Phenomenological Analysis of the Meson Theory of Nuclear Force

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Taking into account new development of meson theory of nuclear force both from field theoretical and from phenomenological point of view, $\rho$ -$\rho$ scattering upto 100 Mev is analyzed. Namely, adopting the pseudoscalar meson theoretical potential in the singlet states and subtracting their effect from the experimental data, magnitudes of the central and tensor potential in the triplet odd state are obtained. This analysis shows that the central potential must be very weak and that the tensor potential is not so strong as potentials of the other states. These features are also favoured to $n$ -$p$ scattering. The meson theoretical potentials of this state are very similar to this phenomenological one and consequently it is concluded that no corrections are requisite that alter the characteristics of the meson theoretical potential of the triplet odd state. The properties of the meson theoretical potentials of the other states are also discussed.

§ 1. Introduction

The problem of the meson theory of nuclear forces has recently been attacked by several authors. On the one hand many attempts\(^1\)\(^2\)\(^3\)\(^4\) have been made to derive the nuclear forces from the symmetrical pseudoscalar meson theory by improving the approximation method and taking into account various corrections. On the other hand such meson theoretical potentials have been shown to reproduce low energy nucleon-nucleon scattering data including deuteron parameters provided physically reasonable prescriptions such as the introduction of phenomenological potentials when the two nucleons are close together or the reduction of the meson pair terms are made.\(^1\)\(^2\)\(^3\)\(^4\) Also the nucleon-nucleon scattering upto 100 Mev has been investigated by us and others\(^5\) (hereafter referred to as II) adopting the static potentials of the second and fourth order derived from the symmetrical pseudoscalar meson theory with pseudovector coupling\(^1\)\(^2\)\(^3\)\(^4\) (hereafter referred to as TMO potentials), where they have been shown to be satisfactory on the whole for such high energy phenomena. In reference II, however, we have pointed out two problems to be investigated anew. One is concerning to the $S$-wave: Because only $S$-waves are affected by the phenomenological potentials of the region where two nucleons are close together, it is necessary to reinvestigate this phenomenological character more in detail adopting the meson theoretical potentials far from the origin. The other is concerning to the triplet odd state: Owing to the cancellation of many terms derived from the meson theory, the potentials of this state have very delicate features. So it is rather desirable to find out the potentials of this state from the phenomena.

Brueckner and Watson\(^6\) have also derived nuclear potentials of the pseudoscalar meson
theory reducing the effects of the meson pair terms which arises in the non-relativistic approximation to the pseudoscalar coupling (hereafter referred to as BW). The nuclear potentials thus obtained are different from TMO's in the treatment of non-adiabatic correction to the second order terms.

Let us call attention to the region $x \leq 0.6$, where the sixth order potentials or multiple scattering effect are not important. The difference between the potentials of TMO and those of BW in this region is quite negligibly small in the singlet states irrespective of their spacial parity, while it is remarkable in the triplet states. In the triplet even state the non-adiabatic correction term of BW yields the attractive central force instead of the repulsive one of TMO and makes the tensor force less strong. In the triplet odd state this correction term is numerically one ninth of the even state.

Putting all that is mentioned above together, it seems reasonable for us at this stage to attack the problem of $p$-$p$ scattering in the following way: Confine ourselves to the nucleon-nucleon scattering of the limited region of energy upto 100 Mev for the reasons discussed in detail in II. Divide the inter-nucleon distance significantly into two regions, i.e., the outside region (which means the region of $x \geq 0.6$). and the inside one ($x \leq 0.6$). Take for the singlet state TMO's potentials in the outside region regarding them to represent the true features of the meson theoretical potentials, while in the inside region various types of phenomenological potentials are taken. Recalculate the low energy parameters and $^1S$-wave phase shift whereby paying attention to the effect of the inside phenomenological potentials because only $^1S$-wave is affected by them (Sec. 2). The inside phenomenological potentials are assumed to be energy independent as the first approximation. Then subtract this singlet even scattering from the experimental result of $p$-$p$ scattering at 18.3 Mev to get the triplet odd scattering, which is analyzed according to the procedure explained below assuming that only central and tensor forces are present in the two-nucleon interaction as predicted by the meson theory (Sec. 3).

This analysis shows that the central potential of this state must be very weak and that the tensor potential is not so strong as potentials of the other states. Such salient features of the phenomenological triplet odd potentials are favourable not only for very low and high energy $p$-$p$ scattering but also for $n$-$p$ scattering (Sec. 4). It is to be emphasized that weak triplet odd central force, which is favourable in $n$-$p$ scattering, is the only one that is allowed by $p$-$p$ scattering.

To find out the triplet odd potentials, $P$-waves are required to play a certain role to determine the angular distribution of $p$-$p$ scattering. However, as energy goes higher, $F$-waves begin to take part in the angular distribution mainly through the mixture parameter $\tan \epsilon_2$ of the two channeled odd state with $J=2$ and the situation becomes very delicate. This is the reason why we should choose the 18.3 Mev experiment as the object of our analysis.

The triplet odd state potentials due to both TMO and BW treatment are very similar to our phenomenological ones as far as the region $x \geq 1$ concerns and it seems

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* $x$ is the inter-nucleon distance in the unit of the meson Compton wave length $\frac{\lambda}{\mu c} = 1.40 \times 10^{-13}$ cm.
quite possible for them to fit the experimental data very well if the proper value of the coupling constant is chosen. Consequently no corrections that alter the characteristics of the TMO or BW potentials of this state are requisite.

