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Momentum Transfer between Quantum Vacuum and Anisotropic Medium

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An anisotropic electromagnetic environment that can be created inside a Faraday chiral material may cause breaking of the universal symmetry of vacuum mode structure and hence lead to a nonzero electromagnetic momentum density of the quantum vacuum. A novel quantum vacuum effect (i.e., transfer of linear momentum from an anisotropic quantum-vacuum fluctuation field to a Faraday chiral material) is predicted. This is a macroscopic quantum vacuum mechanical effect that may provide us with new insight into the electromagnetic structures of quantum vacuum fluctuation fields inside anisotropic artificial materials.

§1. Introduction

The quantum vacuum (i.e., the ground state of quantum fields) has attracted the attention of many physicists in various areas, such as quantum field theory,[1)–3)] quantum optics[4),5)] and condensed matter physics.[6)] This vacuum can exhibit a number of intriguing properties, effects and phenomena, including vacuum topological structures,[1)] vacuum polarization (causing the Lamb shift, and hence the hyperfine structure of hydrogen atomic spectra),[2),3)] anomalous magnetic moment of the electron, Casimir effect,[7)–9)] and dramatic modification to atomic spontaneous emission (decay) rates in a QED cavity.[4)] Recently, some peculiar quantum vacuum effects arising in electromagnetic media (and artificial composite materials) have been predicted and observed. These effects include spontaneous emission inhibition in EIT (electromagnetically induced transparency) media,[10)] magnetoelectric birefringence of the quantum vacuum[11)] and vacuum-induced Berry's phases.[12),13)] But most of these vacuum effects occur in the microscopic domain, and in the literature, the influence of the quantum vacuum on the mechanical properties (e.g., motion) of a macroscopical medium have received less attention than it deserves, i.e., only the quantum vacuum contribution to the momentum of a medium, such as a magnetoelectric material with a simple constitutive relation, has been considered by some authors.[14),15)] One of the reasons for this situation is that the change in the vacuum mode distribution (particularly, the breaking of the universal symmetry of the quantum vacuum) inside conventional materials is not conspicuous enough to create macroscopically observable mechanical effects. However, with recent developments in the technology of artificial composite materials,[16)–20)] macroscopic

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quantum phenomena caused by an anisotropic quantum vacuum could be possibly realized experimentally. Here, we present an effect of the quantum vacuum contribution to the macroscopic mechanical properties of an anisotropic material (Faraday chiral material), in which an anisotropic electromagnetic environment could be built up, and hence the universal symmetry of the quantum vacuum could be broken. This means that it could be possible to manipulate the quantum vacuum by appropriately choosing the optical parameters of anisotropic electromagnetic materials. It is shown below that the produced anisotropic quantum vacuum would possess a small but nonzero linear momentum density. Thus, the anisotropy of the quantum vacuum may lead to the transfer of momentum from a quantum vacuum to an anisotropic material (e.g., Faraday chiral material whose constitutive relation is more complicated than that of magnetoelectric material[14)]).

The anisotropy characterization of Faraday chiral material can lead to some intriguing effects on wave propagation. For example, Mackay and Lakhtakia first suggested the possibility of negative phase velocity propagations in arbitrary directions in a gyrotropic (Faraday) chiral medium and obtained general and complete results.[17)] This can be regarded as an interesting classical electromagnetic effect of the anisotropic Faraday chiral material. Apart from such classical effects, the anisotropic quantum field effects in a Faraday chiral material also warrant consideration. In this paper, we consider the anisotropic characteristics of the eigenmodes of electromagnetic fields (including the breaking of the universal symmetry of vacuum modes) inside a Faraday chiral material, and then we treat the problem of field quantization in the material. Based on this, we study the electromagnetic momentum of an anisotropic quantum vacuum and obtain an explicit expression for the velocity acquired by the Faraday chiral material due to momentum transfer at quantum vacuum level. The physical significance and potential applications of such a quantum vacuum mechanical effect are also discussed.

