Di-Neutron Correlation in Soft Octupole Excitations of Neutron-Rich Ni Isotopes beyond \( N = 50 \)

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We investigate the low-lying octupole response of neutron-rich Ni isotopes beyond the \( N = 50 \) shell closure using the Skyrme-Hartree-Fock-Bogoliubov mean fields and the continuum quasi-particle random phase approximation. Performing detailed numerical analyses employing the Skyrme parameter set SLy4 and a density-dependent delta interaction of the mixed type, we show that a neutron mode emerges above the neutron separation energy as a consequence of the weak binding of neutrons and that it strongly influences the di-neutron correlation.

Subject Index: 211, 213

\section{Introduction}

The di-neutron correlation, a spatial pair correlation with a small correlation length among neutrons, has been one of the central themes of the physics of two-neutron halo nuclei such as \(^{11}\)Li\(^1\)–\(^7\)\) Although an affirmative experimental signature of the neutron spatial correlation in \(^{11}\)Li has been obtained recently\(^8\), many theoretical predictions have been accumulated, concerning not only two-neutron halo nuclei\(^1\)–\(^3\),\(^9\)–\(^14\) but also other neutron-rich systems, including predictions on the surface area of medium- and heavy-mass neutron-rich nuclei\(^15\),\(^16\) and dilute neutron matter\(^17\)–\(^19\). Indeed the analysis of the spatial structure of the neutron Cooper pair in dilute matter\(^17\)–\(^19\) has revealed a mechanism by which the di-neutron correlation may emerge generically. Namely, the spatial di-neutron correlation originates from a strong coupling feature of the neutron pair correlation, which becomes significant at low densities because of the strong momentum dependence of the attractive nuclear force in the \(1S\) channel. Furthermore, the induced pairing interaction caused by the exchange of surface phonons is claimed to provide an additional contribution to the possible enhancement of the spatial correlation in finite nuclei\(^20\).

The influence of the di-neutron correlation on nuclear structure may be multi-fold. Namely, it may influence not only the ground state but also various modes of excitations and dynamics. Attention, however, has been focused so far mostly on soft dipole excitation in two-neutron halo nuclei\(^1\)–\(^8\),\(^12\) and recently, in medium- and heavy-mass neutron-rich nuclei\(^15\),\(^18\). Concerning the latter case, we have shown in

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our previous study\textsuperscript{15}) the strong influence of the di-neutron correlation in the soft dipole excitation of proton semi magic neutron-rich Ca and Ni nuclei. Considering the possible generality of the di-neutron correlation, it is expected that this correlation may also emerge in other multipole modes of excitation. In this paper, we shall investigate octupole modes of excitation because our preliminary study\textsuperscript{18}) showed that the di-neutron correlation is not as strong in the quadrupole response as in the dipole case.

Only a few investigations of low-lying octupole excitations in neutron-rich nuclei have been carried out whereas in stable nuclei the low-lying $3^{-}$ state with the character of octupole surface vibration is well established.\textsuperscript{21,22} The experimental properties of the low-lying octupole modes in neutron-rich nuclei are little known, with exceptions such as $^{20}$O.\textsuperscript{23} On the theoretical side, it is predicted\textsuperscript{24–26} that neutron halo nuclei exhibit a neutron mode with large octupole strength in the continuum region above the neutron threshold, resulting from transitions from weakly bound orbits to continuum orbits, and thus having a completely different character from the surface vibration. Analyses based on the random phase approximation (RPA),\textsuperscript{27–30} which can describe both the continuum states and the collectivity, have shown the coexistence of the collective surface vibration and neutron continuum strength near the threshold in medium-mass neutron-rich nuclei. Note however that models of Refs. 27–30 do not include the pair correlations, and analyses are limited to nuclei with doubly closed shell configurations, such as $^{60}$Ca, $^{28}$O and $^{68,78}$Ni. The quasi-particle RPA including the pair correlation effect is employed in Ref. 23) to describe oxygen isotopes $^{18–24}$O. In this study, however, the Hartree-Fock+BCS approximation is adopted instead of the Hartree-Fock-Bogoliubov (HFB) scheme, and the effects of the pair correlation are not discussed in detail. In contrast to these preceding works, in the present paper we focus on the roles of the pair correlation and the possibility of the di-neutron correlation, especially in the low-lying octupole correlation. For this purpose we use the continuum quasi-particle RPA (the continuum QRPA),\textsuperscript{15,18,31} which can describe the continuum, the pairing and the collectivities in neutron-rich medium-mass nuclei. We choose nickel isotopes $^{80–86}$Ni for numerical analysis, and perform a detailed investigation of $^{84}$Ni, which is chosen as a representative example. A preliminary report of this work can be seen in Ref. 32).

