Iterative LP water system optimal operation including headloss, leakage, total head and source cost

Eyal Price and Avi Ostfeld

ABSTRACT

Linear water balance optimal operation models are common with relative short solution times but suffer from a lack of certainty whether the given solution is at all hydraulically feasible. Introducing hydraulic headloss, water leakage and changing pump energy consumption, effect the resulting system optimal operation but also create a non-linear problem due to the convex relation between flow, headloss, water leakage and total head. This study utilizes a methodology published by the authors for linearization of convex or concave equations. An iterative linear programming (LP) minimal cost optimal operation supply model is solved including the Hazen–Williams headloss equation, pressure related water leakage equation, changing pump energy consumption and source cost. The model is demonstrated using an example application. ‘Greater than’ or ‘less than’ water head constraints at nodes may force the system to maintain certain water levels in water tanks reducing the available operating volume forcing pumping stations to operate in peak tariff periods as less storage is available in low tariff periods. Operationally, reducing water leakage may be achieved by reducing water heads along the system by means of shifting pump operation periods and maintaining low water levels in water tanks. Source costs may serve as penalties or rewards discouraging or encouraging the use of certain water sources.

Key words | convex, headloss, successive linearization, water distribution system optimal operation, water leakage

NOTATION

The following symbols are used in this paper:

- $A_{ht}, B_{ht}$: headloss linear equation coefficients, constants
- $A_{lt}, B_{lt}$: water leakage linear equation coefficients, constants
- $A_{pt}, B_{pt}$: energy consumption linear equation coefficients, constants
- $AH_{t}^{i,j}$: artificial head gain or loss in pipe, variable (m)
- $AH_{t}^{i,j}$: previous iteration steps artificial water head gain in at pipe $(i,j)$, constant (m)
- $C_{ij}$: pipe roughness coefficient, constant (–)
- $D$: group of all demand at nodes
- $De_{i,j}$: consumer demand at node constant (m$^3$/hr)  
- $D_{i,j}$: pipe diameter constant (mm)
- $d_{t}$: demand node index
- $dH_{t}^{i,j}$: pipe headloss, variable (m)
- $E$: group of all arc indexes
- $E_{p}^{t}$: pump’s energy consumption, variable (kWhr)
- $H_{t}^{i,j}$: water head maximum and minimum head, constants (m)
- $H_{t}^{i}$: previous iteration steps water head at junction $i$, constant (m)
- $H^{i}_{i,j}$: water head node $i$, variable (m)
- $i_{j}$: index of pipe origin and destination nodes
- $q_{i,j}$: leakage at node $j$ from pipes leading to $i$, variable (m$^3$/hr)
**INTRODUCTION**

Linear water balance optimal operation models are common with relatively short solution times but suffer from a lack of certainty whether the resulting solution is at all hydraulically feasible. Such models overlook water system properties such as pipe diameter and length, elevation of water tanks and other relevant properties. As an example: undersized pipe sections may limit gravitational flow rates from a water tank limiting the amount of water which may be supplied while maintaining a minimal water head to a consumer. As a result, a pumping station may be forced to work during peak water consumption periods to boost the amount of water and water head supplied to the consumers to meet constraints.

**Headloss and water head**

The headloss relationship between flow and head (i.e., the Hazen–Williams or Darcy–Weisbach formulas) hold a non-linear convex relation, resulting in a non-linear optimization model, which at times may be cumbersome and slow to solve. Minimal or maximal head/pressure constraints at nodes may affect the operation of a water distribution system, forcing pumping stations to work directly to consumption or forcing maintenance of high or low water levels in water tanks.

**Water leakage**

Water leakage may be a major or minor issue in a water distribution system, depending on size, age, and level of system maintenance. Water leakage increases operation costs due to excess water pumped at pumping stations. A direct approach to minimizing water leakage is a costly high-level of system maintenance. In large and complex systems this is hard or uneconomical to maintain and the assumption is that there is always some amount of leakage in the system. The amount of leakage from a given pipe may be non-linearly related to the average water pressure along the pipe, the equivalent size of an orifice through which the water leaks and pipe length. For a given water distribution system, the operational approach to minimizing
water leakage is to minimize water head/pressures throughout the system when not operationally required. Lowering the heads may be achieved by utilizing such means as: time dependent pressure reducing valves (PRVs), maintaining low water levels in water tanks or by increasing hourly flow rates throughout the system to raise dynamic headloss and lower water heads. The subject of PRVs is not addressed in the current research. Maintaining low water levels in water tanks, in order to reduce leakage, also reduces the available storage volume which may be used during low electrical tariff hours, forcing pumping stations to work more hours in expensive electrical tariff periods. Increasing hourly water flow rates may be achieved by operating pumping stations, which pump water from the problematic zone, during the peak consumer hours in which demands occur, increasing total hourly water consumption from the zone.

**Pump station total head**

At any given time, a pump’s energy consumption is dependent upon the pump’s total head (TH), the water flow through the pump and the pump’s overall efficiency at a given flow rate. To deal with the non-linear relation between flow and a pump’s energy consumption, linear programming (LP) models may assume an average fixed linear energy consumption dependent only on the hourly flow through the pump. In well-sized water systems where the pump’s TH changes very little, relative to the flow, this assumption may suffice. In poorly-sized water systems or for pumping stations with a TH which is comprised mainly of dynamic losses, hourly changes in the TH may have a major effect on the pump energy consumption and on the water system’s minimal cost optimal operation.

**Source cost**

Source cost is a fixed price per unit of water passing through a certain pipe leaving a certain source. The source cost penalty may represent the overhead cost of conveying the water along an external water system to the connection point which acts as a source node to the water system being optimized. A positive source cost (penalty) will cause the optimized water system to prefer other lower cost water sources. A negative source cost (reward) may be imposed to encourage the water system to prefer the rewarded source. Note that if the negative source cost is too rewarding, then the system may aim to maximize the amount of water taken from the rewarded source by means of increasing system water leakage and even operating the pumping stations in peak electrical tariff periods.

**Current study**

This study explores the effects of headloss, leakage, TH and source cost on minimal cost optimal system operation. The non-linear nature of the relation between flow, headloss, leakage and TH, creates a model with convex equations, thus forming a non-linear optimization model. The model is solved using an improved version of the iterative convex linearization algorithm presented by Price & Ostfeld (2013) for linearization of exponential convex or concave equations with ‘greater than’ and ‘less than’ constraints. Currently, fixed speed pumps are not handled by the model as this would transform the original problem from a smooth NLP (non linear programming) into a discrete MIP (mixed integer programming). The method may be applied to a pumping station supplying water to consumers via a water tank or directly to the consumer using a variable frequency drive (VFD) as the pumping station. The method is also valid for low lift pumps, where the water level changes at the destination tank are comparable with the head increase across the pump station. The objective function aims to minimize the operating cost which is a combination of pump flow, TH and the electric tariff. The model minimizes the multiplication of energy and tariff. If the objective function was to minimize only the energy consumption, given in terms of TH, regardless of the electrical tariff, then the resulting system operational cost, \( TH \times \text{Tari}f \), would be higher than the current objective function.

