High Energy Heavy-Ion Collisions in a RBUU-Approach with Momentum-Dependent Mean-Fields

Tomoyuki MARUYAMA,** Wolfgang CASSING,* Ulrich MOSEL* and Stefan TEIS*

Department of Physics, Kyoto University, Kyoto 606-01, Japan
*Institut für Theoretische Physik, Universität Giessen
D-35392 Giessen, F. R. Germany

We introduce momentum-dependent scalar and vector fields into the Lorentz covariant RBUU-approach in line with the empirical proton-nucleus relativistic optical potential. Within this extended RBUU-approach we perform numerical simulations for heavy-ion collisions and calculate the transverse flow of nucleons as well as subthreshold production of $K^+$ mesons. By means of these observables we discuss the particular role of the momentum-dependent forces and their implications on the nuclear equation of state. We find that only a momentum-dependent parameter-set can explain the experimental data on the transverse flow from $150 - 1000\text{ MeV}/u$ and the differential $K^+$-production cross sections at $1\text{ GeV}/u$ at the same time.

The main aim of the high energy heavy-ion physics is to determine the equation of state (EOS) of nuclear matter under extreme conditions far from the ground state. Any conclusion on the properties of hot and dense matter must rely on the comparison of the experimental data with theoretical predictions based on nonequilibrium models. Among these, the BUU-approach\(^1\)\(^2\)\(^3\) is a very successful approach in describing the time-dependent evolution of the complex system. As a genuine feature of transport theories it has two important ingredients: the mean-fields or self-energies for nucleons and an in-medium nucleon-nucleon cross-section that accounts for the elastic and inelastic channels. By varying the mean-fields which reflect a certain EOS and comparing the theoretical calculations with the experimental data, one expects to be able to determine the nuclear EOS.

Within the framework of BUU-simulations we have succeeded to predict/reproduce particle production data in heavy-ion collisions and to clarify their reaction processes.\(^4\) In spite of this success the nuclear EOS has not been determined completely, yet. The mean fields cannot be uniquely determined by the equation of state alone, and, in addition, it is not always possible to extract the nuclear EOS from the results of the BUU calculations without ambiguities for other model inputs.

The most important model inputs besides the nuclear incompressibility are the momentum-dependence\(^5\) and the Lorentz covariance\(^6\) of the mean-fields in the high energy region. Thus we introduce an explicit momentum-dependence of the mean-
fields into this Relativistic BUU (RBUU) approach, which is constructed in a Lorentz covariant way by combining the BUU-approach with the $\sigma$-$\omega$ model.\textsuperscript{8}

In order to determine the nuclear EOS without ambiguities, furthermore, we must also investigate several kinds of observables, at least two independent ones, at the same time as well as to improve the transport approach to satisfy other experimental requires. Thus we calculate the transverse flow and the subthreshold $K^+$-production with the same model input and compare with experimental data.

Now the $\sigma$-$\omega$ model predicts two kinds of independent mean-fields: an attractive scalar field $U_s$ and a repulsive four-vector field $U_\nu$. We parametrize the momentum-dependent (MD) parts of these mean-fields in analogy to Fock terms of nucleon self-energies to formulate a conserving theory.\textsuperscript{9} For the actual simulations we use a polynomial approximation for the relativistic nucleon-nucleon interaction.\textsuperscript{10,11} In this work we present two kinds of parameter-sets (POL6, POL7) which yield a saturation density $\rho_0 = 0.17$ fm$^{-3}$, a binding energy per nucleon of $E_b/A = -16$ MeV at $\rho_0$ and an effective mass $M^*/M = 0.65$ at the Fermi surface. The incompressibility is $K = 400$ MeV for POL6 and 200 MeV for POL7. For comparison we also present three kinds of momentum-independent parametrizations (NL6, NL7, NL21); NL6 and NL7 have same nuclear matter properties of POL6 and POL7, respectively, and NL21 gives the incompressibility $K = 200$ MeV and as effective mass $M^*/M = 0.83$.