§2. Skyrme-HFB plus continuum QRPA method

In the present study we assume the spherical symmetry of the ground state because we analyze semi magic nickel isotopes. We first construct the spherically symmetric ground state and the associated self-consistent mean fields by the coordinate-space Skyrme-Hartree-Fock-Bogoliubov (Skyrme-HFB) method,\textsuperscript{33,34} with which we can accurately describe the spatially extended wave functions of weakly bound quasiparticle orbits. This is in contrast to our previous study,\textsuperscript{15} where a Woods-Saxon potential was adopted. We employ the Skyrme force parameter set SLy4,\textsuperscript{35} which has been extensively used for studies of neutron-rich nuclei.\textsuperscript{36–43} We shall also use another parameter set SkM$^{+43,44}$ for comparison. For the effective pair interaction,
we use the density-dependent delta-type interaction (DDDI),\textsuperscript{(2),36),43)} which is given by
\[
v_{\text{pair}}(r, r') = \frac{1}{2} V_0 \left( 1 - P_\sigma \right) \left( 1 - \eta \frac{\rho(r)}{\rho_0} \right) \delta(r - r').
\] (2.1)

We choose the so-called mixed-type pairing,\textsuperscript{38)} i.e., $\eta = 1/2$, $\rho_0 = 0.16$ fm$^{-3}$, for the density dependence, and we determine the strength of the pair interaction $V_0$ in the same way as that adopted in Ref. 15). Namely, we obtained $V_0 = -315$ MeV fm$^3$ for nickel isotopes so that the average pairing gap for neutrons is in overall agreement with the experimental odd-even mass difference evaluated at odd values of $N$ using the three-point formula\textsuperscript{45)} (cf. Fig. 1).

We describe excitation modes using the continuum QRPA. The continuum QRPA formalism is essentially the same as those in Refs. 31) and 15), except that we here formulate it on the basis of the Skyrme-HFB mean fields and different residual interactions. For the residual interaction in the pairing channel, we adopt the one derived from the functional derivatives of the pairing energy functional, as in our previous study.\textsuperscript{15)} We neglect the rearrangement term arising from the density dependence of the pairing interaction. Concerning the residual interaction in the particle-hole channel, we use the same Skyrme interaction as that employed in constructing the ground state, but we apply a Landau-Migdal approximation to the Skyrme interaction\textsuperscript{46)–48)} in order to make the systematic numerical calculations feasible. In practice, the particle-hole interaction is given by $v_{\text{ph}}(r - r') = \{(F_0/N_0)(r) + (F'_0/N_0)(r) \tau \cdot \tau'\} \delta(r - r')$, where $F_0$, $F'_0$ and $N_0$ are the Landau-Migdal parameters\textsuperscript{49)–51)} and the associated normalization factor, evaluated for the Skyrme interaction, respectively. Their expressions are given, for example, in

Fig. 1. The average neutron pairing gap of nickel isotopes. The solid line shows the values obtained in the present HFB calculation using SLy4 and the mixed-type DDDI. The dashed line shows the experimental values extracted from the nuclear masses evaluated at odd values of $N$ using the three-point formula.
Ref. 52) and 53). The Landau-Migdal parameters are usually defined for symmetric nuclear matter, but we treat the Fermi momentum \( k_F \) in the expressions as a local quantity \( k_F (r) = \left(3\pi^2 \rho(r)/2\right)^{1/3} \) related to the nucleon density \( \rho(r) \).\(^{53}\) Thus, the force strength is density-dependent, and hence depends on the spatial coordinate \( r \).

The above approximations to the residual interaction may be justified for a multipole response with a natural parity as far as the qualitative features are concerned (see Ref. 54) for the dipole and quadrupole responses). This is not the case for unnatural parity excitations since we neglect the spin-spin interactions. Also our model has the limitations common to all the HFB plus QRPA approaches, such as low single-particle level density near the Fermi energy due to the low effective mass.