§2. Wave propagation and eigenmodes in a Faraday chiral material

Faraday chiral material can be understood as a combination of natural optical activity (as realized in an isotropic chiral medium[21)]) with Faraday rotation (as realized in a gyroelectric or gyromagnetic medium[22)]). The frequency domain constitutive relation for the Faraday chiral material is given by[16),17)]

$$\begin{aligned}\underline{D}(\underline{r}) &= \underline{\underline{\epsilon}} \cdot \underline{E}(\underline{r}) + \underline{\underline{\xi}} \cdot \underline{H}(\underline{r}),\\ \underline{B}(\underline{r}) &= -\underline{\underline{\xi}} \cdot \underline{E}(\underline{r}) + \underline{\underline{\mu}} \cdot \underline{H}(\underline{r})\end{aligned} \tag{2·1}$$

with the constitutive dyadics[17)]

$$\begin{aligned}\underline{\underline{\epsilon}} &= \epsilon_0 \left[\epsilon \underline{\underline{I}} - i\epsilon_{\rm g}\underline{\hat{z}} \times \underline{\underline{I}} + (\epsilon_z - \epsilon)\underline{\hat{z}}\underline{\hat{z}}\right],\\ \underline{\underline{\xi}} &= i\sqrt{\epsilon_0\mu_0} \left[\xi \underline{\underline{I}} - i\xi_{\rm g}\underline{\hat{z}} \times \underline{\underline{I}} + (\xi_z - \xi)\underline{\hat{z}}\underline{\hat{z}}\right],\\ \underline{\underline{\mu}} &= \mu_0 \left[\mu \underline{\underline{I}} - i\mu_{\rm g}\underline{\hat{z}} \times \underline{\underline{I}} + (\mu_z - \mu)\underline{\hat{z}}\underline{\hat{z}}\right].\end{aligned} \tag{2·2}$$

Such a material can be fabricated using a homogenized composite medium (HCM)[16),17)] obtained from the blending together of an isotropic chiral medium with either a magnetically biased ferrite[18)] or a magnetically biased plasma.[19)] With the development of polymer synthesis techniques, much attention has been given to both fundamental theoretical problems (i.e., optical and electromagnetic properties) and experimental realizations (including potential applications) of Faraday (gyrotropic) chiral media.[20),23)–25)]

In this section we study the anisotropy features influencing the eigenmodes inside a Faraday chiral material and elucidate the manner in which the universal symmetry of the quantum vacuum modes is broken. Consider the propagation of a plane wave with field amplitudes $\underline{E}(\underline{r}) = \underline{E}_0 \exp(ik_0\tilde{k}\underline{\hat{u}} \cdot \underline{r})$ and $\underline{H}(\underline{r}) = \underline{H}_0 \exp(ik_0\tilde{k}\underline{\hat{u}} \cdot \underline{r})$, where $k_0 = \omega/c$ is the free-space wave number and $\underline{\hat{u}} = \underline{\hat{x}} \sin\theta\cos\phi + \underline{\hat{y}} \sin\theta\sin\phi + \underline{\hat{z}} \cos\theta$ is a unit vector. Here, $\underline{\hat{x}}, \underline{\hat{y}}$, and $\underline{\hat{z}}$ denote the unit vectors of the coordinate system. Using Maxwell's equations and constitutive relation (2·1), one can verify that the relative wave number $\tilde{k}$ satisfies a quartic equation

$$a_4\tilde{k}^4 + a_3\tilde{k}^3 + a_2\tilde{k}^2 + a_1\tilde{k} + a_0 = 0 \tag{2·3}$$

whose four roots correspond to the relative wave numbers of the four eigenmodes (including the forward and backward wave modes, as well as their respective left- and right-handed polarization components) in the anisotropic material. Obviously, the wave numbers of the eigenmodes in the Faraday chiral material exhibit a characterization of anisotropy, i.e., the magnitudes of the wave numbers depend upon the spherical angles θ and ϕ. We point out that the coefficients in Eq. (2·3) have explicit physical meanings, which correspond to some nontrivial effects. For example, the nonzero quantity a_1 in Eq. (2·3) unavoidably gives rise to a non-degenerate $\tilde{k}^2$. This may lead to the possibility of backward wave propagation and negative refraction in the anisotropic material. Mackay and Lakhtakia considered the propagations with negative phase velocity in arbitrary directions in Faraday chiral media,[17)] and we investigated the quantum effects involved in such negative phase-velocity propagations.[26)] The coefficient a_3 has a physical meaning related to the summation of the relative wave numbers over the four eigenmodes:

