Table 1 Shaking force and shaking moment before balancing

j	φ_1 (θ)	$_AF_x$ (F)	$_AF_y$ (F)	M_A (FL)
1	0.00000	0.49957	1.54370	0.40278
2	0.26180	0.21336	1.01900	1.50340
3	0.52360	-0.10066	0.45425	2.40980
4	0.78540	-0.43382	-0.11425	3.07550
5	1.04720	-0.76883	-0.65784	3.49250
6	1.30900	-1.07920	-1.15370	3.67780
7	1.57080	-1.33210	-1.58210	3.65810
8	1.83260	-1.49430	-1.92370	3.45790
9	2.09440	-1.53860	-2.15860	3.09410
10	2.35619	-1.45110	-2.26810	2.57720
11	2.61799	-1.23420	-2.23680	1.91610
12	2.87979	-0.90807	-2.05630	1.12460
13	3.14159	-0.50665	-1.7280	0.22681
14	3.40339	-0.07236	-1.26570	-0.73905
15	3.66519	0.35130	-0.69561	-1.71870
16	3.92699	0.72602	-0.05615	-2.64440
17	4.18879	1.02350	0.60468	-3.43730
18	4.45059	1.22760	1.23290	-4.01450
19	4.71239	1.33430	1.77390	-4.29970
20	4.97419	1.34920	2.17840	-4.23690
21	5.23599	1.28440	2.40960	-3.80530
22	5.49779	1.15490	2.44920	-3.03150
23	5.75959	0.97483	2.30000	-1.99210
24	6.02139	0.75437	1.98550	-0.80310

Table 2 Shaking force and shaking moment after balancing

j	φ_1 (θ)	$_AF_x$ (F)	$_AF_y$ (F)	M_A (FL)
1	0.00000	1.17460	0.10814	2.40660
2	0.26180	1.23690	-0.19294	3.12250
3	0.52360	1.20170	-0.45151	3.53380
4	0.78540	1.05860	-0.65206	3.62780
5	1.04720	0.81199	-0.79104	3.43550
6	1.30900	0.48221	-0.87324	3.01540
7	1.57080	0.10344	-0.90711	2.4354
8	1.83260	-0.28233	-0.90008	1.75820
9	2.09440	-0.63291	-0.85621	1.03330
10	2.35619	-0.91329	-0.77567	0.29573
11	2.61799	-1.10100	-0.65805	-0.43055
12	2.87979	-1.18850	-0.49491	-1.12730
13	3.14159	-1.18170	-0.29244	-1.77700
14	3.40339	-1.09590	-0.05371	-2.35810
15	3.66519	-0.95113	0.21015	-2.84270
16	3.92699	-0.76647	0.48166	-3.19680
17	4.18879	-0.55731	0.73788	-3.3803
18	4.45059	-0.33375	0.95246	-3.3521
19	4.71239	-0.10128	1.09880	-3.0770
20	4.97419	0.13720	1.15480	-2.53720
21	5.23599	0.37868	1.10720	-1.74450
22	5.49779	0.61718	0.95671	-0.75005
23	5.75959	0.84163	0.71921	0.35450
24	6.02139	1.03480	0.42413	1.4489

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DISCUSSION

Trevor Davies[2]

The method presented is an ingenious and valuable contribution to the art of balancing. It is probably the best available technique for balancing with a single mass, and is an improvement on the method discussed by Crossley [7][3] in that account is taken of the effect of the shaking moment. However, in the particular example illustrated, Crossley's method would have been almost as effective. The polar diagram of the force vector before balancing is roughly elliptical with a major/minor axis ratio of about 5. A single balance mass 90 deg out of phase of magnitude equal to $^3/_5$ of this major axis reduces this polar diagram to an approximate circle of radius equal to about $^2/_5$ of the major axis. A mass rotating at crank speed in the opposite sense to the crank would further reduce the out of balance force by a factor of about 5. The author in his conclusion does mention that additional masses will cause a greater reduction, but he is not specific.

The title chosen by the author might lead one to believe that the method is of general application to high speed machinery. This is not true of this method, or Crossley's, or any other method of balancing that employs balance masses rotating only at crank speed. In the example illustrated, as in Crossley's, the force vector is predominately first harmonic, hence the nearly elliptical form on the polar diagram. For other mechanisms, and notably those in which the loci of the centers of mass of the parts deviate far from circles, ellipses, and straight lines, balance masses rotating at constant speed will be relatively ineffective in producing balance.

For these mechanisms a more fundamental approach is needed, directed toward bringing the center of mass of the entire mechanism to rest during motion. Briefly, there appear to be two approaches. Firstly, the obvious but inconvenient device of duplicating the mechanism with a dummy mechanism having axial symmetry. This eliminates any unbalanced force but doubles the unbalanced moment. (Reflection, or mirror symmetry, eliminates the moment and one component of the force, but doubles the other component). For perfection, both axial and reflection symmetry are necessary.

Secondly, by a redistribution of mass in each link the center of mass can be transferred to a fixed point. This technique requires detailed explanation, but its application to the simple plane four-bar linkage was illustrated in 1941 by Talbourdet and Shepler [8], and it is also described by Artobolevski [9]. The major drawback of this technique is that it leads to a large increase in mass and rotational inertia.

In view of the large amount of careful preparation involved in using Dr. Han's method, even if the computer program is available, it is as well for the designer to be forewarned that the results of using this technique could be disappointing.

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[3] Numbers in brackets designate Additional References at end of this discussion.

Additional References

7 F. R. E. Crossley, "The Balancing of High-Speed Oscillating Feed Mechanisms," ASME Paper No. 64-Mech-28.

8 G. J. Talbourdet and P. R. Shepler, "Mathematical Solution

of 4-Bar Linkages, Part IV—Balancing of Linkages," *Machine Design*, vol. 13, July, 1941, pp. 73–77.
9 I. I. Artobolevski, *Theory of Mechanism and Machines*, third edition, Ministry of Culture of USSR, Moscow, 1953.

A. B. O. Soboyejo[4]

While I wish to congratulate the author of this paper for his magnificent effort, I wish to comment that the title of this paper is misleading in that it could be pointed out that the method developed by the author provides a specific method of solving the problem of minimizing the mechanical vibration of a punch and reader unit at idle condition, and that the method cannot be applied directly to the solution of the balancing of high speed machinery in general.

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Author's Closure

The author wishes to thank Dr. Davies and Dr. Soboyejo for their discussions. Equation (7) can be generalized to three-dimensional case. In the event that the optimized solution so obtained does not bring down the disturbing action on the machine support to a tolerable extent, more than one balancing weight can be mounted on different shafts from which the major part of disturbing action is generated. Thus a set of equations (8) and (9) can be obtained by principle of minimum; the solution of mass moments and phase angles of balancing weights can be generally obtained by Powell's method.[5] Generally speaking, using more than one balancing weight can achieve better balance.

5 M. J. D. Powell, "A Method for Minimizing a Sum of Non-Linear Functions Without Calculating Derivatives," *Computer Journal*, London, 1965.