Given the HFB mean fields and the residual interactions, we solve the linear response equation

\[
\begin{pmatrix}
\delta \rho_{qL}(r, \omega) \\
\delta \tilde{\rho}_{+,qL}(r, \omega) \\
\delta \tilde{\rho}_{-,qL}(r, \omega)
\end{pmatrix} = \int_0^{r_{\text{max}}} dr' \begin{pmatrix}
R_{0,qL}^{\alpha\beta}(r, r', \omega) \\
\kappa_{\text{ph}}(r') \delta \rho_{qL}(r', \omega)/r'^2 + v_{qL}^{\text{ext}}(r') \\
\kappa_{\text{pair}}(r') \delta \tilde{\rho}_{+,qL}(r', \omega)/r'^2
\end{pmatrix}
\times \begin{pmatrix}
\sum_q \kappa_{\text{ph}}^q(r') \delta \rho_{qL}(r', \omega)/r'^2 + v_{qL}^{\text{ext}}(r') \\
\kappa_{\text{pair}}(r') \delta \tilde{\rho}_{+,qL}(r', \omega)/r'^2 \\
-\kappa_{\text{pair}}(r') \delta \tilde{\rho}_{-,qL}(r', \omega)/r'^2
\end{pmatrix}
\tag{2.2}
\]

to describe the correlated multipole response of a nucleus to an external field. Here \( \delta \rho_{qL}(r, \omega) \), \( \delta \tilde{\rho}_{+,qL}(r, \omega) \) and \( \delta \tilde{\rho}_{-,qL}(r, \omega) \) are the responses for the normal and abnormal densities with multipolarity \( L \), and \( \kappa_{\text{ph}} \) and \( \kappa_{\text{pair}} \) are related to the interaction strengths \( (F_0/N_0)(r), (F_0'/N_0')(r) \) and \( V_0(1 - \eta \rho(r)/\rho_0) \). Note that we construct the response function \( R_{0,qL}^{\alpha\beta}(r, r', \omega) \) in terms of the products of two single-quasi-particle HFB Green’s functions summed over not only the discrete quasi-particle states but also the continuum quasi-particle states.\(^{15,31} \) This is possible because we use the exact HFB Green’s function\(^{56} \) consisting of the regular and outgoing wave solutions of the HFB equation. We consider the external field

\[
V^{\text{ext}}(r) = \sum_i r_i^3 Y_{30}(\hat{r}_i) \quad \text{and} \quad \sum_i \frac{1 \pm \tau_3 i}{2} (r_i^3 Y_{30}(\hat{r}_i),
\tag{2.3}
\]

for the isoscalar, neutron and proton strength functions, \( dB(\lambda L)/dE = \sum_i B(\lambda L; 0_{gs} \rightarrow 3_i^-) \delta(E - E_i) (\lambda L = IS3, n3 \text{ and } p3, \text{ respectively}), \) which can be evaluated as \( dB(\lambda L)/dE = -\frac{2}{\pi} \text{Im} \int \sum_q dv_{\text{ext},q}(r) \delta \rho_{qL}(r, \omega) \) using the solution \( \delta \rho_{qL}(r, \omega) \) of Eq. (2.2).