$$\sum_{\sigma=1}^{4} \tilde{k}_\sigma = -\frac{a_3}{a_4}, \tag{2·4}$$

where σ corresponds to the four roots of the quartic equation (2·3), and a_3 and a_4 are given by[17)]

$$\begin{aligned} a_3 &= 2\{[\mu_{\rm g}(\epsilon\xi_z - \epsilon_z\xi) + \epsilon_{\rm g}(\mu\xi_z - \mu_z\xi) + \xi_{\rm g}(\mu\epsilon_z + \epsilon\mu_z - 2\xi\xi_{\rm g})]\sin^2\theta \\ &\quad +2\xi_{\rm g}(\epsilon_z\mu_z - \xi_z^2)\cos^2\theta\}\cos\theta, \\ a_4 &= (\epsilon\sin^2\theta + \epsilon_z\cos^2\theta)(\mu\sin^2\theta + \mu_z\cos^2\theta) - (\xi\sin^2\theta + \xi_z\cos^2\theta)^2. \end{aligned} \tag{2·5}$$

In general, for isotropic materials and most anisotropic media, in which the temporal and spatial symmetries involved in the constitutive relations are preserved, the sum of the wave numbers of the four eigenmodes vanishes (i.e., $a_3 = 0$). However, for

Faraday chiral material characterized by the constitutive relation (2·1), such a sum (2·4) does not vanish (because $a_3 \neq 0$). This implies that there may be a breaking of the universal symmetry of the four eigenmodes (the counter-propagating modes as well as their respective mutually perpendicular polarization components) and that this anisotropic material would exhibit some unusual classical and quantum optical effects (including an influence on the mechanical properties of the anisotropic material itself). One of the most remarkable effects is the macroscopic mechanical effect of the anisotropic quantum vacuum eigenmodes on the rotational or linear motion of the material. Because in the case considered here, the dynamical contributions of all the eigenmodes (including the vacuum eigenmodes) do not cancel (because $a_3 \neq 0$), the quantum vacuum in an anisotropic electromagnetic environment may possess a nonzero linear momentum density. It is thus possible for momentum transfer between a quantum vacuum and an anisotropic electromagnetic material to take place. In the section that follows, we consider the nonzero momentum of the vacuum eigenmodes (zero-point fluctuation fields) in a Faraday chiral material.

§3. Nonzero linear momentum of anisotropic quantum vacuum

In the literature, there was an Abraham-Minkowski controversy concerning the definition of the electromagnetic momentum inside materials.[27)–31)] Here, we tentatively adopt Nelson's viewpoint,[30)] according to which the electromagnetic momentum density of the ω-mode is $\boldsymbol{p} = \epsilon_0 \boldsymbol{E} \times \boldsymbol{B}$,[30)] where $\boldsymbol{B} = \boldsymbol{k} \times \boldsymbol{E}/\omega$. Then the linear momentum density $\boldsymbol{p}$ can be rewritten as

$$\boldsymbol{p} = \frac{\epsilon_0}{\omega}\left[\boldsymbol{k}E^2 - \boldsymbol{E}\left(\boldsymbol{k}\cdot\boldsymbol{E}\right)\right]. \tag{3·1}$$

It is readily verified that the total electromagnetic momentum density at the quantum vacuum level in an isotropic material vanishes, because the contribution of the forward and backward waves (including the left- and right-handed polarized modes) exactly cancel. In other words, an isotropic vacuum has exactly a zero momentum because the vacuum possesses a universal symmetry. But this may not be true for the vacuum in an anisotropic electromagnetic environment created inside an anisotropic medium, such as Faraday chiral material. Here, the noncompensation effect of a pair of counter-propagating vacuum modes will arise (i.e., the momenta of oppositely propagating vacuum modes will not cancel)[14)] due to the broken universal symmetry of the quantum vacuum.