**LITERATURE REVIEW**

Pudar & Ligget (1992) proposed that leak detection in water distribution systems can be accomplished by solving an inverse problem using measurements of pressure and/or flow. The problem is formulated with equivalent orifice areas of possible leaks as the unknowns. Minimization of
the difference between measured and calculated heads produces a solution for the areas. The quality of the result depends on the number and location of the measurements. Vairavamoorthy & Lumbers (1998) suggested an optimization method to minimize leakage in water distribution systems through the most effective settings of flow reduction valves. The method showed a significant advantage compared to previously published techniques with respect to robustness and computational efficiency. A particular feature of the approach is the use of an objective function that allows minor violations in the targeted pressure requirements.

Water loss quantification, having a direct effect on system operational cost and minimizing system operational cost, was the focus of leak-related research (e.g., Arreguín-Cortes & Ochoa-Alejo 1997; Lambert 2002; Buchberger & Nadimpalli 2004) and leak detection (e.g., Hunaidi et al. 2000; Brunone & Ferrante 2001; Kapelan et al. 2005). The connection between water leakage and water quality in conjunction with the transient intrusion phenomenon was also explored (e.g., Besner et al. 2002; Karim et al. 2003). The direct link between leakage and energy was explored by Colombo & Karney (2002) who developed analytical expressions linking energy consumption to leak size and location and the influence of leaks on energy costs for simple distribution systems.

Colombo & Karney (2005) proposed a conceptual examination of the energy impact of leaks in systems with storage inclusion. Consideration of how leakage is experienced at the pump is followed by an analysis of how different leakage levels alter energy costs for a rudimentary system. EPANET 2.0 (United States Environmental Protection Agency (USEPA) 2012) simulations are used to determine system pressures, storage tank levels, energy costs, power consumption, and leakage volumes for all scenarios at five levels of leakage. Leaks increase operating costs in terms of lost water and extra energy consumption for all systems, and when a price pattern is implemented, the financial cost of energy can sometimes be traded off with actual energy consumption. Storage in a system does not guarantee lower energy use relative to direct pumping, and in some cases it may promote higher leakage due to elevated system pressures.

Giustolisi et al. (2008) proposed a steady-state network simulation model that fully integrates a classical hydraulic representation, pressure-driven demand and leakage at the pipe level.

Nicolini & Zovatto (2009) addressed the problem of optimal location and regulation of flow-controlled PRVs under two conflicting objectives: minimization of the number of valves (a surrogate for installation costs) and minimization of the total leakage in the system. This problem was solved by implementing a real-coded multi-objective genetic algorithm, with a new mutation operator allowing a noticeable increase in the performance of the algorithm.

**METHODOLOGY**

The three non-linear convex equations of headloss, water leakage, and pump energy consumption are introduced as three linear equations into an LP minimal cost model which is iteratively solved using an improved version of the iterative convex linearization method proposed by Price & Ostfeld (2013). Following an initialization solution step, the optimization model is repeatedly solved, such that prior to each solution step the coefficients of the linear equations are modified, based upon the results of the previous solution, relative to the non-linear convex equations. The optimal solution gradually converges closer to the non-linear equation results. The iterative process stops once both an optimal solution is attained and a satisfactory approximation is received. The iterative model was solved using GAMS/CLP which is an open-source LP solver ([https://projects.coin-or.org/Clp](https://projects.coin-or.org/Clp)). The model solves an annual time period at an hourly level (8,760 hr per year), an annual water balance closure constraint on the water tanks is enforced. A smaller weekly version of the model was developed (168 hr per week), in which an average week was solved for each month and the annual operation cost derived from multiplying each weekly result by the number of weeks in each month, a weekly water balance closure constraint on the water tanks was enforced.

**LINEARIZATION OF THE HAZEN–WILLIAMS HEADLOSS EQUATION**

The Hazen–Williams (HW) equation holds the non-linear relation between flow and headloss \(dH = f(Q^{1.852})\) and is
replaced by the general linear relation of \(dH = aQ + b\). Equation (1) shows the non-linear HW headloss equation along pipe \((i,j)\) and Equation (2) and Equation (3) hold the linear representation of the HW equation, subject to the coefficients \(Ah\) and \(Bh\) and pipe resistance \(R\).

**HW headloss equation, non-linear form:**

\[
dH_{ij} = 1.131 \times Q_{ij}^{1.852} \times C_{ij}^{-1.852} \times D_{ij}^{-4.87} \times 10^8 \times L_{ij} \quad \forall (i, j) \in E, \quad \forall t \in T
\]

where: \(t = \) time index (hr), \(T = \) group of all time indexes (1 ... 8,760). \(i, j = \) index of pipe origin and destination nodes. \(E = \) group of all arc indexes. \(dH_{ij} = \) headloss in pipe, variable (m). \(Q_{ij} = \) flow rate in pipe, variable (m³/hr). \(C_{ij} = \) pipe roughness coefficient, constant (-). \(D_{ij} = \) pipe diameter, constant (mm). \(L_{ij} = \) pipe length, constant (m).

\[
R_{ij} = 1.131 \times C_{ij}^{-1.852} \times D_{ij}^{-4.87} \times 10^8 \times L_{ij} \quad \forall (i, j) \in E
\]

where: \(R_{ij} = \) pipe resistance, constant.

**HW headloss equation, linear form:**

\[
dH_{ij}^l = \left( Ah_{ij}^l \times Q_{ij}^l + Bh_{ij}^l \right) \times R_{ij} \quad \forall (i, j) \in E, \quad \forall t \in T
\]

where: \(Ah_{ij}^l, Bh_{ij}^l = \) headloss linear equation coefficients, constants.

### LINEARIZATION OF THE WATER LEAKAGE EQUATION

Water leakage along a pipe \((i,j)\) is given by Equation (4) as suggested by Vairavamoorthy & Lumbers (1998). The leakage equation holds the non-linear relation between water head along the pipe and water leakage \(\dot{q} = f(H^{1.18})\) and is replaced by the general linear relation of \(\dot{q} = aH + b\). Equation (5) presents the non-linear leakage equations used in the current study and Equation (6) presents the linear version used in the optimization model. Relative to Equation (4) an artificial head gain or loss variable was added, \(AH\), representing head gain along the pipe given by a pumping station (positive value) or head loss resulting from a control valve (negative value). The artificial head variable is constantly set to zero in a pipe with no pumping station or valve. The leakage along a pipe is dependent upon the average water pressure along the pipe, pipe length and pipe leakage coefficient. The average water pressure along the pipe is defined as the average difference between the water head and ground elevation at both ends of the pipe.

In the current work, a general water leakage coefficient is used. When applied to real life water networks, the leakage coefficient may be calibrated by means of field tests and historic network data. The water leakage along a pipe is concentrated as water consumption at the pipe’s terminal node, see Equation (7). The total water leakage at a node is equal to the sum of all water leakages in all pipes leading to the node.

