A mathematical model of line bisection behaviour in neglect

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Summary
Subjects with left hemispatial neglect frequently demonstrate an array of abnormal behaviours on line bisection tasks. They misbisect long horizontal lines to the right of true midline. They bisect short lines to the left of true midline. They exaggerate the left-sided length of lines when placing the endpoints for ‘invisible’ lines, and they underestimate the length of the left side of long lines that are shown to them bisected accurately. No current theory of neglect explains all these features of line bisection behaviour. A mathematical model of line bisection behaviour in neglect is presented that proposes that subjects bisect lines at the point where they perceive the ‘salience’ of the two line segments created by their bisection mark to be equal. Salience is determined by the brain’s attentional systems which map salience amplitude to spatial position following a bell shaped distribution. Right hemisphere strokes simulated by decreasing the ‘height’ and ‘breadth’ of the right hemisphere salience to position function produced all of the above features of clinical neglect subjects’ line bisection behaviour. Neglect may be conceived of as damage to brain systems performing mappings between stimulus characteristics (such as spatial location) and salience.

Keywords: neglect; cognition; stroke; right hemisphere syndrome

Introduction
In this paper, we present a theory of line bisection behaviour in neglect. After reviewing some of the reliable but confusing behaviour of neglect subjects on line bisection tasks, the implications of these behaviours for current theories of neglect will be discussed. From this background, the empirical results to be accounted for by the current model will be established and the model itself presented. The results of simulations employing the model will be shown to account for the specified empirical data and additional results, not yet empirically evaluated, will also be set forth. Finally, the implications of the mathematical model for a theory of neglect will be established.

Hemispatial neglect is a clinical syndrome of decreased responsiveness to stimuli in the hemispace contralateral to a brain lesion, a lesion classically of the right cerebral hemisphere (Heilman et al., 1993). The term neglect encompasses behaviours ranging from the extinction of one of a pair of simultaneous stimuli to a complete denial of a body part or illness. It is not surprising that a term used so widely lacks nosologic precision and is of debatable heuristic value (Halligan and Marshall, 1992). However, even in more defined subdomains of neglect, such as line bisection behaviour, there is theoretical disarray. Researchers have repeatedly demonstrated that neglect subjects behave on line bisection tasks in consistent but bizarre ways.

The prototypical line bisection task employs a black horizontal line of moderate length centred on a sheet of white paper. The line is centred at the subject’s midsaggital plane. With their right hand, and no constraint on head or eye movements, the subject is asked to place a mark that will cut the line into two equal halves. Subjects who show hemispatial neglect on this task place their marks to the right of the true midline.

The earliest explanation for this type of behaviour was that subjects were hemianopic and did not see some of the left portion of the line. On its surface this argument is implausible since people frequently attend to space that they do not initially see (e.g. the area behind one’s back). Experimentally it has been disproved: hemianopic patients without neglect will search in their blind fields (Ishii et al., 1989; Hornak, 1992) and rare subjects are found who have neglect but no hemianopia (Bisiach et al., 1986).

Heilman and colleagues developed a theory that the behaviour of neglect subjects on line bisection tasks could
be explained by a directional hypokinesia (Watson et al., 1978) in which failure to place the bisection mark as far to the left as necessary was secondary to a 'motor' disorder and not to a 'sensory' problem. That non-sensory neglect occurs has been established, but it also appears that directional hypokinesia is not the most common explanation for line bisection errors in neglect (Tegnér and Levander, 1991). This has been further demonstrated by varying the line bisection task so that the direction of movement of the hand when making the mark is dislocated from the location of the bisection mark; i.e. moving the hand to the left is required to make a bisection error to the right of midline (Bisiach et al., 1990).

Both before and since the motor theory of neglect, many theorists have implicated spatial attention in the neglect syndrome. Frequently, either an inattention to left-sided material or a difficulty disengaging attention from right-sided material is invoked (for review, see Heilman et al., 1993). Problematic for this variety of neglect theory is the finding of leftward bisection errors with short lines.

