2(d) shows the two extremes of tip shapes that were checked. One is a standard drop-feed oiler and the other a glass burette. The effect of rate of drop size is shown in Fig. 4 where drops per minute are plotted against cu in/min.

### TABLE 4 DATA ON OIL-DROP SIZE IN TERMS OF RATE OF DROPPING AND VISCOSITY OILS USED, SAE 10 AND SAE 40 AT 80°F

<table>
<thead>
<tr>
<th>Drops/min</th>
<th>Cubic inches per minute</th>
<th>Average value, cu in/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>20</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td>25</td>
<td>0.048</td>
<td>0.052</td>
</tr>
<tr>
<td>30</td>
<td>0.058</td>
<td>0.065</td>
</tr>
<tr>
<td>35</td>
<td>0.062</td>
<td>0.070</td>
</tr>
<tr>
<td>40</td>
<td>0.078</td>
<td>0.085</td>
</tr>
<tr>
<td>45</td>
<td>0.080</td>
<td>0.092</td>
</tr>
<tr>
<td>50</td>
<td>0.089</td>
<td>0.095</td>
</tr>
<tr>
<td>55</td>
<td>0.096</td>
<td>0.100</td>
</tr>
<tr>
<td>60</td>
<td>0.107</td>
<td>0.115</td>
</tr>
<tr>
<td>65</td>
<td>0.110</td>
<td>0.117</td>
</tr>
<tr>
<td>70</td>
<td>0.121</td>
<td>0.124</td>
</tr>
<tr>
<td>75</td>
<td>0.129</td>
<td>0.130</td>
</tr>
<tr>
<td>80</td>
<td>0.138</td>
<td>0.138</td>
</tr>
<tr>
<td>85</td>
<td>0.145</td>
<td>0.145</td>
</tr>
<tr>
<td>90</td>
<td>0.153</td>
<td>0.153</td>
</tr>
<tr>
<td>95</td>
<td>0.161</td>
<td>0.161</td>
</tr>
<tr>
<td>100</td>
<td>0.176</td>
<td>0.174</td>
</tr>
<tr>
<td>105</td>
<td>0.182</td>
<td>0.180</td>
</tr>
</tbody>
</table>

### Sample Calculation

To determine the required minimum feed rate to maintain a fluid film in a 360-deg bearing, 2/₄ in. diam × 2/₄ in. long, diametral clearance 0.0045 in., speed 1230 rpm, load 40 psi based on projected area.

Using Equation [5]

\[ Q_M = K_{urml} \]

\[ u = \frac{1230 \times \pi \times 2.125}{60} = 137 \text{ ips} \]

\[ r = \frac{2 \times 0.0045}{2} = 0.00212 \text{ in/in.} \]

\[ m = \frac{2.125}{0.00212} = 999.1 \text{ in.} \]

\[ l = 2.125 \text{ in.} \]

Average value of \( K_M \) from Equation [6]

\[ K_M = 0.0043 + 0.0000185p \]

\[ = 0.0043 + 0.0000185 \times 40 \]

\[ = 0.005 \]

Then in Equation [5]

\[ Q_M = 0.005 \times 137 \times 1.062 \times 0.0092 \times 2.125 \]

\[ = 0.00927 \text{ cu in. per sec} \]

\[ Q_M = 0.156 \text{ cu in. per min} \]

Then from Fig. 4

\[ Q_M = 84 \text{ drops per min, minimum feed rate} \]

### Conclusion

The solution of the general Reynolds hydrodynamic equation by Kingsbury and Needs has been used to obtain the form of an equation for the analysis of side flow in a journal bearing. Coefficients for this general equation have been obtained, by experimental means, for 360-deg journal bearings at the extreme condition where the oil supply has been reduced to the minimum to maintain a complete fluid film in the bearing. Bearings 2/₄ in. diam with two \( t/d \) ratios and three clearances have been used in this determination. A simple relationship is obtained which should enable the prediction of the minimum oil-feed rate to just maintain fluid-film conditions in these and similar bearings. It is expected that since the equation for minimum-flow requirement is in general form, the experimental coefficients may apply to a wide range of bearing sizes, although this has not yet been verified. Some qualitative corroboration, however, has been obtained with journal bearings of other dimensions.

### Discussion

J. A. Cole and C. J. Hughes. The estimation of the oil flow through a journal bearing is an important and difficult problem, and the authors are to be congratulated on their useful and pioneering investigation of this aspect of it.