**Water leakage equation, non-linear form:**

\[
q_t^j = \frac{1}{2} \left( H_{ij}^t - Z_i \right) + \frac{1}{2} \left( H_{ij}^t - Z_j \right) \times L_{ij}^t \quad \forall (i, j) \in E, \quad \forall t \in T
\]

where: \(q_t^j = \) leakage at node \(j\), variable (m³/hr). \(Z_i, Z_j = \) ground elevations at nodes \(i\) and \(j\), constants (m). \(L_{ij} = \) orifice leakage coefficient, constant. \(AH_{ij} = \) artificial head gain or loss in pipe, variable (m) due to pump station TH (positive value) or PRV (negative value), \(AH_{ij} = 0\) (as constant) if there is no pumping station or valve on pipe.

**Water leakage equation, linear form:**

\[
q_t^j = \frac{N}{2} \left( H_{ij}^t + H_{ij}^t + AH_{ij}^t - Z_i - Z_j \right) \times L_{ij}^t \quad \forall (i, j) \in E, \quad \forall t \in T
\]

where: \(q_t^j = \) leakage at node \(j\), variable (m³/hr). \(Z_i, Z_j = \) ground elevations at nodes \(i\) and \(j\), constants (m). \(L_{ij} = \) orifice leakage coefficient, constant. \(AH_{ij} = \) artificial head gain or loss in pipe, variable (m) due to pump station TH (positive value) or PRV (negative value), \(AH_{ij} = 0\) (as constant) if there is no pumping station or valve on pipe.
Water balance at nodes equation with leakage:

\[ \sum_{i} Q_{ij}^{t} = \sum_{k} Q_{jk}^{t} + q_{j}^{t} \]

\[ \forall (i, j; k, l) \in E, \; \forall k \notin D, \; \forall j \notin R, \; \forall t \in T \tag{7} \]

LINEARIZATION OF THE PUMP STATION ENERGY CONSUMPTION

The model formulation presented in the paper deals with a pumping station as a whole, referring to the station’s total combined flow rate assume all the pumps have VFD. The formulation may be altered so as to address each of the pumping units or each of the pump combinations separately. In the following text, the term pumps is equivalent to the pumping units or each of the pump combinations separately.

Pump energy consumption, non-linear form:

\[ E_{p}^{t} = Q_{pj}^{t} \times TH_{p}^{t} \times \eta_{p}^{-1} \times \gamma \; \forall (p, j) \in E, \; \forall t \in T \tag{8} \]

where: \( p \) = index of pump nodes. \( E_{p}^{t} \) = pumps energy consumption, variable \((\text{kWhr})\). \( TH_{p}^{t} \) = pump’s \( TH \), variable \((\text{m})\). \( \eta_{p} \) = pump’s overall efficiency (pump, motor and mechanical losses). \( \gamma \) = unit conversion, constant \((\gamma = 0.736/270)\).

\[ TH_{p}^{t} = H_{j}^{t} - H_{p}^{t} + dH_{pj}^{t} \; \forall (p, j) \in E, \; t \in T \tag{9} \]

\[ dH_{pj}^{t} = Q_{pj}^{t} \times 2.852 \times \gamma \; \forall (p, j) \in E, \; \forall t \in T \tag{10} \]

\[ E_{p}^{t} = \sum_{j=1}^{N} \left[ \left( H_{j}^{t} - H_{p}^{t} \right) \times Q_{pj}^{t} + R_{pj} \times Q_{pj}^{t} \times 2.852 \right] \times \eta_{p}^{-1} \times \gamma \]

\[ \forall (p, j) \in E, \; \forall t \in T \tag{11} \]

where: \( H_{p} \) and \( H_{j} \) are water heads at pump suction side \((p)\) and at downstream node \((j)\), variables \((\text{m})\). \( dH_{pj} \) = pipe headloss, variable \((\text{m})\), see combination of Equation (1) and Equation (2). \( R_{pj} \) = pipe resistance, constant, Equation (2).

The variables \( H_{p} \) and \( H_{j} \) are dependent upon the sum of head losses along the pipes leading to and from the pumping station to and from nodes with known water heads. As the headloss along the upstream and downstream pipes is related to the flow \( [dH = RQ^{2.852}] \), the formulation in Equation (11) may be written generally in the form of \([E = f(Q^{2.852})]\) and is replaced by the general linear relation of \([E = aQ + b]\).

Pump energy consumption, linear form:

\[ E_{p}^{t} = \sum_{j=1}^{N} \left[ A_{pj}^{t} \times Q_{pj}^{t} + B_{pj}^{t} \right] \times \eta_{p}^{-1} \times \gamma \]

\[ \forall (p, j) \in E, \; \forall t \in T \tag{12} \]

where: \( A_{pj}^{t}, B_{pj}^{t} \) = energy consumption linear equation coefficients, constants.
**SOLUTION ALGORITHM – INITIALIZATION STEP**

In the first initialization step, the linear coefficients are set with initial values in a general form similar to that presented by Price & Ostfeld (2013). After the initial values are set, the optimization model is solved and the results of the initialization step are used to adjust the linearization coefficients to be used in the first of the following iteration steps. A modified version of the Price & Ostfeld (2013) algorithm is presented in Figure 1.

The $Ah$ and $Bh$ coefficients of the headloss linearization equation are found for a line passing through the origin $[0, 0]$ and the point $[\gamma_iQ_{ij}^{\max}, (\gamma_iQ_{ij}^{\max})^{1.852}]$ using Equation (13a), see Table 1 and Figure 2 (Line A). $Q_{ij}^{\max}$ is the maximum expected flow rate (m$^3$/hr) in pipe $ij$, dependent on pipe diameter and a maximum expected flow velocity in the pipe (m/s). A maximum flow velocity of 2 m/s was assumed. In a similar manner the leakage linear coefficients $Al$, $Bl$ are found using Equation (13b) relative to $P_{ij}^{\max}$ which represents the maximum expected water pressure in a pipe (m). A maximum pressure of 160 m (about 16 atm) was assumed, equivalent to ISO PN16 nominal hydraulic equipment rating. The 160 m is not a constraint and the pressure may exceed this value. As the model may be implemented on any water system, it seemed that 160 m was a good upper point for the linearization. Other upper pressure values were examined and the result was that there was no significant difference in solution time or quality. If, for example, the examined water system has an average pressure of say 300 m, the maximum pressure should be raised to about 400 m. For pump energy consumption, the linear coefficients $Ap$, $Bp$ are found using Equation (13c) relative to $TH_p^{\max}$ and $Q_p^{\max}$ which are the pumping station’s maximum designed TH (m) and flow rate (m$^3$/hr) defined by the user. The constants $Qi,j^{\max}$, $Pi,j^{\max}$, $TH_p^{\max}$, $Q_p^{\max}$ are not constraining upper bounds for the flow and water head variables $Q$, $H$.