Of late, a new method treating the nuclear force problem of the meson theory has been proposed by Fukuda, Sawada and Taketani (hereafter referred to as FST) in which the dissociation probability of nucleons is properly taken into account. Their result goes to that of BW when the dissociation probability is put identically zero while the treatment of TMO gives the divergent dissociation probability. As to the potentials of FST, they lie between those of TMO and of BW. This result on the one hand supports our adoption of the singlet meson potential of TMO (which is quite the same as BW's) in the region \( x \geq 0.6 \) and on the other hand agrees with our conclusion that the correction to the triplet odd state potentials of TMO or BW is not so large as to change its characteristic features.

In the triplet even state the potentials of FST are close to BW's rather than TMO's. A phenomenological research to this state assuming the potentials similar to BW's has been made by Matsumoto and Watari reporting that such potentials are favourable to give the \( n-p \) total cross section of the magnitude known experimentally at high energies. Neither phenomenological potentials nor meson theoretical one have ever been successful to reduce the \( n-p \) total cross section to the experimental values. In view of all these phenomenological properties of the meson theoretical potentials, one may conclude that the symmetrical pseudoscalar meson theory is consistent with the main features of nucleon-nucleon scattering up to 100 Mev concerning both angular distributions and total cross sections (Sec. 5).

2. Singlet even state

We adopt the potential of TMO in the outside region with various values of coupling constant \( g^2/4\pi \). The results are summarized in Table I and II. In Table I we take as the phenomenological inside potential hard core surrounded by square well, the core radius being a parameter, while in Table II we subdivide the inside region into two parts at \( x=0.3 \), the depth of the potential for \( 0 \leq x < 0.3 \) being a parameter.

It is to be noted that the magnitude of \( ^1S \)-wave phase shifts obtained here is not so large as in II. As mentioned above, we can expect for the potentials of BW and FST not to yield results so different from Table I and II.

2—1. Low energy scattering

In Table I and II, we show the values of effective range \( r_e \) corresponding to scattering length \( a = -23.69 \pm 0.06 \times 10^{-13} \text{ cm} \), which is obtained from the \( n-p \) experimental data. The experimental values of some of the low energy parameters such as singlet \( n-p \) and \( p-p \) effective range \( r_{np} \) and \( r_{pp} \), and singlet \( p-p \) scattering length \( \alpha_{pp} \) depend more or less on the shapes of the potential themselves assumed to analyze the data. Taking this fact into consideration, the experimental data available at present are, including the experimental errors:
\[ r_{ep} = 1.9 \sim 2.7 \times 10^{-13} \text{cm}, \quad \]
\[ r_{en} = 2.5 \sim 2.8 \times 10^{-13} \text{cm}. \]

In this paper, we expect for meson theoretical potential also to yield effective range of the values above, which is quite reasonable, since the shape factor \( P \) of the meson theoretical potential is as small as those of the usual phenomenological potentials as can be seen from the phase shifts at 18.3 Mev in Table I and II.

**Table I.** Properties of the singlet even TMO potential together with phenomenological inside potentials such as

\[ V'(x) = V', \quad \text{for } 0.6 > x \geq x_0, \]
\[ V'(x) = \infty, \quad \text{for } x_0 > x. \]

They give the scattering length \( a = -23.7 \times 10^{-13} \text{cm}. \)

<table>
<thead>
<tr>
<th>( g^2/4\pi )</th>
<th>( x_0 )</th>
<th>( V'(\text{Mev}) )</th>
<th>( r_e(10^{-13}\text{cm}) )</th>
<th>( \delta_6(18.3\text{Mev}) )</th>
<th>( \delta_6(40\text{Mev}) )</th>
<th>( \delta_6(90\text{Mev}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.0</td>
<td>3.5</td>
<td>2.04</td>
<td>52.4°</td>
<td>41.4°</td>
<td>1.0°</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>(-2.8 \times 10^2)</td>
<td>2.15</td>
<td>49.6°</td>
<td>36.1°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>(-3.5 \times 10^3)</td>
<td>2.27</td>
<td>47.2°</td>
<td>31.8°</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.0</td>
<td>(1.4 \times 10^2)</td>
<td>2.46</td>
<td>57.4°</td>
<td>\sim 48°</td>
<td>\sim 33°</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>(-1.1 \times 10^2)</td>
<td>2.52</td>
<td>55.6°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>(-3.5 \times 10^2)</td>
<td>2.58</td>
<td>53.7°</td>
<td>\sim 43°</td>
<td>\sim 27°</td>
</tr>
<tr>
<td>0.12</td>
<td>0.0</td>
<td>(4.0 \times 10^2)</td>
<td>2.76</td>
<td>51.5°</td>
<td>\sim 0.1°</td>
<td>0.5°</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>(2.2 \times 10^2)</td>
<td>2.77</td>
<td>51.5°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>(-2.8 \times 10)</td>
<td>2.81</td>
<td>51.5°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table II.** Properties of the singlet even TMO potential with \( g^2/4\pi = 0.08 \), together with phenomenological inside potentials such as

\[ V'(x) = \tilde{V}', \quad \text{for } 0.6 > x \geq 0.3, \]
\[ V'(x) = \tilde{V}'', \quad \text{for } 0.3 > x. \]

They give the scattering length \( a = -23.7 \times 10^{-13} \text{cm}. \)

<table>
<thead>
<tr>
<th>( \tilde{V}'(\text{Mev}) )</th>
<th>( \tilde{V}''(\text{Mev}) )</th>
<th>( r_e(10^{-13}\text{cm}) )</th>
<th>( \delta_6(40\text{Mev}) )</th>
<th>( \delta_6(90\text{Mev}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>3.5</td>
<td>2.04</td>
<td>52.4°</td>
<td>41.1°</td>
</tr>
<tr>
<td>(1.0 \times 10)</td>
<td>(-7.6\times 10)</td>
<td>2.10</td>
<td>51.1°</td>
<td>37.2°</td>
</tr>
<tr>
<td>(\infty)</td>
<td>(-2.8 \times 10^2)</td>
<td>2.15</td>
<td>49.6°</td>
<td>36.1°</td>
</tr>
</tbody>
</table>

**Table III.** Phase shifts yielded by the phenomenological potentials fitted to the low energy parameters, taken from references 21 and 23.