Do the authors consider it likely that the successive determinations of limiting film conditions may have damaged the bearing and shaft surface finish and geometry, thus affecting the results?

If critical \( ZN/P \)-values are calculated from the data given, very high values are obtained for the lighter loads. This presumably indicates that the measured temperatures were not representative of the limiting film conditions.

Experiments on starved-film lubrication have been performed at the Mechanical Engineering Research Laboratory as part of an investigation using a new transparent-bearing technique, and some of the results may be relevant.

The authors mention that their limiting film conditions gave

\[ 1 \text{ Mechanical Engineering Research Laboratory, Lubrication Division, Thorntonhall, Glasgow, Scotland.} \]

1\( \text{ in.} \times 1 \text{ in.} \times 0.002 \text{ in., 1250 rpm, 40 psi load} \)

Fig. 5 Film Extent in a Complete Journal Bearing (Single oil hole opposite load; Loaded region in center.)
unsteady friction readings. This may be ascribed to the drip feed giving a fluctuating film extent rather than to the intrinsic nature of limiting hydrodynamic lubrication.

With a continuous pressure feed, film extent varies relatively slightly over a wide range of supply pressures, and the characteristic convergent and divergent film patterns for a single oil hole at 180 deg are as shown in Fig. 5(A) of this discussion. Lowering the feed pressure shortens the continuous film.

On shutting off the oil supply, the film is fed from the oil meniscus at each end of the bearing, and the film pattern is inverted, as in Fig. 5(B).

Once the meniscus breaks, air enters and the film shortens drastically. Gradually the meniscus supply is lost by end leakage, and the clearance space empties as in Figs. 5(C, D).

The time scale for these successive stages varies widely with load, speed, and meniscus stability. The sample photographs cover a period of 160 sec from the instant of cutting off the oil supply, but with a bearing shell suitably designed to retain end-leakage oil, it may be possible to obtain safe starved operation for long periods.

The end meniscus also has been observed to permit a bearing pumping action, by preventing the entry of air from the sides of the bearing. With a single hole entry at 180 deg, oil may then be drawn from a level 12–18 in. below the bearing, giving the characteristic pattern of Fig. 6(A).

M. D. HERSEY.

The present investigation goes far toward solving a problem that had been proposed to the ASME Research Committee on Lubrication by the late Prof. George B. Karelitz, a former chairman. This problem of determining the rate of supply of lubricant required to keep a pair of rubbing surfaces separated was further described by the writer in a recent publication.

Reference was made therein to his experiments at M.I.T. on a full journal bearing 1 in. diam X 3 in. long. It was observed that the coefficient of friction decreased, at first rapidly and then much more slowly, when the rate of oil feed was increased from 2 to 60 drops per min. Ten drops per minute were found sufficient for good lubrication, with steady frictional values, over the speed range from 300 to 1500 rpm with loads from 40 to 250 psi on the projected area, and viscosity from about 3 to 15 millionths of a pound-second per square inch at the bearing temperature. Both fatty and mineral oils were used and no systematic difference was observed. When the rate of oil feed was increased from 10 to 60 drops per min there was only a slight drop in friction, between 10 and 20 per cent, but a much greater increase in the minimum film thickness as shown by electrical resistance measurements.

Another early experiment for comparison is that of Bissell at Cornell, using a mineral engine oil. Loads from 70 to 140 psi were applied to the partial bearings in a Thurston railroad lubricant tester, at speeds close to 320 rpm. Although the results can hardly be accepted as quantitative, it was concluded that 0.003 cc of oil per min per sq in. of projected area would be ample to insure fluid-film lubrication. This amount, it was noted in the discussion, corresponds to “three quarters of a cupful of oil on a locomotive crankpin per about 100 miles of service.”

M. W. MUeller. The paper seems to present a simple and satisfactory approach to the problems of journal-bearing lubrication. The results of the presented tests are in reasonable agreement with actual field experience.

There are, perhaps, a few factors which might at the same time be considered more carefully. Such items as bearing materials, bearing design, and service factors, and the effects of higher viscosity lubricants, might all introduce new and significant variations in the present feed-rate equations.

There is information available which would indicate that the selection of the bearing materials may alter the bearing modulus (ZN/p) by over 100 per cent (Fig. 6 of this discussion). This variation may show a considerable difference in the point at which the frictional forces in the test journal are found to be erratic.