A value $\gamma_1$ was found for each of the three equations to best fit the above domain linearly using least squares between the linear and the convex equation; see Table 1 for $\gamma_1$ values. As an example $[A_{H_{ij}}^{\ell} = (0.7412 \times 160)^{1.18}/(0.7412 \times 160)]$ and $[B_{H_{ij}}^{\ell} = 0]$.

**Initialization step – calculating linear coefficients ($Ah$, $Al$, $Ap$, $Bh$, $Bl$, $Bp$):**

\[
A_{H_{ij}}^{\ell} = \left(\frac{\gamma_1Q_{ij}^{\max}}{\gamma_1Q_{ij}^{\max}}\right)^{1.852} \quad ; \quad B_{H_{ij}}^{\ell} = 0 \quad \forall (i, j) \in E, \quad \forall t \in T
\]  
(13a)

\[
A_{L_{ij}}^{\ell} = \left(\frac{\gamma_1P_{ij}^{\max}}{\gamma_1P_{ij}^{\max}}\right)^{1.18} \quad ; \quad B_{L_{ij}}^{\ell} = 0 \quad \forall (i, j) \in E, \quad \forall t \in T
\]  
(13b)

\[
A_{P_{ij}}^{\ell} = \frac{TH_p^{\max} \times \gamma_1Q_p^{\max} + R_{p,i,j} \times \left(\frac{\gamma_1Q_p^{\max}}{\gamma_1Q_p^{\max}}\right)^{2.852}}{\gamma_1Q_p^{\max}} \quad ; \quad B_{P_{ij}}^{\ell} = 0 \quad \forall (p, j) \in E, \quad \forall t \in T
\]  
(13c)
where: $Q_{i,j}^{\text{max}} = \text{maximum expected flow rate in pipe}$ $[Q_{i,j}^{\text{max}} = D_{i,j}^4/4 \times v \times \gamma_h]$ $(v = 2 \, \text{m/s})$, $\gamma_h = \text{unit conversion}[\gamma_h = 0.0036]$ $(\text{s/hr} \times \text{m}^2/\text{mm}^3)$. $P_{i,j}^{\text{max}} = \text{maximum expected pressure in pipes}$ $[P_{i,j}^{\text{max}} = 160 \, \text{m}]$. $Q_{p,j}^{\text{max}} = \text{maximum nominal pump station flow rate (m}^3/\text{hr})$. $TH_{p,j}^{\text{max}} = \text{maximum nominal pump station TH (m)}$.

### SOLUTION ALGORITHM – FIXED POINT ITERATIVE LINEARIZATION

The hourly flow rates $Q_{i,j}^*$, water heads $H_{i,j}^*$ and artificial head gain $AH_{i,j}^*$ resulting from the initial solution stage are passed as input for the calculation of the linear coefficients in the first of the following iterations.

The $AH_{i,j}^*$ and $B_{i,j}^*$ coefficients are found for a line passing through two points on the convex curve, the first a constant fixed point and the second a point dependent upon the results of the previous iteration stage. For headloss, the fixed point is $[\gamma_{H}Q_{i,j}^{\text{max}}, (\gamma_{H}Q_{i,j}^{\text{max}})^{1.852}]$ and the varying point is $[Q_{i,j}^*, (Q_{i,j}^*)^{1.852}]$, using Equations (14a) and (15a) and as shown in Figure 2 (line C). The $\gamma_{H}$ value was found to give minimal least squares between the $Q^{1.852}$ curve and the two lines connecting the origin [0,0] to $[\gamma_{H}Q_{i,j}^{\text{max}}, (\gamma_{H}Q_{i,j}^{\text{max}})^{1.852}]$ and connecting $[\gamma_{H}Q_{i,j}^{\text{max}}, (\gamma_{H}Q_{i,j}^{\text{max}})^{1.852}]$ to $[Q_{i,j}^*, (Q_{i,j}^*)^{1.852}]$ as shown in Figure 2 (line B) and Table 1.

In a similar manner, the $A_{i,j}^*$ and $B_{i,j}^*$ coefficients are found for a line intersecting the point $[P_{i,j}^*, (P_{i,j}^*)^{1.18}]$ and the fixed point $[\gamma_{P}P_{i,j}^{\text{max}}, (\gamma_{P}P_{i,j}^{\text{max}})^{1.18}]$ where $[P_{i,j}^* = (H_{i,j}^* + H_{p,j}^* + AH_{i,j}^* - Z_i - Z_j)/2]$. The $A_{p,j}^*$ and $B_{p,j}^*$ coefficients are found for a line intersecting the point $[Q_{p,j}^*, (Q_{p,j}^*)^{2.852}]$ and the fixed point $[\gamma_{H}Q_{p,j}^{\text{max}}, (Q_{p,j}^*)^{2.852}] + R_{p,j}^{\text{st}}]/(\gamma_{H}Q_{p,j}^{\text{max}})^{2.852}]$ and the fixed point $[\gamma_{H}Q_{p,j}^{\text{max}}, (Q_{p,j}^*)^{2.852}] + R_{p,j}^{\text{st}}]/(\gamma_{H}Q_{p,j}^{\text{max}})^{2.852}]$. [Figure 2] Linearization of convex $f(\Phi)$.
Iteratively calculating $A_h$, $A_l$, $A_p$, $B_l$, $B_p$ coefficients:

$$A_{h(i,j)} = \left[ \frac{Q_{i,j}^* + \phi^{1.852}}{Q_{i,j}^* + \phi} \right]^{1.852} \left( \gamma_{li} Q_{li}^{max} \right)^{1.852} \forall (i, j) \in E, \forall t \in T$$

(14a)

$$A_{l(i,j)} = \left( \frac{H_{i,j}^t + H_{i,j}^* + AH_{i,j}^t - Z_i - Z_j}{2 + \phi} \right)^{1.18} \left( \gamma_{li} Q_{li}^{max} \right)^{1.18} - \gamma_{li} Q_{li}^{max} \forall (i, j) \in E, \forall t \in T$$

(14b)

$$A_{p(i,j)} = \left[ \frac{Q_{p,j}^* + \phi}{Q_{p,j}^* + \phi} \right]^{2.852} \left( \gamma_{lj} Q_{lj}^{max} \right)^{2.852} - \gamma_{lj} Q_{lj}^{max} \forall (p, j) \in E, \forall t \in T$$

(14c)

where: $Q_{i,j}^*$ = flow rate in pipe $(i,j)$ given by the result of the previous iteration stage, constant (m$^3$/hr). $H_{i,j}^t$ = water head at junction $i$, given by the result of the previous iteration stage, constant (m). $AH_{i,j}^t$ = artificial water head gain in at pipe $(i,j)$, given by the result of the previous iteration stage, constant (m). $\phi$ = constant to prevent division by zero [if denominator = 0 then $\phi = 0.01$ else $\phi = 0$].

$$B_{l(i,j)} = Q_{i,j}^* \left( \frac{0.852}{Q_{i,j}^*} \right) - A_{h(i,j)} \forall (i, j) \in E, \forall t \in T$$

(15a)

$$B_{l(i,j)} = \left( H_{i,j}^t + H_{i,j}^* + AH_{i,j}^t - Z_i - Z_j \right) / 2 \times \left( \frac{H_{i,j}^t + H_{i,j}^* + AH_{i,j}^t - Z_i - Z_j}{2 + \phi} \right)^{0.18} - A_{l(i,j)} \forall (i, j) \in E, \forall t \in T$$

(15b)

$$B_{p(i,j)} = Q_{p,j}^* \left( H_{i,j}^t - H_{i,j}^* \right) + R_{p,j} \times \left[ Q_{p,j}^* \right]^{1.852} - A_{p(i,j)} \forall (i, j) \in E, \forall t \in T$$

(15c)

SOLUTION ALGORITHM – CONVEX LINEARIZATION ERROR

Following each iteration step, a maxErr variable is calculated using Equation (16) to find the maximum relative error between the convex and linear equations for each of the solution points. While maxErr $\geq 0.5\%$ the iterative procedure repeats. The process successfully stops when maxErr $<0.5\%$ or fails if a maximum number of iteration steps is reached.