Halligan and Marshall studied line bisection performance in neglect subjects using line lengths much shorter than had been previously used (Halligan and Marshall, 1988; Marshall and Halligan, 1990). They found that as line length shortens some neglect subjects show a diminution in the magnitude of their rightward error. As the line shrinks further, their bisectons may actually become accurate. Eventually, when the lines become very short, these same subjects may crossover and make leftward bisection errors. In the words of Bisiach, ‘[this finding has been] . . . a repressed pain in the neck of neglect theorists’ (Bisiach et al., 1994; see Fig. 1).

These leftward errors are obviously inconsistent with a motor neglect theory since the errors result from moving too far into contralateral space. This finding also appears inconsistent with all the theories of neglect that suggest an ‘attentional deficit’ for the left side of space. How does one overestimate the extent of an underattended domain? How can one reconcile a switching of neglect from the right to the left as a function of line length on the basis of inattention to left-sided stimuli or difficulty disengaging from right-sided stimuli?

Marshall and Halligan (1990) suggested that line bisection errors were a consequence of an expanded Weber fraction, a right to left search pattern, and a switching of the bisection mark from the beginning of the Weber fraction to the end of the Weber fraction when the stimulus was small enough to fall entirely within foveal vision. The Weber fraction is the largest size of the dimension in which a subject cannot tell that two stimuli differ. Comparing two lines for equal lengths, there is some small amount of length that must be added to one of the lines for a subject to detect a length difference. This 'just noticeable' difference is the Weber fraction.

Marshall and Halligan’s explanation suggests that subjects with neglect scan from the right end of lines to the bisection point. There is empirical evidence that neglect subjects do favour a right to left search strategy (Ishiai et al., 1989).

Once neglect subjects reach the point where they can no longer tell the difference in the lengths of the two sides of the line, they place their bisection mark. If the Weber fraction grows as a result of a brain lesion then neglect subjects will place their marks earlier than appropriate. When the line is small, Halligan and Marshall posit that inertia or a change in strategy will keep subjects going until the end of the Weber fraction. Thus, this theory proposes a discontinuity in response pattern depending on line length. However, behaviour of neglect subjects on line bisection tasks appears continuous (Marshall and Halligan, 1990; Burnett-Stuart et al., 1991; Chatterjee et al., 1994a, b).

Posing an additional challenge for neglect theorists are the recent theoretically stymieing, 'self-eloquent' results of Bisiach et al. (1994). In a brief note, they reported the behaviour of two neglect subjects, on whom they had employed the following variation of the line bisection task. Subjects were presented with lines and asked to bisect them. They made rightward errors. Bisiach then placed two points on the right as necessary was secondary to a 'motor' disorder and not to a 'sensory' problem. That non-sensory neglect occurs has been established, but it also appears that directional hypokinesia is not the most common explanation for line bisection errors in neglect (Tegnér and Levander, 1991). This has been further demonstrated by varying the line bisection task so that the direction of movement of the hand when making the mark is dislocated from the location of the bisection mark; i.e. moving the hand to the left is required to make a bisection error to the right of midline (Bisiach et al., 1990).

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endpoints of a line of the same length as they had been working with.

The subjects generally marked the right endpoint first, often slightly undershooting the correct location. Then, the subjects would place the left endpoint of the line beyond its correct position; that is more to the left than was correct. This presented a parody of line bisection performance. The subjects’ finished lines looked like their original performance on the traditional task; the bisection mark was right of midline defined by the subjects’ endpoints. This behaviour is inconsistent with a directional hypokinesia, errors were too far to the left. Therefore, the performance of subjects on the Bisiach task is, on the surface, inconsistent with attentional neglect theories that propose an underattention to the left side of space, an over-attraction to right-sided stimuli, or a ‘shrinkage’ of left hemispace.

This is the experimental morass stumbled into by neglect theory. No extant theory explains all the data. This failure is not just in the particulars of neglect theories, but also applies to their broad themes, e.g. disengagement defects, intentional deficits, or attentional deficiencies. It is also not apparent where to look for an improved theory.

The puzzling behaviours of line bisection performance in neglect that need to be explained are (i) subjects bisect ‘normal’ length lines to the right of their true midline, (ii) as line length shrinks, the bisection error shrinks until, passing through accuracy, it becomes a leftward error, and (iii) when placing the endpoints of fictional lines, for which they have been shown the midpoint, some neglect subjects place the left endpoint of the line too far to the left. In addition, the persuasiveness of any model or theory would be enhanced if it were consistent with prior mathematical characterizations of line bisection performance, namely that line bisection performance can be fitted to linear or power functions (Marshall and Halligan, 1990; Burnett-Stuart et al., 1991; Chatterjee et al., 1994a, b).