<table>
<thead>
<tr>
<th>potential shape</th>
<th>( \delta_6(18.3\text{Mev}) )</th>
<th>( \delta_6(20\text{Mev}) )</th>
<th>( \delta_6(32\text{Mev}) )</th>
<th>( \delta_6(18.3\text{Mev}) )</th>
<th>( \delta_6(20\text{Mev}) )</th>
<th>( \delta_6(32\text{Mev}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>square\textsuperscript{21}</td>
<td>48.5°</td>
<td>41.99°</td>
<td>0.26°</td>
<td>0.770°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yukawa\textsuperscript{21}</td>
<td>47.54°</td>
<td>1.20°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>singular Yukawa\textsuperscript{21}</td>
<td>54.2°</td>
<td>51.15°</td>
<td>0.7°</td>
<td>1.40°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lévy\textsuperscript{21}</td>
<td>52.8°</td>
<td>44.85°</td>
<td>0.35°</td>
<td>0.97°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2—2. 18.3 Mev p-p scattering

The singlet S-wave phase shift $^1\delta_0$ and D-wave phase shift $^1\delta_2$ are given* in Table I and II.

$^1\delta_0$ increases as effective range $r_e$ or coupling constant $g^2/4\pi$ decreases as can easily be seen from the shape independent approximation

$$k \cot^1\delta_0 = - (1/a) + (1/2) r_e k^2.$$  \hspace{1cm} (1)

It is necessary to modify the values of phase shift $^1\delta_0$ in Table I and II if we adopt them to analyze the 18.3 Mev p-p scattering data. The first modification comes from the difference $a_{pp}$ and $a_{np}$. This is estimated according to the fact that $a_{np} < a_{pp} \lesssim -16 \times 10^{-19}\text{cm}$. The second modification comes from the presence of Coulomb force which amount, however, small. Thus, the resultant lower bound for true $^1\delta_0$ of p-p scattering due to the meson theoretical potential is estimated to be about 49°. The upper bound for true $^1\delta_0$ for p-p scattering is determined from the differential cross section of p-p scattering and is about 54°. (See Fig. 2). The same modifications to $^1\delta_2$ are quite negligible.

2—3. High energy scattering

The phase shifts $^1\delta_0$ of 40 Mev and 90 Mev are also given in Table I and II, which are apparently smaller than those obtained in II, i.e., 64.8° and 47.3° respectively. The origin for this differences seems to consist in the two facts that, first, we have adopted the rather small value of $r_e=2.26 \times 10^{-13}\text{cm}$ in II and second, obtained the core radius corresponding to the coupling constant $g^2/4\pi=0.08$ by the linear interpolation from Table 2b in the reference 1.

As can be seen in Fig. 1, the potential of this state derived from the meson theory is characterized by weak attractive force in the region $x<1.2$ and by rapidly varying strong attractive force in the region $0.6<x<1.0$. It suggests the existence of a interaction of very strong repulsive nature in the region $x<0.6$, as it

Fig. 1. Singlet even potentials that are equivalent at low energies:

\[ a = -23.7 \times 10^{-19}\text{cm} \text{ and } r_e = 2.77 \times 10^{-13}\text{cm}. \]

M: meson theoretical potential of TMO with $g^2/4\pi=0.12$,
S: square well potential,
Y: Yukawa potential.

* We denote the singlet and the triplet phase shift by $^1\delta_L$ and $^3\delta_L$ respectively.
must reproduce experimental effective range. If the concept "potential" were allowed to keep its validity above the energy of 100 Mev, such a strong repulsive inside potential might begin to contradict with scattering data, which forces us to introduce energy dependent interaction. (Note that the core radius of Jastrow's potential in the singlet state is 0.43 in our unit.) But this might be only a conjecture because the behavior of the potential in the neighbourhood of $x=0.6$ is not finally determined, for example, on account of the multiple scattering effect.

As pointed out in II, the magnitude of $\delta_2$ is exceedingly small compared with the phase shifts $\delta_2$ given by the usual phenomenological potentials, since the meson potential is extremely weak in the region $x>1.2$. (See Table III and Fig. 1.) This situation is very desirable for isotropic $p-p$ scattering data.

§ 3. Phenomenological analysis of the triplet odd state

3—1. Analysis of the experimental data of $p-p$ scattering at 18.3 Mev

The differential cross section of $p-p$ scattering can be regarded to consist of the five terms:

$$\sigma_{pp}(\theta) = \sigma_{M\text{OTT}} \text{ (due to Coulomb force only)}$$

$$+ \sigma_N \text{ (due to nuclear force of the singlet even state only)}$$

$$+ \sigma_{CN} \text{ (due to interference of Coulomb and singlet nuclear force)}$$

$$+ \sigma'_N \text{ (due to nuclear force of the triplet odd state only)}$$

$$+ \sigma_{CN} \text{ (due to interference of Coulomb and triplet nuclear force)}.$$  (2)