Calculating convex linearization error:

$$\text{maxErr} = \max \left( \left| \frac{Q_{i,j}^*}{Q_{i,j}^*} \right|^{1.852} - \left| A_{l(i,j)} Q_{i,j}^* + B_{l(i,j)} \right|, \right. \left| \frac{Q_{p,j}^*}{Q_{p,j}^*} \right|^{2.852} - \left| A_{p(i,j)} Q_{p,j}^* + B_{p(i,j)} \right| \right)$$

$$\forall (i, j; p, j) \in N, t \in T$$

(16)

where: maxErr = maximum linearization error, variable (%).

PREVENTING SOLUTION OSCILLATION

Price & Ostfeld (2013) reported that in some cases, in the final iteration steps, the optimal solution oscillated indefinitely between two similar solutions preventing the convergence of the linearization process, eventually ending in failure when a maximum iteration step count is reached. The oscillation occurs when for each solution, part of the hourly flow rates are switched between hours with the same electrical tariff rate. For the optimization process, the changes in annual operating costs are minor but for the linearization process each change in the flows wildly changes the linear coefficients preventing convergence. Price & Ostfeld (2013) address this problem by initiating a flow change penalty when solution oscillation is detected by the algorithm.

It is proposed that the flow change penalty be introduced in all the iterations steps following the initial iteration step. It was found that the flow change penalty
helps the solution to converge in less iteration steps; see
Equations (17) and (18).
\[
Q_t^{i,j} = Q_t^{i,j} + v_t^{i,j} - v_t^{i,j} \quad \forall (i, j) \in E, \ \forall t \in T
\]  
(17)
where: \( v_t^{i,j}, v_t^{i,j} \) = positive flow change penalty slacks, variables (m³/hr) \([v_t^{i,j}]^2 \geq 0\).

**MODEL DESCRIPTION**

Following is a simplified description of the LP model used in this study. In the initialization step, Equation (23) is not used and the variables \( v_t^{i,j}, v_t^{i,j} \) are set to zero. All the variables are continuous, and in each iteration step a linear problem is solved.

**OBJECTIVE FUNCTION**

The objective function given in Equation (18) minimizes the annual water system’s operating costs and water flow change penalty given in NIS (New Israeli Shekel). The objective function consists of three parts. The first part is the pumping station’s annual operating cost defined by Equation (11) and by the electrical tariff rates associated with the pumping station. Note that system water leakage affects the water balance at nodes resulting in an increased volume of water to be pumped by the pumping stations, directly affecting the objective function, causing the model to aim at reducing system water leakage. The second part is the source cost penalty, which acts as a finite (positive or negative) on the water supplied from defined sources. The source cost penalty may be modified to operate on selected pipes acting as a finite on water flow in different parts of the system. The third part is the flow change penalty which acts as a finite on the water flow rates as they change relative to the previous iteration steps’ resulting flow rates.

Minimize annual operation cost, water cost and minimize flow change penalty:

\[
\min \left\{ \sum_{t=1}^{T} \left( \sum_{p=1}^{P} \left( A_{p,i}^{t} + B_{p,i}^{t} \right) \times Q_{p,i}^{t} + C_{p,i}^{t} \times v_{p,i}^{t} \times \gamma \times \eta_{p,i}^{t} \right) + \right. \\
+ \sum_{i} \sum_{j} Q_{i,j}^{t} \times \text{SC}_{i,j} + \sum_{i} \sum_{j} \left( v_{i,j}^{t,1} + v_{i,j}^{t,2} \right) \times 100 \right\} 
\]  
(18)
where: \( P = \) group of all pump nodes. \( Tar_{p} = \) hourly electrical tariff charge (NIS/kWhr). \( SC_{i} = \) source cost per source node (NIS/m³).

**CONSTRAINTS**

Water balance at nodes with leakage (same as Equation (7)):

\[
\sum_{i}^{N} Q_{i,j}^{t} = \sum_{k}^{N} Q_{i,k}^{t} + q_{j}^{t} \quad \forall (i, j) \in E, \ \forall k \in E, \ \forall t \in T
\]  
(19)

Demand node water balance:

\[
\sum_{i}^{N} Q_{i,d}^{t} = D_{d}^{t} + q_{d}^{t} \quad \forall d \in D, \ \forall t \in T
\]  
(20)

Pump node input:

\[
\sum_{i}^{N} Q_{i,p}^{t} \leq Q_{p}^{\max} \quad \forall (i, p) \in E, \ \forall t \in T
\]  
(21)

Water leakage equation (same as Equation (6)):

\[
q_{j}^{t} = \sum_{i}^{N} \left[ A_{i}^{t} \left( H_{i}^{t} + H_{i}^{t} + A_{i}^{t} H_{i}^{t} - Z_{i} - Z_{i} \right) / 2 + B_{i}^{t} L_{i}^{t} \right] \times L_{i}^{t} \quad \forall (i, j) \in E, \ \forall t \in T
\]  
(22)

Flow change penalty (same as Equation (17)):

\[
Q_{i,j}^{t} = Q_{i,j}^{t-1} - v_{i,j}^{t,1} - v_{i,j}^{t,2} \quad \forall (i, j) \in E, \ \forall t \in T
\]  
(23)

Water tank hourly and annual water balance:

\[
V_{r}^{t} = V_{r}^{t-1} + \sum_{i}^{N} Q_{i,r}^{t} - \sum_{i}^{N} Q_{i,r}^{t-1} \quad \forall (i, r, j) \in E, \ \forall t \in T
\]  
(24a)

\[
V_{r}^{t-1} = V_{r}^{t-1,760} + \sum_{i}^{N} Q_{i,r}^{t-1} - \sum_{i}^{N} Q_{i,r}^{t-1} \quad \forall (i, r, j) \in E, \ \forall t \in R
\]  
(24b)
Dynamic headloss in pipes:

\[ H'_t = H'_t - (AH'_{ij} \times Q'_{ij} + Bh'_{ij}) \times R_{ij} + AH'_{ij} \]
\[ \forall (i, j) \in E, \ t \in T \]  

