Using equations (6) to (11), the following equation for iteratively solving for $a_i$ and hence $a_i$, and hence $a_i$, is obtained:

$$a_i = \frac{(e_i - e_{ai})/\varphi}{(e_i - e_{ai})/\varphi}$$

where values of $a_{ai}$, $a_i$ and $e_i$ correspond to the $i$th iteration. The actual steps involved in the iteration process are shown in Fig. 3. Thus for a given $e_i$, the values of $a_i$ are obtained.

Substituting the values of $a_i$, $a_i$, and $e_i$ in the moment equilibrium equation, the moment curvature relation is obtained:

$$M = EI\varphi - P\varphi Q_i \int_0^{e_i} (e_i - a_i + a_i) a Q_i \cdot b \cdot d a - P\varphi Q_i \int_0^{e_i} (e_i - a_i + a_i) a Q_i \cdot b \cdot d a$$

In a general case "b" is a known function of "a". For a rectangular section, "b" is constant.

We have, therefore, a method for computing moment-curvature values from known surface strains.

**DISCUSSION**

M. R. Mitchell

My experience with gray iron stems from my former association with Ford Motor Co., Scientific Research Staff, and was primarily concerned with the cyclic deformation and fracture behavior of this “heavily defected” structure. The approach employed considered the size, shape, and distribution of these graphite “flaws” as controlling the deformation response of gray iron. This resulted in an extension of the Neuber analysis by considering the graphite “notches” as having a fatigue strength reduction factor ($K_f$) associated with various morphologies and matrix hardnesses.

G. N. J. Gilbert in “Factors Relating to the Stress-Strain Properties of Cast Iron,” BCIRA Report No. 459, appearing in BCIRA, Vol. 6, No. 11, April, 1957, pp. 546–587, presents a review of several papers which deal with modeling the monotonic stress-strain curves of gray irons. Of these, a representation presented by M. Misiaszek, Pracze Glownego Instytutu Odlewnictwa, 1951, No. 1, pp. 19–28, in which the tensile stress-strain curve is represented by:

$$f = \frac{ax}{1 + bx}$$

where $f$ = stress, $x$ = strain, $a$, $b$ = constants and the compression stress-strain curve is represented by:

$$f = ax - bx^2$$

adequately described this particular mechanical behavior. Why does Dr. Jhansale choose a “two-part” description of the monotonic curves and how is it any more accurate than the above description?

It would appear that designs based on the methods presented in Dr. Jhansale’s paper would anticipate a single application of strain or load. In reality, most components of any material experience repeated strains or loads. Can the “Factors of Safety” presented in this paper be employed or extended to incorporate

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Authors’ Closure

The authors wish to thank Mr. Mitchell for his contribution to the discussion. The queries raised by Mr. Mitchell are helpful in clarifying a number of points made in the paper. The queries are discussed below:

1. The stress-strain model chosen by the authors has the following advantages:

   (a) The tension and compression stress-strain curves for gray iron are adequately described by a single model. The material constants do differ for tension and compression. However, the use of a single model simplifies the calculations illustrated in the Appendix of the paper.

(b) The model for the compression stress-strain curve mentioned in the discussion is a special case of equation (1) in which $e_c = 0$ and $Q = 2$.

2. The model for the stress-strain curve applies to monotonic loading situations. Further analysis is required for more complex loading conditions, e.g., cyclic loading and fatigue. Analyses applied to ductile metals, whereby a complete loading history can be modelled, could be extended to “brittle” materials, such as gray iron. With such analyses available, stress predictions, and hence factors of safety, could be developed which related material strength and sectional strength parameters to the appropriate loading conditions, e.g., cyclic loading.

3. The data quoted in Figs. 1, 4, and 5 were obtained from [4], and are those of a medium flake graphite pearlitic iron with a carbon equivalent of approximately 4.57 percent. This is a low strength iron. The actual graphite morphology is not known.

4. It is considered that the model (equation (1)) with suitable choice of four constants is capable of describing a wide range of stress-strain characterisics from a linear to completely nonlinear type. Thus, widely different gray irons could be described with the selection of suitable constants $E$, $P$, $e_c$, and $Q$ for any one type of iron.

To clarify Mr. Mitchell’s doubt about “consistent factors of safety,” the term “consistent” has been used to emphasize the rationality of the ultimate strength approach. Such an approach, as used in the paper, ensures equal margins of safety in terms of sectional strength. Thus “consistent” does not relate to the character of gray iron, but to the factor of safety, once $V_s$ and $V_b$ are chosen by a designer, for an assumed reliability.

4. The paper is not a comprehensive statement of a design procedure for gray iron. It points out that the inelastic behavior of the material can be considered in design. If this behavior is combined with sectional strength evaluation and a statistical approach to safety, then it is considered the approach offered has merit over the conventional elastic approach where modes of failure and margins of safety are uncertain.

The computations encountered in an inelastic analysis can be handled with little expense using existing computers. Thus, the complexity of such analyses is no longer a disadvantage. A designer concerned with the behavior of sections and elements formed from inelastic materials can use the design procedures suggested to improve new designs or examine uncertainty in existing designs.

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