Neglecting $F$-waves and waves with higher angular momentum,\textsuperscript{14}

\begin{align*}
\sigma_N &= (1/k^2) \left( 3C_0 + 3C_2 \cos^2 \theta \right), \quad (3) \\
C_0 &= \frac{1}{3} \left( \sin^2 \delta_1 \right)^2 + \frac{3}{4} \left( \sin^2 \delta_1 \right)^2 + \frac{13}{12} \left( \sin^2 \delta_2 \right)^2 - \frac{2}{3} \sin^2 \delta_1 \sin^2 \delta_2 \cos (\delta_1^2 - \delta_2^2) \\
&\quad - \frac{3}{2} \sin \delta_1 \sin \delta_2 \cos (\delta_1^2 - \delta_2^2), \quad (3')
\end{align*}

\begin{align*}
C_2 &= \frac{3}{4} \left( \sin^2 \delta_1 \right)^2 + \frac{7}{4} \left( \sin^2 \delta_2 \right)^2 + 2 \sin \delta_1 \sin \delta_2 \cos (\delta_1^2 - \delta_2^2) \\
&\quad + \frac{9}{2} \sin \delta_1 \sin \delta_2 \cos (\delta_1^2 - \delta_2^2), \quad (3'')
\end{align*}

$$\sigma_{CN} = (1/k^2) \left( \eta / 2 \right) \cos \theta \left\{ f(\theta) \phi + g(\theta) \phi' \right\}, \quad (4)$$

$$\phi = \sin 2^3 \delta_1 + 3 \sin 2^5 \delta_1^5 + 5 \sin 2^7 \delta_1^7, \quad (4')$$

$$\phi' = 9 - (\cos 2^3 \delta_1^3 + 3 \cos 2^5 \delta_1^5 + 5 \cos 2^7 \delta_1^7). \quad (4'')$$

where $k$ is the wave number of the relative motion and $\eta = c^2 / hv$, $v$ being the relative velocity. The functions $f(\theta)$ and $g(\theta)$ are independent of the nuclear phase shifts and
their numerical values are easily to be obtained at an arbitrary energy. After both \( \sigma_{\text{TOT}} \) and \( \sigma_N + \sigma_{\text{CN}} \) calculated assuming \( \Delta_0 \) and \( \Delta_2 \) in Sec. 2 are subtracted from \( \sigma_{pp} (\theta) \), the rest terms \( \sigma_N + \sigma_{\text{CN}} \) are compared with the experimental result at 18.3 Mev.\(^{15}\) Three conditions below follow immediately. (See Fig. 2.)

i) Compare the \( \sigma_{pp} (90^\circ) \):

the allowed \( 3C_9 = 0.08 \rightarrow 0 \) corresponding \( \Delta_0 = 49^\circ \rightarrow 54^\circ \). Cond. (I)

ii) Compare the \( \sigma_{pp} (30^\circ) \) taking the experimental error into account:

\[
0.006 > \theta > -0.018,
\]

where
\[
\theta = \frac{1}{2} \left[ \frac{3\sigma_N + 3\sigma_{\text{CN}}}{3\sigma_{\text{N}} + 3\sigma_{\text{CN}}} \right] \cdot \Delta^2
\]

\[
= 2.250C_2 - 0.111 \phi - 0.0001 \leq 0
\]

\[
\Rightarrow 2.250C_2 - 0.111 \phi.
\]

iii) Compare the \( \sigma_{pp} (\theta) \) in the region \( 40^\circ < \theta < 80^\circ \), where the \( 3\sigma_{\text{CN}} \) is negligibly small:

\[
0.01 > C_2 > 0.
\]

Cond. (III)

\[
\times 10^{-27} \text{ cm}^2
\]

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C}
\end{array}
\]

Fig. 2. \( \rho^+ \rho^+ \) scattering at 18.3 Mev

--- Differential cross section in the absence of triplet odd potentials (\( \sigma_{\text{TOT}} + \sigma_N + \sigma_{\text{CN}} \)) with \( \Delta_2 = 0.1^\circ \) and, A: with \( \Delta_0 = 49.0^\circ \), B: with \( \Delta_0 = 51.5^\circ \), C: with \( \Delta_0 = 54.0^\circ \).

--- Differential cross section when \( C_2 = 0.02 \) with adjusted \( C_0 \) to reproduce the experimental \( \sigma_{pp} (90^\circ) \) and with \( \Delta = 0 \).

Experimental data are taken from reference 15.

3.2. Phenomenological phase shifts

According to Wigner and Eisenbud,\(^{16}\) types of the nuclear two-body non-central interaction are as follows if we limit ourselves to expressions involving no higher powers of the relative momenta \( \rho \) than the first:

\[
S_{12} = 3 \left( \sigma^{(1)} \cdot \mathbf{x} \right) \left( \sigma^{(2)} \cdot \mathbf{x} \right) - \left( \sigma^{(1)} \cdot \sigma^{(2)} \right),
\]

\[
L \cdot S = \left( \mathbf{x} \times \rho \right) \cdot \left( \sigma^{(1)} + \sigma^{(2)} \right),
\]

\[
\left( \mathbf{x} \times \rho \right) \cdot \left( \sigma^{(1)} - \sigma^{(2)} \right), \quad \left( \mathbf{x} \times \rho \right) \cdot \left( \sigma^{(1)} \times \sigma^{(2)} \right).
\]

The latter two of which, however, can be effective only between neutron-proton interaction. Consequently we adopt tensor and \( L \cdot S \) type as non-central interaction.

Following the procedure below, one can generally find all sets of phase shifts (\( \Delta_{ij} \),
 allowed by $p^-p$ scattering experiment assuming the type of the non-central interaction in the triplet state, if one knows the singlet phase shifts by some means. (Note that the force of the singlet state is purely central and is easy to be treated.) Conversely one can find the type and magnitude of the non-central interaction in the triplet odd state if one knows the values of the three phase shifts $(\delta_1^0, \delta_1^1, \delta_1^2)$.