Tank volume to water head constraint, assuming water tank constant cross section:

\[ H'_t \times (V'_r^{\max} - V'_r^{\min}) \]
\[ = (V'_r^{\max} - H'_r^{\max} - H'_r^{\min}) + H'_r^{\min} (V'_r^{\max} - V'_r^{\min}) \]
\[ \forall r \in R, \ t \in T \]  

Pump TH:

\[ TH'_p = AH'_{ij} \forall (i, j) \in E, \ i \in P, \ t \in T; \]
\[ AH'_{ij} = 0 \forall (i, j) \in E, \ i \notin P, \ t \in T \]  

where: \( V'_r \) = water volume in water tank, variable (m³). \( V'_r^{\max}, V'_r^{\min} \) = water tank water volume maximum and minimum constants (m³). \( H'_r^{\max}, H'_r^{\min} \) = water head maximum and minimum tank head constants (m). \( H'_t \) = water head at water tank, variable (m). \( r \) = water tank index, and \( R \) = group of all water tank nodes. \( D_{ij} \) = consumer demand at node constant (m³/hr). \( d \) = demand node index, and \( D \) = group of all demand at nodes. \( TH'_p \) = total pump head variable (m). \( AH'_{ij} \) = artificial head gain or loss in pipe variable (m).

DECISION VARIABLES

\[ Q'_{ij} \] – Flow rate in pipe (i, j), (m³/hr).  
\[ 0 \leq q''_{ij}, q''_{ij} \] – Flow change slacks, (m³/hr).  
\[ 0 \leq q'_j \] – Leakage at node j, (m³/hr)  
\[ V'_r^{\min} \leq V'_r^{\max} \] – Water tank node water volume at node r, (m³)  
\[ H'_i^{\min} \leq H'_i \leq H'_i^{\max} \] – Water head at node i, (m).  
\[ AH'_{ij} \] – Artificial head gain or loss along pipe (i, j), (m).

\[ 0 \leq TH'_p \leq TH'_p^{\max} \] – Total head at pumping station p, (m).  

EXAMPLE APPLICATION

The example application is a hypothetical water distribution system, see Figure 3. The system consists of three pressure zones, each controlled by a water tank \([aR], [bR] \) and \([cR] \). The water sources to the system are from a pumping station \([aP] \) and from a well \([aW] \) supplied into pressure ‘zone a’. From pressure ‘zone a’, two pumping stations \([bP] \) and \([cP] \) pump the water to ‘zone b’ and ‘zone c’. ‘zone a’ consists of four water consumers \([aD1−4] \) while ‘zone b’ and ‘zone c’ include one consumer each \([bD], [cD] \). To challenge the algorithm in solving the flow directions within a ring network, junctions \([aL2−6] \) form two supply rings. The example application resembles water supply in a mountainous area with dense population. Such terrain emphasizes the relation between leakage, varying pump TH and source cost.

Node ground elevation above sea level is +70 m unless stated otherwise in Figure 4. All pipes have a length of 1 km and have a HW roughness coefficient of 120. Annual consumer water demand is shown in Figure 4. Monthly and daily water consumption distribution is presented in Table 2 and Table 3, the average weekly demand is calculated by dividing the monthly demand by the number of weeks in each month, hourly consumption occurs evenly between 07:00 and 16:00 (10 hr per day). Pumping nodes maximum working point (flow and TH), water tank volume and water levels (head) are shown in Figure 4. Water source nodes \([aSw] \) and \([aSp] \) are unrestricted. The water tanks are always open in all of the time steps and in all of the examined cases.

Israeli electrical tariff rates are given in units of NIS/kWhr (tariff rates used 15/02/2010). The tariff is grouped into three annual periods: summer (Jul–Aug), winter (Dec–Feb), intermediate (Mar–Jun, Sep–Nov). The tariff is grouped into three daily periods (Sun–Thu, Fri, Sat). According to the session group and the daily period, the hours of the day are grouped into three hourly periods: low charge (marked L), moderate charge (marked M), and peak charge (marked P).
Case 1: Water balance and headloss

The model was solved including water balance and hydraulic headloss constraints, minimal head at demand nodes [\(aD2\)-4] was not applied. The resulting optimal annual operation cost is 591,669 NIS/year, see Table 4.

Figure 4 presents the resulting water balance and water tank volumes in an average week of July in which maximum consumption occurs. Figure 4 shows the weekly operation of the water tanks, which fully fill from the pumping stations and water well during the low and moderate electrical tariff hours and fully empty to consumption during the peak tariff hours. On Friday, the water tanks are moderately used with little change in the water tank volumes as all hours have a low tariff rate.

Figure 5 presents detailed results including head levels on a Wednesday in July in ‘zone a’. Negative water balance values represent water output from ‘zone a’ to zonal consumption or by pumps [\(bP\)] and [\(cP\)]. Positive water balance values represent water input to ‘zone a’ by pumping station [\(aP\)] or by the water well [\(aW\)]. Priority is given to pump [\(aP\)] over pumps [\(bP\)] and [\(cP\)], which works mainly during the low electrical tariff rates while the second two are forced to work during moderate and peak tariff hours. The prioritization is given due to the greater amounts of water supplied by [\(aP\)] and the large volume of [\(aR\)] allowing greater electrical cost savings than can be achieved by prioritizing [\(bP\)] and [\(cP\)]. The water head (+90 to +100 m) in water tank [\(aR\)] is directly linked to the water volume in the tank, constrained by Equation (26). The head at demand node [\(aD3\)] is proportional to the head at tank [\(aR\)] and to the operation of [\(aP\)] and [\(aW\)] (which, when activated, raise the water head at [\(aD3\)]), and to demand rates and pumps [\(bP\)] and [\(cP\)] (which, when activated, lower the head at [\(aD3\)] due to high...
Minimal head at node \([aD3]\), which is the most distant demand node from \([aR]\), is +72.02 m at hour 15, due to peak output from ‘zone a’ and due to low water levels at \([aR]\), \([aP]\) and \([aW]\) being low.