In order to demonstrate that there is truly a quantum vacuum contribution to the momentum of an anisotropic Faraday chiral material, we first calculate the nonzero linear momentum of the quantum vacuum in the material. For simplicity, we assume that the dimensionless gyrotropic parameters $\epsilon_{\rm g}$ and $\mu_{\rm g}$ and the chirality admittances (chirality dyadic elements) ξ, ξ_z, and $\xi_{\rm g}$ are small compared with the relative permittivity and permeability. Indeed, such a condition is satisfied for the practical Faraday chiral materials. This leads to the relation $|\boldsymbol{E}\left(\boldsymbol{k}\cdot\boldsymbol{E}\right)| \ll |\boldsymbol{k}E^2|$. Therefore, the expression for the electromagnetic momentum density in the material reduces to the form $\boldsymbol{p} \simeq (\epsilon_0/\omega)\boldsymbol{k}E^2$. Thus, the total momentum of the quantum

vacuum corresponding to all ω-frequency modes (including the counter-propagating modes and their respective mutually perpendicular polarization components) in the Faraday chiral material is

$$\boldsymbol{P}_\omega = \int_V \frac{\epsilon_0 E_0^2}{\omega} \left(\sum_{\sigma=1}^{4} \boldsymbol{k}_\sigma \right) dV, \tag{3·2}$$

where E_0 denotes the field strength of quantum vacuum fluctuations and V is the medium volume. In order to obtain an explicit expression for the quantum vacuum linear momentum, we evaluate the quantized electromagnetic field energy in the material.[32] If the dispersion of the optical "constants" of the material is not significant, the relation $\epsilon\epsilon_0 E^2 V \simeq (n_\mathrm{p} + 1/2)\hbar\omega$ may hold for a quantized electromagnetic field. Here, $\hbar$ denotes the Planck constant, and n_p is the total number of ω-mode photons in the material. Thus, the quantum vacuum fluctuation energy (with photon number $n_\mathrm{p} = 0$) in the material is $\epsilon\epsilon_0 E_0^2 V \simeq \hbar\omega/2$. We define $\boldsymbol{P}_\omega = \int_V \boldsymbol{p}_\omega^{(\mathrm{vac})} dV$ and obtain the following nonzero total momentum density $\boldsymbol{p}_\omega^{(\mathrm{vac})}$ of the ω-mode field at the quantum vacuum level:

$$\boldsymbol{p}_\omega^{(\mathrm{vac})} = \frac{\hbar}{2\epsilon V} \sum_{\sigma=1}^{4} \boldsymbol{k}_\sigma. \tag{3·3}$$

The total wave vector in expression (3·3) is the sum of the wave vectors over the four eigenmodes (the counter-propagating modes and their respective polarizations) corresponding to the mode frequency ω. Apparently, the quantized electromagnetic momentum density (3·3) is not that of the vacuum eigenmodes of all frequencies, but only that of the ω-mode field. In the next section, we take into account the quantum vacuum contribution of all the vacuum eigenmodes by means of a phase-space integral, we then show how there arises a nontrivial macroscopically observable mechanical effect of the quantum vacuum field on the anisotropic material.

§4. Momentum transfer at quantum vacuum level

The total momentum density of an isolated electromagnetic system consisting of both a quantum vacuum and an anisotropic Faraday chiral material can be zero, i.e., $\boldsymbol{p}_\omega^{(\mathrm{vac})} + \rho \boldsymbol{v}_\omega = 0$, where ρ is the mass density of the medium and $\boldsymbol{v}_\omega$ is the velocity of the medium resulting from the ω-mode vacuum momentum transfer. Thus, in this case, the Faraday chiral material acquires a momentum density with the same magnitude as $\boldsymbol{p}_\omega^{(\mathrm{vac})}$ but with an opposite sign. It follows that the velocity acquired by the material is

$$\boldsymbol{v}_\omega = -\frac{\hbar}{2\epsilon\rho V} \sum_{\sigma=1}^{4} \boldsymbol{k}_\sigma. \tag{4·1}$$

Thus, we have obtained the ω-mode quantum vacuum contribution to the Faraday chiral material. All the quantum vacuum modes will make contributions to the net

velocity of the material. This is calculated using a phase-space integral in what follows.

