is preferred. Weight set 2, in which less importance has been given to the criterion “flood depth”, produced a ranking of the alternatives as shown in Figure 7(b). It can be noted that alternative “Floodway 2” is suitable for the protection of some buildings and floodplains (a little less compared to weight set 1 ranking). Weight set 3, in which less importance has been given to damage and more importance to flood depth, shows “Floodway 2” being more effective for the left bank floodplain and “Floodway 1” for some locations, while the alternative “Dike” protects most of the area including the right bank floodplain.

Comparison of SFCP with Z-MF for three different weight sets

Results of the fuzzy analyses with Z-MF are shown in Figure 8. Unequal weight assigned to criteria values selected “Dike” and “Floodway 2” as the most effective alternatives for most of the region. However, with equal weight assigned to both of the criteria, SFCP with Z-MF shows a different choice of alternatives. “Floodway 2” is found to be preferred for most of the area, while “Floodway 1” and “Dike” are acceptable only for some scattered locations.

If only the ranked alternatives produced by both SFCP using T-MF and SFCP using Z-MF were to be compared, it is apparent that SFCP use of the triangular membership function seems more appropriate for flood management because there is a difference between the ranking of alternatives using SFCP (T-MF) for weight set 2 (Figure 7(b)) and weight set 3 (Figure 7(c)). On the other hand, the ranking of alternatives using SFCP (Z-MF) for weight set 2 (Figure 8(b)) and weight set 3 (Figure 8(c)) does not show a significant effect of differently assigned weights.

CONCLUSIONS

Existing multi-objective decision-making techniques have limitations in terms of their ability: (a) to address water resources management problems with spatially distributed decision variables and (b) to consider both objective and subjective uncertainties involved in the decision-making. In this study a new multi-objective decision-making technique, called Spatial Fuzzy Compromise Programming (SFCP), has been introduced in order to address shortcomings of the existing techniques. Application of Spatial Fuzzy Compromise Programming to flood management demonstrated the ability of the technique to address spatial decision-making under uncertainty.

The Red River flood management case study and the set of experiments designed to test the new technique generated the following conclusions:

1. Spatial fuzzy multi-objective decision-making provides additional information that is of value in the decision-making process. The most preferred solution is identified for each location in the region of interest. Spatial distribution of the preferred alternative solutions can be easily shown in the form of a map by integrating the information generated for each location. This type of map can be used in the process of determining future flood protection options for the region. The same
information can be used in addition to assess the level of vulnerability of each location in the region under the existence of a particular protection measure. This information can be valuable in the implementation of zoning ordinance or flood insurance non-structural measures.

2. Results obtained from the comparison of different fuzzy membership functions indicate that the shape of this function affects the final ranking a great deal. Therefore, selection of the appropriate membership function is one of the pre-requisites for successful application of the proposed technique. An initial work on the development of fuzzy membership functions for flood management is presented in Despic & Simonovic (2000). Additional research in this area is warranted.

3. Implementation of the new technique through integration of GIS software with the MATLAB tool allows for easy adaptation of the proposed approach to other problems in water resources decision-making.

ACKNOWLEDGEMENTS

Funding from the Institute for Catastrophic Loss Reduction (ICLR) and National Sciences and Engineering Research Council (NSERC) of Canada to carry out this work is gratefully acknowledged. Thanks are also due to Dr K. Ponnambalam, Dr V. Rajasekaram and Mr P. Prodanovic for their valuable help.

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