Numerical details are as follows. The HFB equation is solved using the radial coordinate in a spherical box with size \( r = 0 - r_{\text{max}} \). The Skyrme-HFB code is our original code, whose numerical procedures essentially follow those of Ref. 33). We solve the radial HFB equation using the Runge-Kutta method instead of the Numerov method adopted in Refs. 33) and 55) since we need a consistent evaluation of derivatives of the quasi-particle wave functions to construct the Green’s function used in the continuum QRPA calculation. We have verified that the results of our Skyrme-HFB code agree with those produced by the HFBRAD code.\(^{55} \) The continuum QRPA part is based on our previous version,\(^{15} \) which employed
the Woods-Saxon potential, but here we replace the Woods-Saxon potential by the Skyrme-HFB mean fields. Also we implement the Landau-Migdal approximation of the Skyrme interaction as the residual interaction in the particle-hole channel. As the box size we choose $r_{\text{max}} = 22 \text{ fm}$, and we apply an equidistant discretization with $\Delta r = 0.2 \text{ fm}$. Concerning the cutoff of the quasi-particle waves, we set $l_{\text{max}} = 17\hbar$ for the single-particle partial waves $l_j$, and $E_{\text{max}} = 60 \text{ MeV}$ with respect to the quasi-particle energies. This cutoff energy is a standard choice adopted in many Skyrme-HFB calculations, but is slightly larger than that adopted in our previous Woods-Saxon calculation. Concerning the orbital angular momentum cutoff, we find that the ground state and the octupole strength function already have good convergence at $l = 12\hbar$, but to evaluate the transition densities we need larger values of $l$. Because we adopt the Landau-Migdal approximation, the consistency between the ground state and the excited states is partly broken. In order to minimize the effects of the breaking of self-consistency, we renormalize the strength of the particle-hole residual interaction as $v_{\text{ph}} \rightarrow f \times v_{\text{ph}}$, i.e., by multiplying by a numerical factor $f$ so that the spurious center-of-mass mode in the isoscalar dipole excitation has zero excitation energy. We also calculate the dipole response for comparison. In this case we evaluate the $B(E1)$ strength function using the dipole field $V_{\text{ext}}(r) = \frac{1}{\hbar} \sum_{i=1}^{Z} r_i Y_{10}(\hat{r}_i) - \frac{N}{\hbar} \sum_{i=1}^{N} r_i Y_{10}(\hat{r}_i)$, in which the center-of-mass motion is explicitly removed to circumvent the self-consistency problem.

§3. Numerical analysis

3.1. Octupole strength functions in $^{84}$Ni

We first discuss a representative example, $^{84}$Ni, in order to clarify the basic features of low-lying octupole excitation.

In Fig. 2, we show the isoscalar octupole strength in $^{84}$Ni. Here the strength function is calculated with smoothing parameter $\epsilon = 0.2 \text{ MeV}$, which is introduced in the linear response equation as the imaginary part of the excitation frequency $\omega + i\epsilon$. This means that the strength function is folded with a Lorentzian function with FWHM $= 2\epsilon = 0.4 \text{ MeV}$. It can be seen that there are essentially two groups in the strength distribution: one around $E = 26 - 32 \text{ MeV}$ and the low-lying distributions below $E \sim 10 \text{ MeV}$. We regard the high-energy group as the $3\hbar\omega$ high-frequency vibrational mode. In the low-energy group, the sharp peak at $E \approx 4.15 \text{ MeV}$ is the most prominent, but another structure can also be seen. We focus on the low-energy group in the following discussion.

In Fig. 3 we show a magnification of the isoscalar strength function in the low-lying region $E = 0 - 8 \text{ MeV}$, together with the neutron and proton octupole strength functions. The smoothing parameter is chosen to be a smaller value, $\epsilon = 0.05 \text{ MeV}$, in this case (and also in most of the following calculations unless mentioned explicitly). It is clear in Fig. 3 that the low-lying strength consists of two structures. Apart from the sharp peak at $E = 4.16 \text{ MeV}$, a broad bump exists, which emerges above the one-neutron separation energy (the one- and two-neutron separation energies are $E_{1n,2n} = 1.86$ and $2.37 \text{ MeV}$, respectively, as indicated with arrows). Although the
two structures overlap in the same energy region, their characters can be clearly distinguished as follows. Firstly, the broad bump does not form a well-defined peak, and we consider it as a type of continuum mode that involves two-quasi-particle configurations consisting of continuum neutron orbits. Essentially, it carries only neutron strength. Secondly, the sharp peak has a small width even though it is located above the neutron separation energy. It can be regarded as a narrow resonance. It can also be distinguished from the neutron mode by the fact that it carries sizable proton strength.

In order to provide more characterizations, we examined how the residual interactions influence these modes. Namely, we performed three calculations where either or both of the residual pairing and particle-hole interactions are neglected. Results are shown in Fig. 4. The sharp peak disappears when the residual particle-hole interaction is neglected, and it is influenced rather weakly by the residual pairing interaction. Thus the main origin of the sharp-peak mode is correlation due to the residual particle-hole interaction. This suggests that this mode may be the surface vibrational mode consisting of low-energy $1-h\omega$ particle-hole transitions.\textsuperscript{21} In contrast, the broad neutron mode exists even when the residual particle-hole interaction is neglected; it has a different origin.