In Fig. 1 we show the resulting MD mean-fields $U_s$ (a) and $U_0$ (b) at $\rho = \rho_0$ as a function of the nucleon kinetic energy $\varepsilon_\kappa$. Solid, thick dotted and dotted lines indicate the results of POL6, NL21 and NL6, respectively. In Fig. 1(c) we also present the Schrödinger equivalent potential defined by

$$U_{\text{SEP}} = -U_s + U_0 + \frac{1}{2M} (U_s^2 - U_0^2) + \frac{U_0}{M} \varepsilon_\kappa,$$  \hspace{1cm} (1)

where the shaded area indicates the result of the experimental analysis by Hama et al.\textsuperscript{12} We find that the optical potentials for the parametrization POL6 almost reproduce the experimental result. The $U_{\text{SEP}}$ of NL6 is linear in $\varepsilon_\kappa$ and not so different from the experimental result up to about 300 MeV, but yields too strong repulsion at higher energy. Since this strong repulsion is due to strong vector fields, the NL6
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parameter-set overestimates the Lorentz force on a fast moving particle in the medium.

In Fig. 2 we show the energy per nucleon for nuclear matter (a) and the effective mass at the Fermi surface (b) as a function of density for each parameter-set. The expression for the total energy has been given in Ref. 9). Besides, it is not very much different from that for NL6 (dashed line) since the explicit momentum-dependence does not significantly change the nuclear EOS at moderate densities. In Fig. 2, furthermore, we display the results for POL7 (chain-dotted line), NL7 (dotted line) and NL21 (thick-dotted line). Here we confirm that the momentum-dependence does not strongly change the nuclear matter properties.

Now we report on results of the actual numerical simulation of the RBUU approach with the MD mean-fields and calculate the transverse flow and the subthreshold $K^+$ production for various heavy-ion collisions.

In Ref. 13) we have already discussed that the explicit momentum-dependence of the mean-fields decreases the transverse pressure in the system for the two-Fermisphere geometry which is realized in the early stage of a heavy-ion collision. This fact predicts also a suppression of the transverse flow. To demonstrate this we calculate the mean transverse momentum in the reaction plane $<p_x/A>$ versus the

![Fig. 3](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTP.120.283/1931752)

**Fig. 3.** The mean transverse momentum per nucleon $<p_x/A>$ versus the center-of-mass rapidity (per initial projectile rapidity) for Ar+KCl collisions at $E_{lab}=800$ MeV/u (a), Au+Au at 400 MeV/u (b) and Au+Au at 150 MeV/u (c) at $b=6$ fm. The solid, chain-dotted, dashed and dotted lines show the results for POL6, POL7, NL6 and NL7, respectively. The experimental data (asterisks) are taken from Ref. 14) for (a) and Ref. 15) for (b).
normalized center-of-mass (CM) rapidity $Y_{CM}/Y_{PR}$ ($Y_{PR}$: the projectile rapidity). In Fig. 3 we show the results in Ar+KCl collisions at $E_{lab}=800 \text{ MeV}/u$ (a), Au+Au collisions at $E_{lab}=400 \text{ MeV}/u$ (b) and $150 \text{ MeV}/u$ (c) at $b=6 \text{ fm}$ for the parametrizations POL6 (solid lines), POL7 (chain-dotted lines), NL6 (dashed lines) and NL7 (dotted lines).

The differences between our various parametrizations are negligible at $E_{lab}=150 \text{ MeV}/u$ in the interesting regime around mid rapidity, but become larger with increasing beam-energy. At $E_{lab}=400 \text{ MeV}/u$, both the momentum-dependence and the incompressibility lead to differences at high rapidity $Y_{CM}/Y_{PR}>0.6$ though there are again no sizeable effects at mid rapidity. However, these differences are not very pronounced because all results almost agree with the experimental data.

