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Multidimensional measurements FREE

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Multidimensional measurements

Our world is filled with measurements: Jill runs faster than Jane. Bill throws farther than Ed. Julia is a better mathematician than Josephine. Gary is a better economist than George. Notice that the first two are clearly linear measurements, achievable with a stopwatch and a measurement tape, respectively. The second two, on the other hand, involve a multitude of variables and require assessing their relative weight.

Multidimensional measures are well established in fields where quantification is difficult, such as poverty[1] and immigrant integration.[2] But some people in other fields, such as economics, have the conceit that they can measure things without the multidimensions. My goal here is to suggest a mathematical procedure for handling multidimensional measurements. My conclusion may not be profound, since some subjectivity is unavoidable, but it does illuminate the role that bias plays when people collapse a complex measurement into a linear one, as they so often do—for example, when voting or hiring faculty.

Consider, first, a linear quantity, such as the height one can reach when jumping. Given two choices, such as A and B, we can make a judgment, such as A is greater than B. Since A and B are scalars, we can order them, and there is no ambiguity about that order. They don't need to have predefined amplitude ranges, and they can even be irrational or transcendental. But they cannot be complex numbers or vectors, since those cannot be ordered. They may have uncertainties (error bars), but those can be defined and taken into account, essentially as probabilities for the sense of the inequality. My concern here lies in cases where the uncertainties are small enough to be unimportant.

Now consider instead the vectors $\mathbf{A}$ and $\mathbf{B}$. They belong to the same multidimensional space, which—for example, in the case of a faculty member up for tenure—could consist of a set of variables that are agreed by consensus to have relevance. We might agree on the parameters, and we might even agree on their relative importance, although that can be difficult. But we still need to agree on the rank order of $\mathbf{A}$ and $\mathbf{B}$.

To do so, we must construct scalars. A dot product is a scalar, and my (not unique, but plausible) suggestion is to think of the rank orderings as arising from dot products with normalized bias vectors, such as $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. A large dot product arises when the bias is in a direction similar to the candidate vector's. We can then certainly have, for example, $\mathbf{A} \cdot \boldsymbol{\alpha} > \mathbf{B} \cdot \boldsymbol{\alpha}$, which is the ranking chosen by $\boldsymbol{\alpha}$, and still have $\mathbf{B} \cdot \boldsymbol{\beta} > \mathbf{A} \cdot \boldsymbol{\beta}$, which is the ranking chosen by $\boldsymbol{\beta}$. The two observers ($\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$) disagree on ranking, which is not uncommon. And our observers can be just as multidimensional as the attributes ($\mathbf{A}$ and $\mathbf{B}$) that they are ranking. In these equations, we're dealing only with scalars (dot products), and there is no ambiguity in the comparisons. Ambiguity is in the observers, not the data.

We could even have an "ideal" candidate $\mathbf{I}$ who is better than either $\mathbf{A}$ or $\mathbf{B}$ in the sense that $|\mathbf{I}| > |\mathbf{A}|$ and $|\mathbf{I}| > |\mathbf{B}|$ and yet preferred by neither—that is, $\mathbf{A} \cdot \boldsymbol{\alpha} > \mathbf{I} \cdot \boldsymbol{\alpha}$ and $\mathbf{B} \cdot \boldsymbol{\beta} > \mathbf{I} \cdot \boldsymbol{\beta}$. Of course, that presumes a consensus on relative merits of the attributes (implicit in the use of normalized vectors for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$).

This template could be useful in job hiring, but that is not the only potential application. For example, the attributes of a football team can depend on whether the players are running on muddy or dry ground or whether they are at home or away.

In short, bias can be as important as attributes, and we should never think that we should try to understand the candidate (for example, $\mathbf{A}$) just by their attributes—it is about as important to understand the bias ($\boldsymbol{\alpha}$). Perhaps that is obvious, but a numerical example helps explain it and perhaps even quantify it. Left unresolved and unresolvable is the implicit need to agree on attributes, since without that, there can be no agreement on the vector space.

References

1. Oxford Poverty and Human Development Initiative, "Policy—a multidimensional approach," https://ophi.org.uk/policy/multidimensional-poverty-index.
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Factors for assessing researchers

I enjoyed reading the Issues & Events piece "Global movement to reform researcher assessment gains traction" by Toni Feder (Physics Today, October 2023, page 22). I obtained my PhD in the mid 1990s, and since that time I have heard various degrees of dissatisfaction with faculty evaluation processes. It was nice to see the topic discussed in Physics Today.

While it is important to consider a wide range of contributions when assessing research faculty, I would like to speak out against including social impact and entrepreneurship among the factors considered. Research scientists and institutions ought to achieve their influence and status through their contributions to the altruistic endeavors of knowledge creation (research) and knowledge preservation (education).

Entrepreneurship is frequently antithetical to those goals and is instead aimed at the creation of profit-making enterprises. Likewise, "social impact," as it is normally construed, relates to influencing systems of power and the allocation of resources. While those endeavors are certainly important, they should be distinguished from the research goals of knowledge creation and preservation.

I like that Feder's piece discusses the role of objectively defined metrics versus subjectively defined expert judgment. Both kinds of evaluation are important. By working together and communicating in an open and respectful manner, we can build and sustain the kinds of institutions we want to work in.

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