In this paper, we presents a theory of line bisection errors in neglect that is based on a mathematical model. The model is founded on commonly held concepts of how attentional systems are organized in the brain. This mathematical neglect theory reconciles the above findings, indeed requires their presence, and makes additional specific predictions that are submitted for empirical evaluation. The organization for the paper is first to present the model’s development, providing the rationale behind its form and organization. Once the formal model has been explained, the results of a simulated right hemisphere stroke on the model’s behaviour are presented.

The model
Definitions and premises
This model specifies that an individual places his bisection mark of a line where the ‘salience’ of the left and right halves of the line become equal. The salience of a point is a function of its spatial location along the linear dimension of left to right and is something akin to the weight or attraction a point has as a result of a subject’s attention. In the following paragraphs the implicit premises of the model are reported, the term salience is defined, the mathematical equations specifying the function that maps salience to spatial position is presented, and the mathematical method for determining the placement of bisection marks, as predicted by the model, is illustrated.

In constructing this model of a highly complex system, certain simplifying suppositions and premises were adopted and are implicit in the model. First, it is presumed that, in people with neglect, we are seeing the function of normal, undamaged, neural systems operating in combination with the disordered functioning of damaged systems. Secondly, neglect patients are truthful. When you ask them to cut a line down the middle they do it; placing their bisection mark where they perceive the middle of the line to be. They do not report a disordered world where one half is missing, shrunken or distorted and therefore it is assumed that the world they perceive seems to them quite normal. Thirdly, the excess of neglect occurring after right hemisphere lesions, in comparison to left hemisphere lesions, indicates separate attentional systems for the two hemispheres with different spatial fields (Heilman and Van Den Abell, 1980).

Salience denotes the degree to which something stands out or is conspicuous, prominent, and noticeable (Guralnik, 1972). Unlike attention, salience does not have a long history of specific use in behavioural neurology and neuropsychiatry so it is not hobbled by excessive connotative baggage. Salience can be operationally defined by reference to the proposed model. The word salience does denote the relevant concept of how much a percept stands out from the background. Salience is a modifier and implies that not all sensory percepts stand out equally.

The present model postulates that subjects with neglect see the entire line they are asked to bisect. There is evidence that neglect subjects are covertly aware of neglected stimuli. The change in line bisection and cancellation performances as line lengths and array sizes change, requires a knowledge of the neglected stimuli. Subjects with neglect have been shown to covertly respond to neglected stimuli presented visually and in a tactile manner (Marshall and Halligan, 1988; Vallar et al., 1991; Bisiach and Berti, 1995).

The model uses an object centred reference frame. In its present form it takes no account of eye, head or body movements. It is assumed that if an object is the focus of attention its location in this reference frame is central regardless of where the object falls on the retina. Similarly, which visual field the image falls in and consequently which cerebral hemisphere processes the initial visual percept is not relevant to the calculations required by the model. The reference frame employed by the model is analogous to the ‘word-centred grapheme description’ reference frame employed by Caramazza and Hillis (1990) in their model of neglect dyslexia.
The model presumes that when a subject bisects a line, the subject’s estimation of the midpoint, or the lengths of the two segments of the lines they are creating, is not a function simply of the objective length of the line but rather is determined by the salience of the two segments of the line. For normal subjects length and salience coincide; placement of a bisection point on a line cuts the line into halves of equal length and salience. In the presence of neglect there is a disjunction of salience and length.

**Model description**

The functions mapping salience to spatial position and determining bisection performance can be best visualized graphically (see Fig. 2). The magnitude of a point’s salience is depicted on the y-axis and the linear spatial dimension from far left (0) to far right (1000) on the x-axis. The dimensions for both space and salience are arbitrary. Because the integral of the curves will be seen to yield the arctan function simply of the objective length of the line but rather is determined by the salience of the two segments of the line. For normal subjects length and salience coincide; placement of a bissection point on a line cuts the line into halves of equal length and salience. In the presence of neglect there is a disjunction of salience and length.