Putting

$$\sin^3 \delta_1^0 = \sin^3 \delta_1^1 = x, \quad \sin^3 \delta_1^2 = y,$$

$$x - s = X, \quad y - z = Y,$$

then retaining terms quadratic in $X$, $Y$ and $z$, it follows that

$$C_0 = \frac{1}{3} X^2 + \frac{3}{4} Y^2, \quad (7)$$

$$C_2 = \frac{3}{4} Y^2 + 6 Y z + 2 X z + 9 z^2, \quad (7')$$

$$\beta = 2 X + 6 Y + 18 z, \quad (8)$$

$$\beta = \frac{1}{3} X + 6 Y + 18 z, \quad (7)$$

$$\gamma = \frac{1}{3} X + 6 Y + 18 z + 4 X z + 12 Y z. \quad (8')$$

Plotted on the $X-Y$ plane (Fig. 3), condition (I) $C_0 = \text{a constant}$ corresponds to an ellipse with its center at the origin (for example the curve $a$ and $b$ of the Fig. 3). As allowed $3 C_0 = 0.08 \sim 0$, an arbitrary point inside the ellipse $a$ satisfies the condition (I) independently of $z$. The condition (II) $d = \text{a constant}$ corresponds to a parabola with its axis parallel to the $X$-axis if $z$ is fixed (Fig. 3. curve $c$, $d$, $e$ and $f$, for example). Therefore, if $z$ is fixed, an arbitrary point on the corresponding parabola inside the ellipse $a$ represents the phase shifts $(\delta_1^0, \delta_1^1, \delta_1^2)$ satisfying the condition (I) and (II). The condition (III) $C_2 = \text{a constant}$ corresponds also to a parabola with its axis parallel to the $X$-axis if $z$ is fixed. In the region of interest, the leftward open parabolas (Fig. 3. $f$, for example) satisfying the condition (II) make $C_0 \sim 0.1$ and must be discarded by the condition (III) $C_2 < 0.01$.

Assume only the tensor type as the non-central interaction. Then according to the Born approximation*

$$x \propto 3 V_0^o - 4^3 V_0^t, \quad (9)$$

$$y \propto 3 V_0^o + 2^3 V_0^t,$n

$$z \propto 3 V_0^o - 0.4^3 V_0^t.$$

Therefore

$$x - s = X : Y = - 3.6^3 V_0^t : 2.4^3 V_0^t = - 3 : 2, \quad (10)$$

* We denote, for example, the triplet central odd potential as $3 V_0^o$, and the singlet even potential $1 V_0^e$. 

Eq. (11) means that any set of the phase shifts \((\delta_0^0, \delta_1^0, \delta_1^1)\) yielded by central plus tensor force exists near the straight line \(Y = -(2/3)X\) (Fig. 3, straight line \(s\)). In addition, the distance between the origin and the point representing the phase shifts gives the measure of the magnitude of the tensor force as \(X\) and \(Y\) are proportional to \(^3V_T^t\) independently of \(^3V_0^t\). According to \(^3V_0^t > 0\) or \(< 0\), this point lies in the second or fourth quadrant respectively. The magnitude of the central force presents itself through the parameter \(z\) though indirectly and a little complicately. In the absence of tensor force the point representing the phase shifts lies on the origin, which means that the pure central force in the triplet odd state gives rise no isotropic differential cross section, namely \(C_0 = 0\).

From the relation between the exact phase shifts and those of Born approximation, one can see that the point representing the exact phase shifts lies in general in the upper side of \(Y = -(2/3)X\), i.e., in the shadowed region of Fig. 3.

What is mentioned above holds also true when a hard core is adopted as the inside potential, because \(X\) and \(Y\) are unaffected by the presence of a hard core so far as the first approximation is concerned.

If one assumes only \(L\cdot S\) type as non-central interaction, the point representing the phase shifts lies near and in the upper side of the straight line

\[
Y = + (2/3)X, \tag{12}
\]

(See Fig. 3, straight line \(t\)) instead of \(Y = -(2/3)X\) that corresponds to the tensor force.

Conversely, if \(^3\delta_0^0, \ ^3\delta_1^0\) and \(^3\delta_1^1\) are known by some means, the point representing them on the \(X-Y\) plane shows the relative weight of the types of the non-central interaction—by the

---

Fig. 3. Phenomenological triplet odd phase shifts represented on the \(X-Y\) plane.

- **a**: \(3C_0 = 0.08\) corresponding to \(^3\delta_0 = 49.0^\circ\).
- **b**: \(3C_0 = 0.03\) corresponding to \(^3\delta_0 = 51.5^\circ\).
- **c**: \(A = -0.006\) when \(z = -0.03\).
- **d**: \(\quad\quad\quad \quad z = 0.0\).
- **e**: \(\quad\quad\quad\quad z = 0.049\).
- **f**: \(\quad\quad\quad\quad z = 0.1\).

- **s**: Phase shifts by the Born approximation when central and tensor force are present.
- **t**: Phase shifts by the Born approximation when central and \(L\cdot S\) force are present.

---

\(\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\c...
tangent of the straight line through this point and the origin—and the magnitude of the interaction—by the distance between this point and the origin.

The pseudoscalar meson theory predicts negligibly small $L \cdot S$ force between two nucleons as far as the outside region concerns so that we assume here only tensor force as the non-central interaction, at least up to 100 Mev*. It is impossible to determine the potential uniquely even if the phase shifts are known at certain energies. Therefore, adopting simply square well potentials both for central and tensor part with the range $x = 2$ which corresponds to the considerably long range that the meson potentials for this state seem to have, we obtain the depth allowed by the experimental data and recalculate the phase shifts yielded by them. The results are summarized in Table IV.