It should be noted that in Faraday (gyrotropic) chiral material, ϵ_z and μ_z are different from ϵ and μ, respectively. However, for convenience, we can assume $\epsilon_z \simeq \epsilon$ and $\mu_z \simeq \mu$. This simplifies the following calculation procedure (but does not affect the order of magnitude of the numerical result for the net velocity acquired by the material). Thus, for the coefficient a_4, we take $a_4 \to \mu\epsilon$ [the term $-(\xi \sin^2\theta + \xi_z \cos^2\theta)^2$ is negligibly small compared with $\mu\epsilon$]. The Faraday chiral material under consideration has the general constitutive relation (2·1). However, in most Faraday chiral materials considered in the literature, the parameter $\xi_{\rm g}$ is usually taken to be zero.[20),23)–25)] Thus, the quantum vacuum contribution due to the gyrotropy in chirality admittance $\underline{\underline{\xi}}$ vanishes. It then follows from Eq. (2·3) that the sum of the wave vectors over the four eigenmodes (corresponding to the mode frequency ω) is

$$\sum_{\sigma=1}^{4} \boldsymbol{k}_\sigma = -\frac{a_3}{a_4}\frac{\omega}{c}\boldsymbol{e}_{\boldsymbol{k}}$$
$$\simeq -2(\mu\epsilon)^{-1}[(\mu_{\rm g}\epsilon + \epsilon_{\rm g}\mu)(\xi_z - \xi)]\frac{\omega}{c}\sin^2\theta\cos\theta\boldsymbol{e}_{\boldsymbol{k}}, \tag{4·2}$$

where $\boldsymbol{e}_{\boldsymbol{k}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ is a unit vector. As the result in (4·1) and (4·2) involves only the quantum vacuum contribution of one frequency mode (i.e., the ω-mode), in what follows, we calculate the total contribution of all the vacuum modes with wave numbers satisfying $k \le k_{\rm cut}$, where $k_{\rm cut}$ represents the cutoff wave number in the material. The total net velocity acquired by the material because of the quantum vacuum momentum transfer is defined by $\boldsymbol{v} = (1/2)\sum_{\boldsymbol{k}} \boldsymbol{v}_\omega$. Here, the reason that the factor 1/2 emerges is that we have taken account of both forward and backward vacuum eigenmodes when considering the quantum vacuum contribution to the medium velocity $\boldsymbol{v}_\omega$ [see the expression (4·1)]. Then, with the help of expressions (4·1) and (4·2), we obtain an expression

$$\boldsymbol{v} = -\frac{\hbar}{4\epsilon\rho V}\sum_{\boldsymbol{k}} 2\gamma k \sin^2\theta\cos\theta\boldsymbol{e}_{\boldsymbol{k}} \tag{4·3}$$

for the acquired net velocity, where the parameter γ is defined by

$$\gamma = -(\mu\epsilon)^{-\frac{3}{2}}[(\mu_{\rm g}\epsilon + \epsilon_{\rm g}\mu)(\xi_z - \xi)]. \tag{4·4}$$

Here, the relation $\omega \simeq kc/\sqrt{\mu\epsilon}$ in the phase space has been inserted. Under the continuity approximation condition, the summation over the wave vectors in (4·3) can be replaced with a phase-space integral, i.e., $\sum_{\boldsymbol{k}} \to (2\pi)^{-3}V\int d^3\boldsymbol{k}$. Here, the phase-space volume element is $d^3\boldsymbol{k} \simeq k^2 dk d\varOmega$, where the solid angle element is $d\varOmega = \sin\theta d\theta d\phi$. Therefore, the explicit expression for the velocity acquired by the material is

$$\boldsymbol{v} = -\frac{\hbar\gamma k_{\rm cut}^4}{64\pi^3\epsilon\rho}\int_{\varOmega}\sin^2\theta\cos\theta\boldsymbol{e}_{\boldsymbol{k}} d\varOmega. \tag{4·5}$$