The functions employed in the model are of the general form:

\[
SF = \frac{1}{1 + (x - M)^2 / SD^2}
\]

where \(SF\) refers to the standard deviation, \(x\) refers to the spatial placement of a point along the left-right continuum from 0 to 1000, and \(M\) is the position along the left-right continuum where the peak of the salience to position curve is maximal and is analogous to a ‘mean’; \(SD\) refers to the breadth, or ‘standard deviation’ of the salience to position curves, and \(SF\) is a scaling factor that affects the overall height of the salience to position curves.

This formula leads to a bell shaped curve. In this equation the only variable is \(x\) with all other abbreviations denoting constants: \(x\) refers to the spatial placement of a point along a left-right continuum from 0 to 1000; \(M\) is the position along the left-right continuum where the peak of the salience to position curve is maximal and is analogous to a ‘mean’; \(SD\) refers to the breadth, or ‘standard deviation’ of the salience to position curves, and \(SF\) is a scaling factor that affects the overall height of the salience to position curves.

The functions mapping salience to linear location for both postulated hemisphere attentional systems are of the same general form but differ in their constants. For the left hemisphere attentional system, all the constants bear the \(L\) subscript, for the right hemisphere attentional system all the constants bear the \(R\) subscript. Figure 2 shows the two curves, one for each hemisphere, plotted.

As stated above, this model relies upon the Heilman and Van Den Abell (1980) hypothesis that the right hemisphere attentional system attends to space bilaterally, whereas the left hemisphere system attends primarily to contralateral hemispace. Therefore \(M_L\) was set equal to 750, giving it its peak salience for items in the middle of the right hemispace and \(M_R\) was set equal to 470–480, depending on the simulation, placing its centre near the centre of the spatial dimension with a slight leftward preponderance. The model is also consistent with suggestions reviewed by Halligan and Marshall (1994) that normally the left hemisphere is associated with focused attention and the right hemisphere global attention. This distinction is captured by the difference in their SD terms (\(SD_L = 100; SD_R = 250\)).

The total salience for any point along the left-right continuum is the sum of the left and right hemispheric systems. Computationally, the two curves can be summed.
Fig. 4 Lesions in the model are induced by decreasing the SD and SF terms of the right hemisphere function. In this figure the combined curve of the right and left hemispherical attentional systems is shown after a 'stroke' in the right hemisphere (SF$_R$ = 0.6, SD$_R$ = 75, M$_R$ = 480, SF$_L$ = 0.6, SD$_L$ = 100 and M$_L$ = 750). Note an increase in the 'dip' between the peaks of the two curves. It is the presence and location of this valley that produces the cross-over effect for the short lines. By mentally drawing lines over the tops of the curves one can form an impression of the approximate bisector location that would produce equal salience for the two line segments. In this example a 600 unit line is drawn centred within the reference frame. The model places its bisector point at the location where the two segments of the line created by the bisector mark have equal salience. The salience of a segment is the area under the curve bounded by the end of the line and the bisector mark. The area associated with each 'half' of this line is shown by the two different shades of grey. For this example the model placed its bisector mark at location 563.

and the salience computed from the value of $x$. Figure 3 shows the plot for this summed function.

Lines are composed of an infinite number of points. Therefore, it is not possible to compute the total salience of a line segment by simply computing a few points and adding them together. Determining the salience of a line segment involves finding the area under the curve formed by that segment. Computationally, the composite function of the combined hemispheral attentional systems is integrated over the range from the left endpoint to the right endpoint.

Total salience = \left(\frac{\text{left hemisphere function}}{\text{right hemisphere function}}\right)

When this integration is performed the result is

\[
\text{Total salience} = (\text{SF}_L \ast \text{arctan}(\frac{x_L-M_L}{\text{SD}_L}) + \text{SF}_R \ast \text{arctan}(\frac{x_L-M_R}{\text{SD}_R})) - (\text{SF}_L \ast \text{arctan}(\frac{x_R-M_L}{\text{SD}_L}) + \text{SF}_R \ast \text{arctan}(\frac{x_R-M_R}{\text{SD}_R})).
\]

Computation of a bisector point in the model is lengthy but straightforward. If we have defined all the constants, and we know the starting and ending points of a line segment, the point 'a' where the integral of the above over the interval of $x_L$ to 'a' equals the integral of the above over the interval of 'a' to $x_R$ defines the location of the model's bisector point. The value of 'a' is where the bisection point is placed. $x_L$ denotes the left endpoint of the line and $x_R$ denotes the right endpoint of the line. Figure 4 shows this process schematically.