Table IV. Phenomenological potentials (square well of the range $x = 2$) and phase shifts allowed by 18.3 Mev $p - \bar{p}$ scattering experiment when only central and tensor force are assumed. $^3V_0^c$ and $^3V_0^t$ are their depth respectively. $C_0$, $C_2$ and $\Delta$ determine the angular distribution.

<table>
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<tr>
<th>$^3V_0^c$ (Mev)</th>
<th>$^3V_0^t$ (Mev)</th>
<th>$^3\delta_0$</th>
<th>$^3\delta_1$</th>
<th>$^3\delta_2$</th>
<th>$C_0$</th>
<th>$C_2$</th>
<th>$\Delta$</th>
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§ 4. Potentials of the triplet odd state

4—1. Properties of the phenomenological potentials

From Table IV we can see that the experimental results of the $p - \bar{p}$ scattering at 18.3 Mev require the phenomenological potentials of the triplet odd state such as $^3V_0^c$ is very weak,

\begin{equation}
^3V_0^c \text{ is very weak,}
\end{equation}

\begin{equation}
(13-1)
\end{equation}

* Recently, K. Nishijima and M. Shindo obtained $L \cdot S$ force of the fourth order by canonical transformation method. It is very small compared with tensor force in the region $x \geq 1$. 
First, the central part of them is very weak. It is much weaker than the tensor part except the case of extremely large $^3\delta_0$. Second, the tensor part itself is not so strong as the potentials of the other states. These phenomenological potentials are continuous with $^3V_0'$ where the larger $|^3V_0'|$ means that the contribution of the triplet odd interaction to the isotropic angular distribution is the larger, which is required when $^3\delta_0$ is the smaller.

Physically, possibility of strong attractive central force is excluded as it gives rise to large $C_0$, when the angular distribution gives a minimum at 90°. This excluded possibility corresponds to the curves such as $e$ and $f$ in Fig. 3, which have been laid aside in 3-2 already. These curves require instead of eqs. (13-1) and (13-2)

$$^3V_0' = +5 \sim -8 \text{ Mev.}$$  \hspace{1cm} (13-2)

Another possibility of strong repulsive central force is also ruled out, as first, it gives large $C_0$, and second, it interferes with Coulomb force constructively in the region $\theta \sim 30°$ and definitely contradicts with experiment, since the interference term is determined mainly $\phi = (1/9) (\sin 2^3\delta_0^0 + 3 \sin 2^3\delta_1^1 + 5 \sin 2^3\delta_2^2)$ which is proportional to $^3V_0'^c$ according to the Born approximation. Therefore the central force of the triplet odd state must be weak indispensably.

The properties of the required phenomenological potentials (13-1) and (13-2) are as follows:

i) Some of them are not in disagreement with the information of low energy $p$-$p$ scattering experiment\(^\text{17}\) which tells that the averaged $P$-wave phase shift is negative, for example, at 3.899 Mev, \((1/9) (6^3\delta_0^0 + 3^3\delta_1^1 + 5^3\delta_2^2) = -0.109 \pm 0.20°\). At such a low energies $P$-wave phase shifts are determined mainly by the features of the potential far from the origin (the wave length $\lambda \simeq 3 \times \hbar/\mu c$) so that the square well potential model adopted here is not adequate for quantitative discussion. (See also 4-3 and Note Added in Proof.)

ii) Not only at 18.3 Mev but also at the higher energies upto 100 Mev, there remains the relation $C_0 < C_0'$ and it is satisfactory for the isotropic $p$-$p$ angular distribution. Actually we have performed the same prescription at 40 Mev. The result is quite similar to (13-1) and (13-2) or Table IV, that is, $^3V_0'^c = +1 \sim -2 \text{ Mev}$ and $^3V_0' = +4 \sim -6 \text{ Mev.}$ For the quantitative comparison, however, attention must be called to the fact that: In the presence of tensor force, the mixture parameter $\tan \epsilon_2$ of the two channeled odd state with $J = 2$ ($P_2 = P_2$ state) makes a delicate effect on $p$-$p$ differential cross section. For example, even at 90 Mev, the $P$-wave phase shift is in general indeed much smaller than the $P$-wave phase shift but according to our estimation, the mixture parameter $\tan \epsilon_2 \simeq 0.3$ do have the possibility to change the differential $p$-$p$ cross section due to the triplet odd interactions to the extent of about 30%. Unfortunately, while we scarcely know about the nature of this mixture parameter (What is responsible for determining the sign and magnitude of it? Or which region of $x > 0.6$ or $< 0.6$ is more responsible for it?), the large tensor part of the phenomenological potentials above suggests the importance of this
parameter. Therefore, the higher the energy goes, the more suspicious it is to discuss quantitatively adopting these phenomenological potentials in place of meson theoretical potentials. At low energies \( \tan \epsilon_z \) decreases like \( k^z \) and the \( \gamma \)-wave phase shift (that goes to the \( P \) wave phase shift in the absence of tensor force) decreases like \( k^{z}\) \(^{19}\) and it is certainly unnecessary to take above fact into account.

iii) From the angular distribution of high energy \( n-p \) scattering which is fairly symmetrical about 90\(^\circ\),* the next inequality was concluded by Christian and Hart\(^5\) assuming the same shape and range of potentials for all states:

\[
0 < \left( \frac{1}{4} \right) V_0 + \left( \frac{3}{4} \right) V_o < - \left( \frac{1}{5} \right) V_o. \tag{15}
\]

The singlet odd phenomenological square well potential with the range \( x = 2 \) that gives the same phase shift as the meson theoretical \( V_0 \) of TMO or BW has the depth of about +16 Mev. Combined with this phenomenological \( V_0 \) our phenomenological \( V_o \) gives

\[
\left( \frac{1}{4} \right) V_0 + \left( \frac{3}{4} \right) V_o = 2 \sim 7 \text{ Mev}
\]

and is quite satisfactory.