Another point of consideration which would be desirable is the modification of the equation for \( Q_M \) so that it would contain a term for viscosity apart from the quantity \( K_M \). Experience in the field tends to indicate that, by using higher viscosity oils, the quantity of lubricant may be reduced considerably without materially affecting the life or performance of the bearings.

For example, in the test data, there is one point at which the No. 2 bearing failed by seizing. Assuming a viscosity of about 4 centipoises at the given temperature, we have an equation for expressing the permissible unit pressure as follows:

\[
p = \frac{ZN}{3175 \times 10^4} \left( \frac{D}{L} \right)^2 \left( \frac{D + L}{2.125} \right) \times \frac{0.0034}{2.125 + 2.125} = 83 \text{ psi permissible unit pressure}
\]

where

- \( p \) = pressure, psi
- \( Z \) = centipoise
- \( N \) = rpm
- \( D \) = shaft diameter, in.

10 Trubon Engineering Corporation, Cleveland, Ohio.
The results show that the bearing has been overloaded by 350 per cent. However, by using a heavier oil or by providing a means of cooling the present oil to give a viscosity of, say, 15 or 20 centipoises, the bearing probably would not have failed since the load-carrying capacity is shown to be directly proportional to viscosity. By increasing the viscosity to 16 centipoises the permissible unit pressure would be determined as 332 psi which is greater than the load at which the bearing failed. Also, it would follow that a much smaller quantity of oil would suffice to maintain a fluid film around the bearing in other tests. In its present form, the equation containing \( K \) is somewhat vague and perhaps misleading, and, therefore, might well be scrutinized further.

**Authors' Closure**

The authors are pleased to see the extent of interest in the question of minimum feed rates for journal bearings and wish to thank the discussers for their questions and stimulating comments.

The test procedure was such that the oil feed rate was slowly reduced until an instability was noticed in the friction of the bearing. A sensitive balance-type scale was used for this measurement which, when adjusted to the order of magnitude of the friction force that was being weighted, had a full scale deflection of ± 0.1 lb. The instability in the friction, then, that was observed, was a small and transient one and in most cases represented only a very small percentage change in the total friction force that was being measured. When the bearing and shaft surfaces were examined after these tests, no indication of solid contact could be seen. These fluctuations were erratic and of very short duration and showed no relationship to the rate of dropping of oil from the drop-feed oiler. Actually the feed tube, which was about four inches long, would also tend to smooth out fluctuations in the oil feed rate to the bearing.

The determination of a minimum value of \( ZN/p \) seems to have little correlation with test data for the starvation condition that was investigated. The test data show minimum values of \( ZN/p \) ranging from 70 to 17,000. It may be that this parameter is not a significant measure of operating conditions in journal bearings when feed rates are inadequate.

It is quite true that surface finish, the wearing-in history of the bearing and shaft, and the type of bearing material are influential in establishing the minimum film thickness in a bearing. The investigation reported in this paper was of limited scope and did not include these many variables. However, selection was made of what was considered to be representative materials, surface finishes, bearing construction, and operating conditions, in an effort to obtain equally representative data on minimum feed rates to just maintain a hydrodynamic fluid film.

The expression used for presenting the results of the investigation is Equation [5]. Although no separate symbol in this equation represents the viscosity, the lubricant viscosity is inherently part of the equation because the minimum film thickness in a bearing is developed when the eccentricity of the shaft is at its extreme limit and still able to maintain separation of the surfaces by a fluid film. These conditions are specified by the interdependence of speed, load, clearance, and viscosity. Thus the viscosity is a variable in Equation [5] in the same manner as it is a variable for specified eccentricity ratios in Equations [3] and [4]. Essentially, Equation [5] specifies that the journal has reached its maximum possible eccentricity in the bearing without rupturing the fluid lubricant film, and of course
this maximum tolerable eccentricity ratio depends, among other things, upon the viscosity of the lubricant in the bearing.

A very interesting comment was received by the authors from Prof. G. B. DuBois stating that the authors' results could be plotted, as in Fig. 7, which is based on the NACA TN 3491 written by G. B. DuBois, F. W. Oevirk, and R. L. Webe. This form of plotting the results of the log of the oil flow factor $q$ versus load number $1/C_r$ appears to be very useful.

No doubt the approach suggested in this paper will be modified and refined when further experimental data become available, but even in its present form the authors have found the analysis of frequent value in establishing an order-of-magnitude evaluation for minimum feed rates in journal bearings.

Again, we should like to express our appreciation and thanks to those who have contributed their comments in the discussion of this paper.