Table 1 Bisection mark placement as a function of line length

<table>
<thead>
<tr>
<th>Absolute line length</th>
<th>Perceived* line length 1†</th>
<th>Perceived line length 2‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>100</td>
<td>108</td>
<td>104</td>
</tr>
<tr>
<td>200</td>
<td>212</td>
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<tr>
<td>250</td>
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<td>500</td>
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<td>462</td>
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<td>600</td>
<td>474</td>
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<td>800</td>
<td>596</td>
<td>682</td>
</tr>
<tr>
<td>900</td>
<td>676</td>
<td>772</td>
</tr>
<tr>
<td>950</td>
<td>720</td>
<td>818</td>
</tr>
</tbody>
</table>

*Perceived is used here as it is commonly employed in clinical studies where the line length is assumed to be twice the distance from the right edge of the line to the bisection mark. †The parameters for this simulation were SF$_R$ = 0.6, SD$_R$ = 75, M$_R$ = 480, SF$_L$ = 0.6, SD$_L$ = 100 and M$_L$ = 750. ‡The parameters for this simulation were SF$_R$ = 0.7, SD$_R$ = 125, M$_R$ = 470, SF$_L$ = 0.6, SD$_L$ = 100 and M$_L$ = 750.

For modelling the Bisiach paradigm, a line segment of a defined length was employed (e.g. 500). The value of the integral from 500 to its right endpoint (750 in this example) was computed (the subjects in Bisiach's report tended to place the right endpoints first and were modestly accurate). Next, the value of the left endpoint was solved by determining the point 'a' for which the integral from 'a' to 500 equalled the value of the integral from 500 to 750.

While the above may seem cumbersome and complicated when viewed in its full mathematical formulism, the concepts are few and the general operation of the model for a given set of constants can usually be inferred from simple inspection of the graphs of the functions without any computation (see Fig. 4).
To summarize, the model assumes that every point along the continuum of left to right has a salience that is a function of its spatial position. The right and left hemisphere attentional systems perform the task of assigning salience on the basis of spatial position and their behaviour can be described by a mathematical function mapping the magnitude of salience to spatial location. The left hemisphere curve focuses towards right hemispace. The right hemisphere curve focuses just left of midline. If one mentally draws lines, centred at the 500 mark, over the combined curve resulting from the summation of the two functions, one can get a sense of the salience of different portions of the line and where the model’s bisection marks will fall (see Fig. 4).

**Experimental simulations**

The constants provided for the model are user specified. The SFs and SDs were selected to present a relatively flat surface for the combined curve under ‘normal’ conditions. The curves are shown in Fig. 2 and their summation is shown in Fig. 3.

For the simulation of injury the values SF_R and SD_R were decreased. The combined curve resulting from one such injury is shown in Fig. 4. The mathematical computations were performed with the Mathematica software (Wolfram Research Inc., Champaign, Ill., USA).

**Results**

**Bisection task**

The bisection behaviour of the model for two sets of right hemisphere injury parameters was determined. For the first simulation the parameters for the model were SF_R = 0.6; SD_R = 75; M_R = 480; SF_L = 0.6; SD_L = 100 and M_L = 750. For the second simulation the parameters were SF_R = 0.7; SD_R = 125; M_R = 470; SF_L = 0.6; SD_L = 100 and M_L = 750. The results from these two runs are shown numerically in Table 1. In Fig. 5 the line bisection of the model is depicted graphically for the second simulation.

At ultra short line lengths, bisection is accurate. As the line lengthens the bisection error moves towards the left. As the line grows further the bisection point moves towards the right passing through the true midpoint. When these data are plotted according to the typical convention of clinical studies by comparing the absolute line length with twice the distance from the right endpoint to the bisection mark, then the data are well fit by both linear and power functions (all correlations > r = 0.99, all P values < 0.000).