As was stated before, the conditions (II) and (III) depend on the value of \( \delta_0 \). Qualitatively, for large \( \delta_0 > 0 \), \( \delta \) must be smaller because of the interference between \( 1S \)- and \( 1D \)-waves. Quantitatively, however, even for \( \delta_0 = 0.5^\circ \), \( \delta \sim -0.035 \) and so the characteristic features of the phenomenological potentials (13-1) and (13-2) are not affected.

4—2. Comparison with the other phenomenological potentials

Upto date several \( p-p \) phenomenological potentials have been proposed which consist of central and tensor part. We will discuss here briefly the characteristic features of their triplet odd state and compare them with our phenomenological ones (13-1) and (13-2).

i) Jastrow's potentials\(^9\)

\[
3V_o = 0,
\]

\[
3V_0^t = + (50.8 \text{ Mev}) e^{-x/\alpha}, \quad x_t = 0.535. \tag{16}
\]

Clearly they satisfy (13-1) and owing to the exchange character attached to the tensor part, \( (0.3 + 0.7 P_{\tau}) \), it is weak compared with the potentials of the other states.

ii) Christian and Noyes' potentials\(^20\)

\[
3V_0 = 0,
\]

\[
3V_0^t = \pm (23.5 \text{ Mev}) e^{-x/\alpha} / (x/\alpha)^2, \quad x_t = 1.14. \tag{17}
\]

Also they satisfy (13-1). \( 3V_0^t \) is rather large because their singlet central force is square well and gives the small \( 1\delta_0 \). (See Table III.)

iii) Levy's potentials\(^22\)

\[
\]

* As far as we know, there is one exceptional experimental result that gives the small forward scattering at 145 Mev,\(^{19\circ}\) but recent results\(^{20\circ}\) again give the symmetrical curve about 90\(^\circ\) in the region 5\(^\circ\) to 180\(^\circ\).
Phenomenological Analysis of the Meson Theory of Nuclear Force

$$\frac{3}{3} V^o_{\sigma} = \frac{1}{3} \left( \frac{G^2}{4\pi} \right) \mu^2 \frac{e^{-2\pi}}{x}$$

$$- 3 \left( \frac{G^2}{4\pi} \right) \mu^2 \frac{1}{x^2} \left\{ \frac{2}{\pi} K_1(2x) + \frac{\mu}{2M} \left( \frac{2}{\pi} K_1(x) \right) \right\},$$

(18)

$$\frac{3}{3} V^t_{\sigma} = \frac{1}{3} \left( \frac{G^2}{4\pi} \right) \mu^2 \frac{1}{x^2} \left( 1 + \frac{3}{x} + \frac{3}{x^3} \right) e^{-\pi}.$$  

These potentials have been shown to reproduce 18.3 Mev $p-p$ scattering data well. According to Martin and Verlet, we take the value $G^2/4\pi = 10.36$ corresponding to $g^2/4\pi = 0.058$. The numerical estimate shows that $V^t_0 = |V^o_0|$ at $x \sim 0.8$, $|V^o_0| \ll |V^t_0|$ outside. Furthermore $V^o_0 = 0$ at $x \sim 2.0$ owing to the cancellation of the two terms in the central force. So eq. (13-1) is satisfied as far as the outside region is concerned. $V^o_0$ is the usual symmetrical pseudoscalar meson potential of the second order and so eq. (13-2) is also satisfied as will be discussed in 4-3. However, since $V^o_0$ is attractive in the region $x \lesssim 2$ and rather strong near the origin, Lévy's potentials might give rise too large $C_2$ at higher energies.

As a summary, the salient feature (13-1) is common to all phenomenological potentials and (13-2) depends on the values of $V_0^o$ more or less. It is to be emphasized that the weak central force of the triplet odd state is necessary and sufficient for $p-p$ angular distribution, while it is only sufficient for the $n-p$ angular distribution, as eq. (15) can be satisfied if the central forces of the singlet and triplet odd states have opposite sign and cancel out each other, however strong they may be.

4-3. Comparison with the meson theoretical potentials

The TMO potentials of the triplet odd state with $g^2/4\pi = 0.08$ and the BW potentials with the equivalent coupling constant $G^2/4\pi (\mu/2M)^2 = 0.08$ with reduced meson pair terms are plotted in Fig. 4. They are very similar with the phenomenological potentials (13-1) and (13-2) with the positive tensor part, $V^o_0 > 0$ in the region $x > 1$. Therefore, we may surely say that no corrections altering the characteristic features of both tensor and central part of the triplet odd state potentials are necessary from the phenomena and that it is possible for the meson theoretical potentials to reproduce the $p-p$ scattering data fairly well if the value of $g^2/4\pi$ is adjusted and suitable inside phenomenological potentials are adopted.

On the other hand, from the field theoretical point of view, no corrections to this state are to be put to the potentials in the region $x > 1$ whose dominant part is of the second order. The potentials of TMO and BW are markedly different from each other only in the region $x < 1$.