**Figure 4** | Example application – Case 1: Resulting water balance, tank volume, July.

**Table 2** | Monthly demand multipliers for all demand nodes

<table>
<thead>
<tr>
<th>Month</th>
<th>Multiplier, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>6.3</td>
</tr>
<tr>
<td>FEB</td>
<td>6.3</td>
</tr>
<tr>
<td>MAR</td>
<td>7.0</td>
</tr>
<tr>
<td>APR</td>
<td>7.9</td>
</tr>
<tr>
<td>MAY</td>
<td>9.2</td>
</tr>
<tr>
<td>JUN</td>
<td>9.9</td>
</tr>
<tr>
<td>JUL</td>
<td>10.8</td>
</tr>
<tr>
<td>AUG</td>
<td>10.6</td>
</tr>
<tr>
<td>SEP</td>
<td>9.7</td>
</tr>
<tr>
<td>OCT</td>
<td>8.4</td>
</tr>
<tr>
<td>NOV</td>
<td>7.3</td>
</tr>
<tr>
<td>DEC</td>
<td>6.6</td>
</tr>
</tbody>
</table>

**Table 3** | Daily demand multipliers for all demand nodes

<table>
<thead>
<tr>
<th>Days</th>
<th>Multiplier, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUN</td>
<td>15.7</td>
</tr>
<tr>
<td>MON</td>
<td>14.3</td>
</tr>
<tr>
<td>TUE</td>
<td>12.9</td>
</tr>
<tr>
<td>WED</td>
<td>13.6</td>
</tr>
<tr>
<td>THR</td>
<td>17.9</td>
</tr>
<tr>
<td>FRI</td>
<td>16.4</td>
</tr>
<tr>
<td>SAT</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Dynamic head losses along the system. Minimal head at node \([aD3]\), which is the most distant demand node from \([aR]\), is +72.02 m at hour 15, due to peak output from ‘zone a’ and due to low water levels at \([aR]\), \([aP]\) and \([aW]\) being low.
non-active. The use of pump [aP] is highly preferable to the use of well [aW] due to the higher TH of the well which operates for only 1.5 hr on Wednesday compared to 14.2 hr of the pump. Note that the ground elevation at node [aD3] is +70 m.

During hour 15, the water head at consumer [aD3] is +72.02 m, supplying a water pressure of (2.02 m, gage). As such a low water pressure is not acceptable for supply, a minimum water head is introduced in case 2a.

Table 4  Annual results of examined cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Input (Mm³)</th>
<th>Demand (Mm³)</th>
<th>Leakage (Mm³)</th>
<th>maxErr (%)</th>
<th>Operation cost (M-NIS)</th>
<th>Water cost (M-NIS)</th>
<th>Total cost (M-NIS)</th>
<th>R</th>
<th>C</th>
<th>NZ</th>
<th>Iteration count</th>
<th>Solution time (m:ss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.5</td>
<td>5.5</td>
<td>-</td>
<td>0</td>
<td>0.59</td>
<td>-</td>
<td>0.59</td>
<td>220,082</td>
<td>278,210</td>
<td>681,411</td>
<td>7</td>
<td>1:37</td>
</tr>
<tr>
<td>2a</td>
<td>5.5</td>
<td>5.5</td>
<td>-</td>
<td>0</td>
<td>0.65</td>
<td>-</td>
<td>0.65</td>
<td>220,082</td>
<td>278,210</td>
<td>681,411</td>
<td>7</td>
<td>2:09</td>
</tr>
<tr>
<td>2b</td>
<td>5.5</td>
<td>5.5</td>
<td>0.8</td>
<td>0</td>
<td>0.77</td>
<td>-</td>
<td>0.77</td>
<td>220,082</td>
<td>278,210</td>
<td>681,411</td>
<td>10</td>
<td>4:49</td>
</tr>
<tr>
<td>3</td>
<td>6.3</td>
<td>5.5</td>
<td>0.8</td>
<td>0</td>
<td>0.71</td>
<td>-</td>
<td>0.71</td>
<td>268,466</td>
<td>326,594</td>
<td>778,179</td>
<td>7</td>
<td>4:06</td>
</tr>
<tr>
<td>4a</td>
<td>6.3</td>
<td>5.5</td>
<td>0.8</td>
<td>0</td>
<td>1.21</td>
<td>0.13</td>
<td>1.34</td>
<td>268,476</td>
<td>326,595</td>
<td>780,197</td>
<td>9</td>
<td>6:30</td>
</tr>
<tr>
<td>4b</td>
<td>6.4</td>
<td>5.5</td>
<td>0.9</td>
<td>0</td>
<td>0.74</td>
<td>-6.23</td>
<td>-5.49</td>
<td>268,476</td>
<td>326,595</td>
<td>780,197</td>
<td>7</td>
<td>6:01</td>
</tr>
</tbody>
</table>

*Problem size as reported by GAMS solver during solution time; R – rows (single equations), C – columns (single variables), NZ – non zeros (non zero elements).
Case 2a: Water balance and minimal water head constraints

The same model is solved as in case 1 with the addition of a minimal head constraint of $+85\text{ m} \ (85 \leq H_i)$ at demand nodes $[aD2–4]$ during hours in which demand is not zero. The resulting optimal annual operation cost is 645,008 NIS/year, see Table 4.

Figure 6 shows that the water head at demand node $[aD3]$ is maintained above $+85\text{ m}$. The water levels at water tank $[aR]$ are maintained at higher levels than in case 1, due to the activation of the well and pump during peak tariff hours 10–15. The high water levels maintain a higher water head in ‘zone a’. The operation of pumps $[bP]$ and $[cP]$ are shifted from hours 14–16 to minimize water flows along the system and reduce system head losses raising water heads in ‘zone a’. The raise in annual operating costs relative to case 1 is mainly due to the extended operation of $[aP]$ and $[aW]$ during peak electrical hours.

The resulting hourly water flows from the optimization model for the well, pumping stations and consumers of ‘zone a’ and the starting water level of $[aR]$ were entered as inputs and consumptions to an EPANET2.0 hydraulic simulation model of ‘zone a’. Figure 7 shows a weekly comparison between the resulting water heads and flows at selected nodes and pipes received from EPANET and the optimization model. The water levels of $[aR]$ are the same with a relative error of 0.005%, the difference may be explained due to numerical number rounding. The water head at node $[aD3]$ follows the same pattern with a relative error of 0.26%. The water flow at pipe $[aJ5 – aJ4]$ was selected for comparison being the center of the ring network affected most by water head calculation errors. The flow in the pipe exactly follows the results of the
optimization model, except for several hours on Thursday and Friday when a slight drift occurs with a maximum relative error of 12.6%. This error may be explained by accumulative errors along the network due to the fact that the linearization convergence stops when the relative convergence error is 0.5%.

Case 2b: Water balance, minimal and maximal water head constraints

The same model is solved as in case 2a with the addition of a maximal head constraint of $+90 \text{ m} \ (85 \leq H_i \leq 90)$ at demand nodes [aD2–4] during hours in which demand is not zero. The purpose of this case is to demonstrate that the convex linearization algorithm works with minimum and maximum constraints. The resulting optimal annual operation cost is 771,444 NIS/year, see Table 4.

Figure 8 shows that the water head at demand node [aD3] is maintained between $+85$ and $+90 \text{ m}$. The water levels at water tank [aR] are maintained at minimal water levels to lower water head in ‘zone a’. The well [aW] and pumping station [aP] are operated directly to consumption storing little water in the water tank and maintain the $+85 \text{ m}$ minimal constraint.

Case 3: Water balance, minimal water head constraint and single pipe leakage

The same model is solved as in case 2a with the addition of water leakage in pipe [aJ1] – [aJ2] relative to average
pressure along the pipe, see Equation (22). A leakage coefficient of \((L_{K_{aJ1,aJ2}} = 1.0)\) was used. A large leakage coefficient was chosen to clearly demonstrate the algorithm implementation. When examining an existing water system, the leakage coefficient may be calibrated according to existing water system operational data and field tests. Resulting optimal annual operation cost is 710,543 NIS/year, see Table 4.