**Bisiach’s endpoint task**

In the next simulation an emulation of Bisiach’s endpoint task (Bisiach et al., 1994) was conducted. A series of lines of varying lengths were presented and the location of the model’s placement of the left endpoint of the line was recorded. At long line lengths the model’s estimation of the location of the left endpoint was too far to the left (similar to Bisiach’s two subjects, but at shorter line lengths the model crossed over to place the subjective left endpoint too far to the right. The absolute measures of line length and endpoint placement are shown in Table 2.

**Milner’s landmark task**

In this task (Harvey et al., 1995), a subject is shown a line with the bisection mark placed correctly in the centre. The subject is then asked to point to the side of the line where the end seems closest to the central mark. Milner has reported that for neglect subjects, and using lines of moderate length (20 cm), subjects point to the left side of the line. This finding is obviously inconsistent with motor neglect.

For simulation of this task, lines of varying lengths had the salience of their left and right halves computed. Consonant with the theory introduced in this report it was assumed that the model would indicate the side of the line with the lowest salience as being the shorter segment.

The results show that the left side of lines are less salient at most lengths (consistent with Milner’s findings in human subjects). At shorter lengths the model predicts that the designation will become equal, and that at still shorter lengths human neglect subjects should cross-over and designate the right side as shorter. The numerical results are in Table 3.

As a follow-up, Milner’s cued version of the task (Harvey et al., 1995) was also simulated and submitted to the model in both ‘normal’ and ‘right hemisphere stroke’ conditions. For these simulations, it was assumed that the cue would shift the location of the line within the frame of reference such that the end of the line near the cue would be centred in the reference frame (see Fig. 6).

The results of this analysis are shown in Table 4. In the ‘normal’ condition the model indicates that the side opposite the cue is shorter, similar to Harvey et al. (1995). In the ‘stroke’ condition, the model selects the left side as the shorter side, also in replication of Harvey et al. (1995). (The suggestion to examine the model on this task was made by Dr Anjan Chatterjee.)

**Table 2 Model behavior on Bisiach’s endpoint task**

<table>
<thead>
<tr>
<th>Model’s left endpoint placement</th>
<th>True left endpoint placement</th>
<th>Midpoint location</th>
<th>True right endpoint location</th>
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</thead>
<tbody>
<tr>
<td>–24</td>
<td>250</td>
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<tr>
<td>476</td>
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</table>

*The parameters for this simulation were SF_R = 0.7, SD_R = 125, M_R = 470, SF_L = 0.6, SD_L = 100 and M_L = 750.*

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(Tables and diagrams are not included in the text format, but would typically be included in the full document. The text provided is a representation of the content as it appears in the document.)
Table 3 Model performance on Milner's landmark task*

<table>
<thead>
<tr>
<th>Left endpoint position</th>
<th>Right endpoint position</th>
<th>Left half salience</th>
<th>Right half salience</th>
<th>Model's choice for 'shorter' half</th>
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</thead>
<tbody>
<tr>
<td>100</td>
<td>900</td>
<td>87</td>
<td>181</td>
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<td>375</td>
<td>625</td>
<td>62</td>
<td>55</td>
<td>R</td>
</tr>
<tr>
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<td>R</td>
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<td>32</td>
<td>27</td>
<td>R</td>
</tr>
<tr>
<td>475</td>
<td>525</td>
<td>17</td>
<td>15</td>
<td>R</td>
</tr>
</tbody>
</table>

*The parameters for this simulation were $SF_R = 0.6$, $SD_R = 75$, $MA = 480$, $SF_L = 0.6$, $SD_L = 100$ and $ML = 750$.