Substituting the expression for the unit vector $\boldsymbol{e}_{\boldsymbol{k}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ into (4·5), we can now show that the $\hat{x}$ and $\hat{y}$ component velocity satisfy $v_x = v_y = 0$

and that the $\hat{z}$ component (i.e., the component along the distinguished axis of the Faraday chiral material) is given by

$$v_z = -\frac{\hbar\gamma k_{\rm cut}^4}{120\pi^2\epsilon\rho}. \tag{4·6}$$

It can be easily seen that the acquired velocity depends on both the material parameters and the cutoff wave number. From expression (4·4) for the parameter γ, it is seen that physical origin of the momentum transfer at the quantum vacuum level is the nature of the gyrotropy (i.e., that $\epsilon_{\rm g}$ and $\mu_{\rm g}$ are nonzero) and uniaxial chirality admittance (i.e., that nonzero $\xi_z - \xi$ is nonzero) of the material. In other words, an anisotropic quantum vacuum with a nonzero linear momentum would appear inside a Faraday chiral material with such anisotropic characteristics.

The linear momentum transfer caused by the universal symmetry breaking of the vacuum can be considered a new macroscopic quantum vacuum effect that may be valuable in the development of new techniques for device design in photonics, quantum electronics and other areas (e.g., high-sensitivity sensor for navigation and seismology[14)]). Recently, Jonsson et al. suggested a theory of parametric generation and amplification in artificially gyrotropic media.[36)] Within that scheme, the authors derived the nonzero elements of the second-order optical and third-order magneto-optical susceptibility tensors governing optical and magneto-optical parametric generation.[36)] If such parametric generation and amplification are indeed promising, the macroscopic mechanical contribution of the quantum vacuum to Faraday chiral material could be dramatically amplified.

We have thus suggested the possibility of quantum vacuum *linear* momentum transfer between the vacuum field and an anisotropic material. This leads to the following question: Can a quantum vacuum *angular* momentum transfer take place between the quantum vacuum field and an anisotropic material? We find that only when the macroscopic body (material) has a special geometric shape can such angular momentum transfer take place. For example, let us consider a ring composed of a Faraday chiral material whose symmetric axis is along the circumference of the ring, i.e., the symmetric axis (the distinguished axis of the constitutive matrices of the Faraday chiral material) edges the polar unit vector $\boldsymbol{e}_\theta$ [The two orthogonal unit vectors in the toroidal coordinate on the ring are $\boldsymbol{e}_r$ (the radial unit vector) and $\boldsymbol{e}_\theta$ (the polar unit vector).]. As a velocity is induced (because of vacuum momentum transfer) along the symmetric axis of the material, such ring-type macroscopic body would rotate at a certain angular frequency.

Basically, the presently considered quantum-vacuum momentum transfer is a QED effect. We can also consider this effect in nuclear physics and QCD, where the contribution to the vacuum momentum transfer would increase if an anisotropic nuclear or QCD medium could be fabricated. It follows from (4·6) that the magnitude of the acquired velocity is proportional to $k_{\rm cut}^4/\rho$. As is well known, the cutoff wave number $k_{\rm cut}$ is approximately equal to $1/a$ (with a being the length scale of the microscopic structure units of the medium). Thus, the relation $k_{\rm cut}^4/\rho \simeq 1/(a^4\rho)$ can be obtained. Because $a^3\rho$ can be regarded as the mass of the microscopic structure unit (e.g., nucleon), the acquired velocity is inversely proportional to the structure

unit scale a. Typical values for the dimensionless parameters in the constitutive relation (2·1) are as follows: $\epsilon = 2$, $\mu = 2$, $\epsilon_{\mathrm{g}} = 0.1$, $\mu_{\mathrm{g}} = 0.1$ and $\xi_z - \xi = 0.1$. If the structure unit scale is $a = 10^{-15}$ m, then the acquired velocity in (4·6) is about $0.1 - 0.01$ m/s. We hope that such anisotropic (gyrotropic and chiral) nuclear matter or QCD matter (quark-gluon plasma) could be produced in experiments in the future. The momentum transfer between the quantum vacuum and the anisotropic nuclear or QCD matter (should such exist) might be applicable to some areas of technology (e.g., in possible power for interplanetary flight).