Figure 9 shows the hourly water leakage from pipe \([aJ1] – [aJ2]\). To minimize the water leakage, the operation of pumps \([bP]\) and \([cP]\) was grouped together and concentrated during hours in which pump \([aP]\) is active, mainly at high water levels in water tank \([aR]\).

The activation of pump \([aP]\) raises the water head at node \([aJ1]\) and substantially in nodes downstream. The activation of \([bP]\) and \([cP]\) increases water consumption from ‘zone a’ raising flow rates initiating at \([aP]\) and \([aR]\) causing high dynamic headloss in the pipes of ‘zone a’ lowering pressures along pipe \([aJ1] – [aJ2]\) reducing water leakage. At all times, the water head at node \([aD3]\) is maintained at or above +85 m. The operating cost in case 3 is higher than in case 2a mainly due to the fact that pump \([aP]\) and well \([aW]\) have to pump additional water to compensate for leakage. Annual network consumer demand is 5.5 Mm³/year while the annual input pumped by \([aP]\) and \([aW]\) is 6.3 Mm³/year, the difference is due to the leakage.

Case 4a: Water balance, minimal water head, leakage and positive source cost

The same model is solved as in case 3 with the addition of a source supply fine (tax) at \([aSp]\), \((SC_{aSp} = 1.0 \text{ NIS/m}^3)\). The resulting optimal annual operation cost is 1,333,688 NIS/year (of which 125,189 NIS/year is source water cost and 1,208,500 NIS/year is electricity cost), see Table 4.
Figure 10 shows the dramatic change in prioritization of the water well \([aW]\) over the pump \([aP]\); the well pumps 6.16 Mm\(^3\)/year and the pump only 0.14 Mm\(^3\)/year. Despite the high energy costs of the well, when considering the additional supply cost at the pump, the overall water cost from the well is cheaper than that of the pump. Because the hourly flow rate from the well is less than that of the pump, the well is operated for almost 23 hr per day compared to about 17 hr per day of the pump in case 3. The almost constant operation of the well causes a sharper volume change in the water tank, mainly due to changes in output from ‘zone a’ and not due to tariff cost operating considerations. The electrical operating cost in case 4a is higher than in case 3 due to the high TH of the well in case 4a relative to that of the pump in case 3 resulting in a higher energy consumption, and due to operating in peak tariff hours.

**Case 4b: Water balance, minimal water head, leakage and negative source cost**

The same model is solved as in case 3a with the addition of a negative source supply fine (negative tax) at \([aSp]\), \(S_{C_{aSp}} = -1.0\) NIS/m\(^3\). Resulting optimal annual operation cost is -5,491,619 NIS/year (of which -6,234,259 NIS/year is source water cost and 742,640 NIS/year is electricity cost), see Table 4. The negative annual operation cost means the system is profitable. The resulting negative operation cost system is an extreme case relative to real world system operation, and it emphasizes the example and algorithm.

The meaning of the negative fine at source \([aSp]\) is that the system is rewarded for water taken from the source. The negative fine causes the system to maximize the volume of water taken from the source in order to maximize profit even at the expense of higher electrical operating costs.
Figure 11 demonstrates the operation of the system when the system is rewarded for taking water from source \([aSp]\). The system operation in case 4b is completely opposite to the operation in cases 4a and 5. The volume of water tank \([aR]\) is kept at a maximum level to raise the water heads in ‘zone a’ resulting in higher water leakage. The leakage is maximized to increase overall water output volume from the system and maximize the water input volume from the rewarded source \([aSp]\). Well \([aW]\) is seldom used and pump \([aP]\) which is linked to source \([aSp]\) is preferred. The operation of pumps \([bP]\) and \([cP]\) is spread throughout the day in order to lower hourly water consumption from ‘zone a’ allowing the water tank \([aR]\) to be kept at peak level. Note that although operating pump \([aP]\) is profitable, in hours 7–9 and 13–15 the well \([aW]\) is operated to maintain water heads in ‘zone a’, due to low water levels in water tank \([aR]\) and the minimum head constraint of consumers \([aD2–4]\). Water head at consumer \([aD3]\) is maintained above +85 m at all hours according to the constraint.

**CONCLUSIONS**

A minimal cost optimal operation iterative LP model, including the non-linear HW headloss, water leakage, variable pump energy consumption and linear source cost, was demonstrated on an example water supply system. Minimum and maximum water head constraints at selected demand nodes cause the shifting of pump operation so as to maintain a minimum and maximum water level in the water tanks to meet the head constraints. Also, hourly flow rates were reduced to lower dynamic headloss and increase water heads at constrained nodes. Introducing pressure-dependent water leakage along pipes causes the grouping of pump stations’ operation...
throughout the day such as to create maximum hourly flow rates creating higher dynamic headloss, lowering water heads and by that reducing leakage, while maintaining minimum head constraints at selected nodes. Last, a source cost penalty was introduced at a pumping station with lowest TH (energy cost). The source cost penalty caused the system to prefer the use of a water well, with higher energy consumption which was previously seldom used, and abandon the use of the pumping station. When a negative penalty at the pumping station was introduced, encouraging the use of the pumping station even more, the pumping station was operated such as to reduce flow rates in the system in order to lower dynamic headloss and create higher water head and increased water leakage, thus increasing the overall amount of water supplied from the pumping station and maximizing the reward. The penalty may be applied on any of the pipes in the system. The intuitive result is that the model will aim to lower the amounts of water supplied through a fined pipe and maximize the amounts of water supplied through pipes with a rewarding water fine.

The algorithm was successfully demonstrated with ‘greater than’ and ‘less than’ constraints. The algorithm successfully terminates with an optimal operation solution, following 7–10 iteration steps with an average total solution time of about 2 minutes. Currently, fixed speed pumps are not handled by the model as this would transform the original problem from a smooth NLP into a discrete MIP. The method may be applied to a pumping station supplying water to consumers via a water tank or directly to the consumer using a VFD as the pumping station. The method is also valid for low lift pumps, where the water level changes at the destination tank are comparable with the head increase across the pump station. The objective function aims to minimize the operating cost which is a combination of pump flow, TH and

Figure 11 | Example application – Case 4b: Resulting flow, node head, tank volume, July, Wednesday, ‘zone a’.

Downloaded from https://iwaponline.com/jh/article-pdf/15/4/1203/387129/1203.pdf by guest
the electric tariff. The model minimizes the multiplication of energy and tariff. If the objective function was to minimize only the energy consumption, given in terms of TH, regardless of the electrical tariff, then the resulting system operational cost, TH × Tariff, would be higher than the current objective function. Further research may include incorporating varying pump efficiency curves and discrete pump station operation into the presented iterative algorithm.

REFERENCES


First received 25 July 2012; accepted in revised form 24 January 2013. Available online 21 February 2013