Table 4 Model performance on landmark task-cued version*

<table>
<thead>
<tr>
<th>Condition</th>
<th>Cue</th>
<th>Left side salience</th>
<th>Right side salience</th>
<th>Left endpoint</th>
<th>Mid-point</th>
<th>Right endpoint</th>
<th>Model's choice</th>
</tr>
</thead>
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<tr>
<td>Stroke</td>
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<td>94</td>
<td>500</td>
<td>700</td>
<td>900</td>
<td>L</td>
</tr>
<tr>
<td>Stroke</td>
<td>L</td>
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<td>83</td>
<td>500</td>
<td>650</td>
<td>800</td>
<td>L</td>
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<tr>
<td>Stroke</td>
<td>R</td>
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<td>74</td>
<td>100</td>
<td>300</td>
<td>500</td>
<td>L</td>
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<tr>
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<td>R</td>
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<td>67</td>
<td>200</td>
<td>350</td>
<td>500</td>
<td>L</td>
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<tr>
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<td>204</td>
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<td>500</td>
<td>700</td>
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<tr>
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<td>148</td>
<td>200</td>
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<td>500</td>
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</table>

*The model parameters for the 'normal' condition were $SF_R = 1.0$, $SD_R = 250$, $MA = 470$, $SF_L = 0.6$, $SD_L = 100$ and $ML = 750$. The parameters for the right hemisphere stroke condition were $SF_R = 0.6$, $SD_R = 75$, $MA = 480$, $SF_L = 0.6$, $SD_L = 100$ and $ML = 750$.

Discussion

Neglect is a fascinating clinical syndrome that encompasses an array of bizarre and puzzling behaviour. No satisfactory theory exists to explain why people with hemispatial neglect behave as they do. It may be that a term like neglect, encompassing such a wide array of behaviour, will never be satisfactorily explained by a unitary theory. However, it seems much less ambitious to hope to satisfactorily explain line bisection errors in neglect patients with a single theory.

The line bisection task should be more theoretically tractable. The task itself is limited. It deals with a very defined behaviour that can be precisely and repeatedly measured. Further, the behaviour is generally consistent across neglect subjects, differing more in the quantitative measures than the qualitative patterns of response.

Despite this optimistic impression, line bisection behaviour in neglect has not been satisfactorily explained. Heilman et al. (1993) appear to have adequately captured one subtype of this syndrome with their demonstration of motor neglect but, unfortunately, this still leaves the behaviour of many neglect subjects unexplained.

Numerous theories, quite logical and reasonable at the time of their proposal, have become suspect as clever experimentalists have found subject performances inconsistent with these theories [the representational (Bisiach et al., 1979), motor (Heilman et al., 1993), attention deficit, the disengagement (Posner et al., 1984) and the spatial shrinkage (Milner, 1987)]. It is into this void that the current offering is cast.

While complicated when viewed in its full mathematical formalism, the model presented here is direct in its logic and theoretical tenets. The model proposes that each cerebral hemisphere possesses its own attentional system. The left hemisphere is focused on right hemispace and the right hemisphere is focused towards the centre of space. Each hemisphere's attentional system performs the task of establishing the salience of a point in space based on the point's left–right spatial position; the performance of each hemisphere attentional system on this task can be mathematically described. In this current version of the model the function is similar in shape and contour to the familiar 'normal' distribution that describes much biological data.

When asked to bisect a line the model bisects where the
salience of the right and left segments are equal. The salience of any object is the total salience of all the points that comprise the object.

After simulated right hemisphere injury the model produces line bisection behaviour reminiscent of human neglect subjects. At long line lengths the model bisects to the right, as do human subjects. As line length shrinks the model’s bisection marks moves to the left. At a moderate length the model bisects accurately and at short line lengths the error is actually to the left. These are precisely the result reported by Halligan and Marshall (1988) for lines, and Chatterjee (1995) for letter strings and words, and readily reproduced by anyone who sees neglect patients (see Fig. 1). The model also predicts that at very short line lengths the inaccuracy should shrink again and achieve accurate bisection. When Marshall and Halligan (1990) had subjects attempt to bisect dots, the ultimate short line, the subjects were more accurate in their bisections.

The model also reproduces the clinical observations of Bisiach et al. (1994) and Harvey et al. (1995). At line lengths that yield rightward bisection errors subjects will point to the left side of an accurately bisected line as shorter and will, when given a bisection mark, place the leftward end too far out.

The simulations of the model employed in this paper go beyond the clinical data and offer predictions that can be tested empirically. The model predicts that at varying line lengths a variation of the Marshall and Halligan effect will be seen. There will be line lengths for individual neglect subjects where performance on both the landmark and endpoint tasks will be accurate; at shorter lengths it is predicted that a cross-over in the direction of errors will occur.