It should be noted that for a Faraday medium, an static external magnetic field must be applied. At this stage, it is necessary to clarify the role of such static external fields. The nonzero momentum of the vacuum presented here results from the universal symmetry breaking of the electromagnetic medium, which is caused by the static external fields; that is, the applied external fields can give rise to anisotropy of the medium and thereby lead to an anisotropic mode distribution structure of the quantum vacuum field.

As we know, a (strong) electromagnetic interaction can be used to manipulate (control) a (weak) electromagnetic or optical responses: specifically, the presence of the static fields can result in electro-optic and magneto-optic effects, or lead to spatial symmetry breakings (i.e. inhomogeneity, anisotropy and certain boundary conditions). For example, periodicity in the dielectric constant can create a range of forbidden frequencies (a photonic bandgap) and lead to atomic spontaneous emission inhibition,[41] and proper boundary conditions (e.g. two parallel mirrors, metallic plates, and waveguides) can dramatically modify vacuum mode structures.[4),42] For an isotropic medium, optical and electromagnetic properties can be said to be symmetric among the four eigenmodes (the counter-propagating modes as well as their respective mutually perpendicular polarization components). However, asymmetry among these four eigenmodes can be caused by applied external static fields via electro-optic and magneto-optic effects (e.g., in magnetized plasma/ferrite and magnetoelectric material, the gyrotropy characterization emerges in their constitutive relations).[14] In general, the gyrotropy parameters depend on the applied static external field strengths, and furthermore, the external fields provide a preferred direction for the vacuum mode structure inside the anisotropic electromagnetic environment. Optical and electromagnetic effects and phenomena in anisotropic materials, which are produced by the applied static fields, have been extensively considered in the literature, but less attention has been paid to the vacuum zero-point fluctuation field in these materials.[14),20),23)–25] In this paper, we discussed the effects associated with the anisotropic vacuum mode structure that is created in an anisotropic material under the influence of the applied static fields. As the static fields exhibit a zero Poynting vector, and therefore make no contribution to the medium momentum, the momentum acquired by the medium is the contribution of the anisotropic quantum vacuum field.

Though the static external fields make no contribution to the medium momentum, they create an anisotropic electromagnetic environment for the vacuum modes, and this leads to noncompensation effect among the backward and forward modes of the vacuum (including the left- and right-handed circularly polarized components).[14]

It is this non-compensation effect that gives rise to a nonzero quantum-vacuum electromagnetic momentum.

§5. Concluding remarks

As the quantum vacuum in an anisotropic electromagnetic environment has a nonzero momentum, the linear momentum transfer between the quantum vacuum and a gyrotropic chiral material can take place. In most conventional electromagnetic media, the quantum vacuum inside possesses a universal symmetry and hence has no influence on the motion of the media. However, for a Faraday chiral material, the macroscopically observable mechanical effect, due to the breaking of the universal symmetry of the quantum vacuum may appear. It is also conjectured that quantum vacuum may exhibit other interesting effects and phenomena because of anisotropy of the electromagnetic environment. Here, the mechanical effects related to dynamical quantities (e.g., energy, momentum, spin, and polarization[13)]) of the anisotropic quantum fluctuation fields may not be exactly cancelled among the counter-propagating vacuum modes (including the mutually perpendicular polarized fields). The transfer effect of momentum from the quantum vacuum fluctuation fields in other artificial composite materials, such as anisotropic photonic crystals and uniaxial bicrystals[37),38)] may also take place, and this warrants investigation. We believe that the scheme presented here may have significance in both pure and applied physics. For example, this vacuum effect may provide us with new insight into electromagnetic structures of quantum vacuum fluctuation inside artificial anisotropic materials, and we may be able to utilize this mechanical effect to develop sensitive, accurate measurement technologies. In addition, such quantum vacuum effects may lead to new topics regarding fundamental physical problems, such as field quantization, inertia of photon's spin (in spin-rotation coupling[39),40)]) and some relevant quantum optical effects inside composite materials. For these reasons, we hope that the quantum vacuum momentum transfer investigated in this paper can be tested experimentally in the near future.

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