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Discussion

Discussion: "Fin (On a Pipe) Effectiveness: One-Dimensional and Two-Dimensional" (Look, Jr., D. C., 1999, ASME J. of Heat Transfer, 121, No. 1, pp. 227–230)

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The author in his Introduction gives the reasons why one should consider the two-dimensional analysis and non-insulated tip. Briefly, he states the "*mathematics in the one-dimensional solution are not difficult, because of large heat transfer coefficients, the tip should no be considered insulated, and the two-dimensional solution is more difficult*." However, the following discussion will show that the above are rather superficial statements without any scientific basis. The dimensionless variables employed by the author are not appropriate, this leads to *n* confusing results.

Given the fin geometry r_1, r_2, and L, the parameters k, h, h_e, and the temperatures T_w, T_∞, these quantities should be combined to form the dimensionless parameters that characterize the heat transfer process. In the heat transfer literature these non-dimensional parameters are

$$\text{the ratio } \beta = \frac{r_2}{r_2} \tag{1a}$$

$$\text{the } \frac{\text{conductiom}}{\text{convection}} \text{ coefficient } u^2 = \frac{(r_2 - r_1)^2 h}{kL} = \frac{(\beta-1)^2 h r_1^2}{kL} = (\beta-1)^2 w^2 \tag{1b}$$

$$\text{the parameter} \left(\frac{h_e}{h}\right)\frac{h r_1}{k} = \left(\frac{h_e}{h}\right) B_r \tag{1c}$$

The parameters w^2 and B_r are usually called the surface *conduction/convection* coefficient and *surface Biot number*, respectively. B_r enters the solution through the boundary condition, author's equation, (2), which is written as

$$\xi = \beta, \quad \frac{d\theta}{d\xi} + (h_e/h) B_r = 0. \tag{2}$$

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Similarly, in the two-dimensional solution B_r is introduced via the author's Eqs. (7*a*) and (7*b*). Using the above non-dimensional parameters the fin dimensionless temperature is

$$\theta = \theta(u, \beta, \xi, (h_e/h) B_r) = \theta(w, \beta, \xi, (h_e/h) B_r) \quad \text{one-dimensional solution} \tag{2a}$$

$$\theta = \theta(u, \beta, \xi, \nu, (h_e/h) B_r, B_r) = \theta(w, \beta, \xi, \nu, (h_e/h) B_r, B_r) \quad \text{two-dimensional solution} \tag{2b}$$

The heat dissipated by the fin is

$$Q_{\text{fin}} = Q_{\text{fin}}(u, \beta, (h_e/h) B_r) = Q_{\text{fin}}(w, \beta, (h_e/h) B_r) \quad \text{one-dimensional} \tag{3a}$$

$$Q_{\text{fin}} = Q_{\text{fin}}(u, \beta, (h_e/h) B_r, B_r) = Q_{\text{fin}}(w, \beta, (h_e/h) B_r, B_r) \quad \text{two-dimensional.} \tag{3b}$$

From the above equations one can readily derive the following fin efficiency and effectiveness:

$$\text{efficiency} \quad \eta = \eta(u, \beta, (h_e/h) B_r) = \eta(w, \beta, (h_e/h) B_r) \tag{4}$$

$$\text{effectiveness} \quad R = R(u, \beta, (h_e/h) B_r, B_r) = R(w, \beta, (h_e/h) B_r, Br). \tag{5}$$

Considering the tip to be insulated the above equations become

$$\eta = \eta(u, \beta) \tag{4a}$$

$$R = R(u, \beta, B_r). \tag{5a}$$

One can choose any set of parameters involving u or $w = u/(\beta - 1)$, and B_r or $B_i^{1/2} = B_r/w = (\beta-1) B_r/u$. Gardner (author's reference) used Eq. (4*a*) to produce graphs of the fin's efficiency versus u for different values of β. Razelos and Imre (author's reference) and Netrakanti and Huang [1] employed w, and β to determine the optimum fin dimensions with variable thermal parameters.

The author's two-dimensional solution, of which that no reference is given, contains some confusing non-dimensional variables Γ, B_i, and the roots λ_n, that he states are determined from Eq. (6*f*), not (8*e*) that do not exist. He never noticed that the solution always contains the product $\lambda_n L$. The author should have consulted Carslaw and Jaeger [2]. The roots of equation (6*f*) are

$$\rho_n = \lambda_n L = \rho_n(B_i). \tag{6}$$

These roots and are tabulated in [2]. One key observation is the fact that the magnitude of the roots for $n > 1$, irrespectively of the B_i value are

$$(n-1)\pi \leqslant \rho_n \leqslant (2(n-1)+1)\pi/2. \tag{7}$$

Moreover, the author should have followed [2] and express the $\sin(\rho_n)$ as a function of $\tan(\rho_n) = B_i/\rho_n$ to obtain the solution given by Eq. (2). It has been shown by Razelos and Georgiou (author's reference) that due to the large values of ρ_n, the predominant

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term in the series for $n>1$ is the first one. Therefore, the two-dimensional solution consists of only one term and is not more difficult than the one-dimensional solution. In addition, in the above reference, graphs of the effectiveness are presented for different values of u and β versus $B_i^{1/2}$, which show that two-dimensional solution actually reduces to the one-dimensional solution.

In concluding, we should point out that the author's comments such as ''*large heat transfer coefficients, thin and thick fins*,'' should be disregarded because all these quantities are introduced through the proper non-dimensional parameters. Also, the author's last statement ''*the major difference is that the one-dimensional solution is less restrictive*'' whatever that means should be ignored.

Today fin designers are using more sophisticated programs to design optimum or nearly optimum fins, taking into account variable thermal parameters. Therefore this paper does not offer any help due to an inappropriate set of dimensionless variables and the misleading statement ''*Use Fin*'' that appears in the Figs. 2 and 3.

References

[1] Netrakanti, M. N., and Huang, C. L. C., 1985, ''Optimization of Annular Fins With Thermal Parameters by Invariant Imbedding'' ASME J. Heat Transfer, **107**, pp. 966–968.

[2] Carslaw, H. S., and Jaeger, J. C., 1959, *Conduction of Heat in Solids*, Oxford at the Clarendon Press, London, p. 222.

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