It is useful to evaluate the integrated sums of octupole strengths associated with the two modes. It is, however, not easy to evaluate them separately since the sharp-peak mode and the broad neutron mode overlap in the same energy region. Nevertheless, we estimate them in the following manner. For the broad neutron mode we integrate the strength functions in an energy region above the one-neutron
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Fig. 3. The low-energy part of the calculated octupole strengths in $^{84}$Ni. The isoscalar, neutron and proton strengths are plotted with the solid, dashed and dotted curves, respectively. The smoothing parameter is $\epsilon = 0.05$ MeV. See also the caption of Fig. 2.

The separation energy, $E_{1n} < E < E_{1n} + 1.5$ MeV, with an interval of 1.5 MeV, where the neutron strength dominates and the collective vibrational mode barely overlaps. The boundary energies are $E_{1n} = 1.86$ and $E_{1n} + 1.5 = 3.36$ MeV in the case of $^{84}$Ni. Since the choice of the energy interval, 1.5 MeV, is rather arbitrary and probably too small to cover the whole strength of the broad neutron mode, we expect that this may be an underestimate by a factor of up to about 2. For the strength of the surface vibrational mode, we define an energy interval, $E_1 < E < E_2$, where the strengths are integrated by noting that this mode carries the proton strength, which is a character clearly distinguishable from the broad neutron mode. In practice, we define the boundaries $E_1$ and $E_2$ to be where the proton strength function is 1.0% of the value at the peak energy $E_{\text{peak}}$. In $^{84}$Ni, $E_1$ and $E_2$ are 3.50 and 5.00 MeV, respectively, and there is no overlap between the two energy intervals. In Table I, we list the integrated isoscalar octupole strengths of the broad neutron mode and the surface vibrational mode in $^{80-86}$Ni. The obtained isoscalar octupole strength for the broad neutron mode is $5.85 \times 10^4$ fm$^6$ in $^{84}$Ni. Note that the energy-weighted sum of this strength is 1.7% of the classical isoscalar octupole sum rule value$^{21,57}$,

$$S_{\text{EW}} = \frac{147}{4\pi} \frac{h^2}{2m} \left( N \langle r^4 \rangle_n + Z \langle r^4 \rangle_p \right).$$

The isoscalar strength of the surface vibrational mode is $6.74 \times 10^5$ fm$^6$ in the same nucleus, and it accounts for 29% of the classical sum rule value. The strength of the broad neutron mode is smaller than that of the surface vibrational mode by about a factor of ten.

3.2. Transition densities of low-lying octupole modes in $^{84}$Ni

The natures of the sharp-peak mode and the broad neutron mode become more evident by investigating their transition densities. We here analyze three types of...
Fig. 4. The effects of the residual interactions on the low-energy part of the isoscalar octupole strength in $^{84}$Ni, shown in Fig. 3. The solid curve shows the full calculation while the dashed and dotted curves show the results neglecting the residual pairing and particle-hole interactions, respectively. The dot-dashed curve shows the unperturbed strength when both of the residual interactions are neglected.

transition densities,
\[ \rho_{iq}^{\text{tr}}(r) = \langle \Phi_i | \sum_\sigma \psi_q^\dagger(r \sigma) \psi_q(r \sigma) | \Phi_0 \rangle = Y_{LM}^* \rho_{iqL}^{\text{tr}}(r), \] (3.1)
\[ P_{iq}^{\text{pp}}(r) = \langle \Phi_i | \psi_q^\dagger(r \uparrow) \psi_q^\dagger(r \downarrow) | \Phi_0 \rangle = Y_{LM}^* P_{iqL}^{\text{pp}}(r), \] (3.2)
\[ P_{iq}^{\text{hh}}(r) = \langle \Phi_i | \psi_q(r \downarrow) \psi_q(r \uparrow) | \Phi_0 \rangle = Y_{LM}^* P_{iqL}^{\text{hh}}(r), \] (3.3)