At higher energy ($800 \text{ MeV}/u$) the MD mean-fields POL6, POL7 nicely reproduce the experimental data (a) while the flow for the MID parameter-sets NL6 and NL7 is overestimated at $800 \text{ MeV}/u$; the results are quite insensitive to the incompressibility $K$.

With increasing beam energy the momentum-dependence dominates in the transverse flow. Note that this holds in spite of the fact that the densities reached here are even higher than at the lower energies. This demonstrates that it is not the density compression but the Lorentz force which drives the transverse flow. At $E_{lab}=800 \text{ MeV}/u$ the MD parametrization POL6 and POL7 can reproduce the experimental results whereas the results of the MID parametrization NL6 and NL7 overestimate the data. The latter result is due to the fact that our MD mean-fields (POL6 and POL7) yield the proper strength of the Lorentz force while the mean-field of NL6 and NL7 yields a too strong repulsion at high energy (cf. Fig. 1(c)).

Now we discuss the $K^+$-production in the subthreshold energy region. We directly show the results of our present calculation for the experimental angle $\theta_{ab}=44^\circ$ in Ne+NaF (a) and Au+Au (b) collisions at $E_{lab}=1 \text{ GeV}/u$ (Fig. 4). Also displayed in Fig. 4 are the experimental data from Au+Au collisions and the preliminary data from Ne+NaF collisions of the KAOS group at SIS.

In the Ne+NaF reaction there is no significant difference among all cases except the results for NL21 which are about 15% larger than the others. In Ne+NaF collisions only moderate densities in the medium ($\rho<2\rho_0$) are achieved such that the results do not reflect the properties of high density
nuclear matter.

From the Au+Au collisions, however, we get some information about the high density matter. The calculated yield is sensitive both to the incompressibility and the effective mass at the Fermi surface. However, the explicit momentum-dependence does not change the spectrum very much except for the slope which is slightly smaller for POL6 than for NL6. This shows that the MD mean-fields generate higher kinetic energies or higher temperature than the MID fields. The amount is ordered as $(NL21) > (NL7) > (NL6) \simeq (POL6)$; this order is equivalent to that in the binding energy of high density nuclear matter. The results for the $K^+$ yield thus reflect the energy density of the dense nuclear system.

The MD parametrization POL6 can reproduce the experimental data for $K^+$-production as well as the transverse flow whereas none of the MID parametrizations can explain both sets of data at the same time. Hence we can expect that the nuclear EOS corresponding to the parameter-set POL6 is very close to the exact one for $\rho \approx 2 - 3 \rho_0$.\textsuperscript{11)

From the comparison of the above results, furthermore, we obtain a clear picture of the effects of the MD mean-fields. The high energy behaviour of the mean-fields is significant in the initial stage, and the strength at the initial energy determines the mean transverse flow and the density distribution in the participant zone. With increasing time, the matter is compressed and the bombarding energy is distributed among the nucleons in the high-density participant zone where the average relative momentum between two baryons becomes much smaller. In the most condensed phase the high energy behaviour of the mean-fields becomes less important and the low energy behaviour of the mean-fields essentially determines the energy-momentum distribution in the participant zone due to energy-momentum conservation. Thus the phase-space distribution in the participant zone is determined by both: the high energy behaviour at $\rho = \rho_0$ and the low energy behaviour of the mean-fields in the whole density region. Fortunately the difference in the maximum densities between POL6 and NL6 does not influence the result for $K^+$-production. In addition, only the nuclear incompressibility $K$ affects the $K^+$ yield significantly; the scalar and vector fields have no separate effect on this quantity.

From the results of the present investigations and the comparison to various experimental data on flow phenomena and kaon production simultaneously we infer that the true nuclear EOS should be close to that of POL6 around $\rho \approx 2 - 3 \rho_0$.\textsuperscript{11)} Essential for this conclusion is the simultaneous analysis of unrelated sets of data within the same model since there might be unknown medium effects. We have, furthermore, applied our covariant RBUU-approach to the subthreshold production of $p\bar{p}$-pairs for which we refer the reader to Ref. 17).

References

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