The tensor part of the meson theoretical potentials in the region $x > 1$ is repulsive and mainly proportional to $g^2/4\pi$. So that for the larger $g^2/4\pi$ we have the smaller $V_0^o$ in the singlet even state and the stronger tensor force of the triplet odd state. This tendency is consistent with the $p-p$ scattering phenomena because the decrease of the singlet cross section due to the smaller $V_0^o$ must be covered with the tensor part of the triplet odd state.
The potentials as pointed out in 4—1.

The fact that the central part of the meson theoretical potential is slightly repulsive in the region \( x > 1 \) is favourable to the low energy data suggesting the repulsive potential effective to the averaged \( P \)-wave phase shift. Note that the wave length \( \lambda = \frac{3}{k} = \frac{3}{(pC)} \).

(See 4—1, i). (See also Note Added in Proof.)

§ 5. Summary of the properties of the meson theoretical potentials

5—1 The triplet even state

The potential of TMO and those of BW are markedly different from each other only in the triplet even state, that is, the central part of TMO is strongly repulsive and in consequence the phenomenological inside potential of attractive nature is necessary for the deuteron to be bound\(^{0} \), while the central part of BW is attractive. The potentials of FST is between those of TMO and BW but close to the latter\(^{0} \). Matsumoto and Watari\(^{7} \) have adopted the "L-shaped" potential similar to those of BW in the region \( x > 0.6 \) and obtained many interesting results. According to them, if suitable energy dependent inside potentials are assumed, not only low energy \( n-p \) scattering data including deuteron parameters are reproduced, but also the total \( n-p \) triplet cross section at 90 Mev is smaller than that of II by about 27%. Consequently, the \( n-p \) total cross section decreases by about 23%. This is very satisfactory because both the meson theoretical\(^{0} \) and the phenomenological potentials\(^{0,9} \) proposed so far have given the larger \( n-p \) total cross sections than experimental ones by about 20-30%. Their success is attributed partly to the particular choice of the inside phenomenological potentials, that is, for \( 0 \leq x < 0.4 \) infinitely repulsive hard core and for \( 0.4 \leq x < 0.6 \) the square well of the depth \( 3V\_e = -500 \) Mev, \( 3V\_e^t = +300 \) Mev. As we have pointed out previously, meson theoretical results for nucleon-nucleon scattering have some phenomenological nature through \( S \)-wave phase shift even at the energies lower than 100 Mev, which, in the presence of tensor force, is itself complicated threefold through \( \alpha \)-wave phase shift, \( \gamma \)-wave phase shift and mixture parameter \( \tan \varepsilon \). So, the problems left in future are first to reexamine the triplet even state potentials derived from pseudoscalar meson theory with FST treatment and then to connect the above
inside potentials with the field theoretical one in the region $0.4 \leq x < 0.6$.

5—2. *The triplet odd state*

The only type of the potentials that the $p-p$ scattering experiment upto 100 Mev requires is such that the central part is very weak and the tensor part is not so strong as the potentials of the other states. The meson theoretical potentials are very similar to these phenomenological ones and it is surely possible to adjust the coupling constant $g^2/4\pi$ and assume suitable phenomenological inside potentials to fit the experimental data very well from 0 to 100 Mev. It is to be noted that as energy goes higher, the details of the potential shape will perhaps present themselves through the mixture parameter $\tan \epsilon$. Quantitatively, it is necessary to reexamine the $p-p$ scattering upto 100 Mev assuming the FST potentials of various values of $g^2/4\pi$ in the region $x > 0.6$ and several types of phenomenological potentials in the region $x \leq 0.6$.

5—3. *The singlet states*

There is no difference between the potentials of TMO and those of BW in these states.

The strong repulsive force in the singlet odd state satisfies, combined with the triplet odd central potential, the Christian-Hart inequality (15) which is necessary to reproduce the $n-p$ angular distribution.

The exceedingly small $1/2\epsilon$ is favourable for $p-p$ angular distribution. The phenomenological inside interaction of strongly repulsive nature in the singlet even state which is required from the low energy experimental data might contradict with the experiments at the energies higher than 100 Mev, if one assumed their energy independence.

5—4. *Conclusions*

The nuclear force derived from the symmetrical pseudoscalar meson theory both with pseudovector coupling and with pseudoscalar coupling (if the meson pair term which arises in the non-relativistic approximation is reduced) is consistent with the main features of nucleon-nucleon scattering upto 100 Mev including both angular distributions and total cross sections.

The authors wish to express their sincere gratitude to the members of the research group of nuclear force, especially to Prof. M. Taketani, N. Fukuda, Messrs. J. Iwadare, S. Machida, K. Sawada, A. Sugie, S. Tani and W. Watari for their discussions and criticism. Some of the results and discussions of Section 5 belong to Messrs. W. Watari and M. Matsumoto. They are also indebted to Prof. M. Kobayasi, Prof. S. Takagi and their colleagues for the encouragement during the work.
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2) K. Nishijima, Prog. Theor. Phys. 6 (1951), 815, 911.
3) K. A. Brueckner and K. M. Watson, Phys. Rev. 92 (1953), 1023. Here referred to as BW.
4) N. Fukuda, K. Sawada and M. Taketani, Prog. Theor. Phys. 12 (1954), 156. Here referred to as FST.

Note added in proof:

Recently, one of the author (S.O.) and collaborator have shown that the meson theoretical potentials actually reproduce negative averaged $P$-wave phase shift at low energies and that the slightly repulsive central potential of the second order in the region $x > 1.5$ is important for it. The coupling constant that fits the experimental data is $g^2/4\pi = 0.08 \sim 0.06$ and agrees with that predicted from the singlet even state data in this paper, i.e. $0.08 \sim 0.12$. We think that this fact gives strong support to the meson theory of nuclear forces.