where $\rho_{iq}^{\text{tr}}(r)$ is the usual particle-hole transition density, while $P_{iq}^{\text{pp}}(r)$ and $P_{iq}^{\text{hh}}(r)$ are the transition densities for particle-pair and hole-pair, respectively, for either neutrons or protons ($q = n, p$). They are evaluated as
\[ \rho_{iqL}^{\text{tr}}(r) = -\frac{C}{\pi r^2} \text{Im} \delta \rho_{qL}(r, \omega_i), \] (3.4)
\[ P_{iqL}^{\text{pp}}(r) = \frac{C}{\pi r^2} \text{Im}(\delta \bar{\rho}_{+,qL}(r, \omega_i) - \delta \bar{\rho}_{-,qL}(r, \omega_i)), \] (3.5)
\[ P_{iqL}^{\text{hh}}(r) = \frac{C}{\pi r^2} \text{Im}(\delta \bar{\rho}_{+,qL}(r, \omega_i) + \delta \bar{\rho}_{-,qL}(r, \omega_i)), \] (3.6)
in terms of the solutions of the linear response equation (2.2). Here $C$ is a normalization constant, which is fixed so that the transition amplitude $M_{iqL} = \int r_{qL}(r) r_{iqL}^\text{tr}(r) x r^2 dr$ gives the integrated isoscalar octupole strength of the mode under consideration by the standard definition $B(\text{IS3}) = 7/2 \sum_q |M_{iqL}|^2$.

In Fig. 5, we show the transition densities associated with the surface vibrational mode. We evaluate them at $E = 4.15$ MeV, approximately at the peak energy. It can
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Fig. 5. The transition densities evaluated at $E = 4.15$ MeV for the octupole response of $^{84}$Ni (cf. Fig. 4). The upper panel shows the particle-hole transition density $r^2 \rho^{tr}_{iqL}(r)$ weighted with the volume element $r^2$. The neutron and proton amplitudes are plotted with the solid and dashed curves, respectively. The particle-pair transition density $r^2 P^{pp}_{iqL}(r)$ and the hole-pair transition density $r^2 P^{hh}_{iqL}(r)$ of neutrons are shown in the bottom panel with the solid and dotted curves, respectively. $P^{pp}_{iqL}(r)$ and $P^{hh}_{iqL}(r)$ for protons (dashed curves) are exactly zero.

It can also be seen that the particle-hole transition densities $\rho^{tr}_{iqL}(r)$ of both neutrons and protons exhibit large and in-phase amplitudes near the nuclear surface (the calculated matter rms radius of $^{84}$Ni is 4.28 fm). We thus confirm that the mode is typical of the surface vibration in which neutrons and protons provide coherent contribution. It can also be seen that the amplitude of the particle-hole transition density $\rho^{tr}_{iqL}(r)$ is significantly larger than those of the particle-pair transition density $P^{pp}_{iqL}(r)$ and the hole-pair transition density $P^{hh}_{iqL}(r)$ for both neutrons and protons. ($P^{pp}_{iqL}(r) = P^{hh}_{iqL}(r) = 0$ for protons is a trivial consequence of the zero proton pairing gap $\Delta_p = 0$.) The dominance of the particle-hole amplitude is consistent with the observation that the mode is generated by the residual particle-hole interaction (cf. §3.1).

Figure 6 shows the transition densities of the broad neutron mode, evaluated at $E = 3.0$ MeV, which have different characters from those of the surface vibrational mode. We observe two distinct features. Firstly, the particle-pair transition density
Fig. 6. The same as Fig. 5 but for the transition densities evaluated at $E = 3.0$ MeV.

$P_{iqL}(r)$ of neutrons is significantly larger than the neutron particle-hole transition density $\rho_{iqL}^r(r)$ in the external region of the nucleus. The ratio between the two transition densities is approximately 3 at $r = 10$ fm and $\sim 10$ at $r = 15$ fm. This indicates that the mode is characterized, especially in the external region, by the motion of neutron pairs rather than by particle-hole excitations. In other words, the neutron pair correlation is the main character of this mode. Secondly, both the particle-hole and particle-pair transition densities of neutrons exhibit a very long tail extending to $r \gtrsim 20$ fm, especially in the particle-pair transition density. This indicates that neutrons in weakly bound and continuum orbits participate in forming this mode. In addition to these two features, we also observe that the proton amplitude of the particle-hole transition density is considerably smaller than that of neutrons. This is in accordance with the dominance of the neutron strength already observed in the previous subsection.