The model also yields data that can be explained by linear and power functions as reported by Chatterjee et al. (1994a). This relationship was not built into the model but emerged from its operation. Similarly, the results on the Landmark task were not known to the investigator at the time of model design, but were subsequently suggested to the author as a useful data set to examine. It is important to emphasize that the current model was not simply built to somehow encompass all known results on line bisection tasks post hoc. In addition to explaining data sets which were not part of the design, the model also predicts specific patterns of behaviour that can be assessed empirically.

From this quantitative start there are additional conceptual understandings that can be gained. The model employs an object centred coordinate system. The line is always centred over the central point of the x-axis. It is known that cueing subjects to the left end of a line often improves their line bisection performance (Riddoch and Humphreys, 1983; Robertson and North, 1992; Robertson et al., 1992; Ishiai et al., 1994). If the presence of a cue resulted in a shifting of the line’s placement within the reference frame so that the end of the line near the cue were centred in the coordinate system, different response patterns would be predicted (see Fig. 6). For a left cue, the entire line would fall into the right hemispase and based on salience computation a significant improvement in bisection performance would be predicted. Rightward cueing would result in a worsening of neglect behaviour but might well eliminate the cross-over to a leftward error, since now the line would not fall over the ‘dip’ in the curve, which is what results in the left-sided error cross-over in the routine task. This conception of how cueing should be incorporated in the model, shifting location of the object rather than modifying the curves, proved successful in explaining the cued landmark task of Milner.

While satisfactorily explaining a wealth of confusing data and making specific, testable empirical predictions there are still significant questions concerning the accuracy of this model. Are ‘normal’ curves the best curves to describe the salience to position function of human subjects? The answer to this question is not established. The bell shaped curves employed were used because they are often successful in explaining biological data and because they are relatively simple and straight forward to use. It should be possible to try and curve fit human subject data to determine the optimal salience-position functions.

Another issue for the model concerns the right way to simulate damage. For the present simulations the M, SF and SD parameters were all changed. It may be that only one or two of these parameters needs to be changed to capture the behaviour on line bisection tasks of neglect subjects. This can be learned by further comparisons of clinical data with model simulations. A deeper question is: what does it mean to alter the SD or SF terms? How do the physical changes of brain injury relate to alterations of the mathematical constants? There are no answers at present.

The current model utilized only a single spatial dimension. Is a one-dimensional model useful? How can it explain performance on cancellation tasks or line bisection tasks conducted vertically, radially or diagonally? How do environment and viewer centred frames of reference fit this coordinate system? In response, it is offered that a one-dimensional model seems a useful start. Based on the results achieved with this one-dimensional prototype, further efforts can be made to expand the basic idea into additional spatial dimensions and deal more fully with the issue of reference frames. Further, other ‘dimensions’ beyond the spatial, such as colour and identity, clearly influence an object’s salience. A full theory would need to develop multi-dimensional coordinate systems and functions to incorporate all the features, spatial and otherwise, that influence an object’s salience.

Is this model truly a theory of neglect or is it just a description of neglect, and is it useful in either case? This model is not merely a description. Data were not simply fit to an equation, such as occurs in fitting bisection data to a power function (Chatterjee et al., 1994b) or a Newtonian model (Burnett-Stuart et al., 1991). This model began with basic fundamental conceptions of how attention is organized...
in the brain and postulated that bisection marks were placed as a result of the output of those systems. From these premises, results consistent with clinical data were produced. The ability to yield these complicated behavioural data with a single curve seems a useful simplification of the phenomena. Since the model is based on postulates of how spatial attention in the hemispheres is focused and the basis by which subjects make their bisection marks (salience versus objective length), it goes beyond a mathematical description to a theoretical explanation.

In summary, a mathematical model of line bisection errors in neglect is presented. Results of simulations computed by this model yield the cross-over effect of leftward bisection errors on short lines reported by Marshall and Halligan, the far left endpoint placement of Bisiach, and the landmark data of Milner. The model makes specific predictions which can be tested empirically. The model has conceptual value and provides a logical and useful base for considering a range of other observations on line bisection tasks. While modification is necessary to expand the model beyond its one spatial dimension, it provides a theoretical and descriptive basis for line bisection behaviour in neglect.

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References


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