We emphasize that the residual pairing interaction acting in the the QRPA equation brings the correlation to this mode, and that the pairing mean field alone is not sufficient. This is seen in the difference between the solid and dashed lines in Fig. 7, where the dynamical pairing effect, i.e., the RPA correlation due to the residual pairing interaction, is either included or neglected. It can be seen that the dynamical pairing effect enhances the particle-pair transition density by a factor of...
Fig. 7. The neutron transition densities, $r^2\rho^{tr}_{iqL}(r)$ and $r^2P^{pp}_{iqL}(r)$, of the neutron mode, evaluated at $E = 3.00$ MeV. Here the effects of the residual interactions are shown. The dotted curve is the case where the residual particle-hole interaction is neglected, while the dashed curve is the case where the dynamical pairing effect, i.e., the residual pairing interaction, is neglected. The full calculation is also shown with the solid curve for comparison. See also the caption of Fig. 5.

\[ \sim 2. \] This indicates a significant configuration-mixing effect originating from the residual pairing interaction. The important role of the pair correlation can be seen even when the residual particle-hole interaction is neglected (the dotted curve in Fig. 7).

The large dynamical pairing effect is analyzed in more detail. In Fig. 8 we show how the transition densities of the neutron mode change if we include only part of the neutron single-particle orbits with lower orbital angular momenta $l$, i.e., only orbits up to a smaller cutoff $l_{\text{cut}}$. This is the same analysis that we performed for the soft dipole mode \(^{15}\). It can be seen that the convergence of the particle-pair transition density $P^{pp}_{iqL}(r)$ of neutrons with respect to $l$ is slow, and orbits with large $l$ contribute coherently to produce the dynamical pairing effect. There is a sizable contribution even around $l \sim l_{\text{max}} = 17$. Note that the precise treatment of the continuum states, guaranteed in the continuum QRPA approach, is essential to describe the correlation, since high-$l$ orbits with $l > 4$ are all continuum states.

On the basis of the above two features, i.e., the large effect of the dynamical pairing and the high-$l$ contribution, we present a parallel argument to the soft dipole
Fig. 8. The dependence of the neutron transition densities of the neutron mode, evaluated at $E = 3.0$ MeV, for different values of $l_{\text{cut}} = 7, 9, \ldots, 17h$. The upper and lower panels show the particle-hole and particle-pair transition densities, $r^2 \rho_{\text{tr}}(r)$ and $r^2 P_{\text{pp}}(r)$, respectively. The transition densities obtained by neglecting the dynamical pairing effect are also plotted with the thin solid curve for comparison.

case, in which a neutron pair moving in the neutron mode exhibits a spatial correlation at small relative distances between the paired neutrons. To this end, let us first recall that the spatial correlation in the HFB ground state at small relative distances of $r < 2 - 3$ fm originates from coherent configuration mixing involving orbits with large angular orbital momenta (up to $l \sim 8$ in the $^{84}$Ni case) and both parities. This is easily understood in terms of the simple angular momentum coupling rule, i.e., two neutrons in an $l$-orbit coupled with total angular momentum $J = 0$ exhibit behavior similar to that of $P_l(\cos \theta_{12})$, which shows a correlation at small relative angles of $\theta_{12} \lesssim 2/l$. This implies that the mixing of high-$l$ orbits corresponds to the spatial correlation at small relative angles. In the case of the soft dipole mode, the $l$-decomposition of the particle-pair transition density $P_{\text{pp}}(r)$ of neutrons also exhibits a coherent contribution up to $l \sim 11 - 13$, indicating the presence of the spatial correlation of the same character in this mode. Because we see a qualitatively similar feature in Fig. 8, we conclude that the di-neutron correlation also appears in the neutron mode in the octupole response. We shall compare the dipole and octupole cases in more detail in a later subsection.
3.3. Isotopic dependence of the neutron mode

The strength of the neutron mode increases significantly as the system approaches the neutron drip line. Figure 9 shows the strength functions calculated for $^{80,82,84,86}\text{Ni}$. The calculation is the same as that in §3.1. The calculated one-neutron separation energies in these isotopes are $E_{1n} = 2.79, 2.54, 1.86$ and $1.32$ MeV in order of increasing mass number, and are shown in Fig. 9 with arrows. It can be seen in all the nuclei that above $E_{1n}$ there exists a broad distribution of predominant neutron strength, which corresponds to the neutron mode. Clearly the magnitude of the strength increases monotonically with increasing $N$. This can also be seen in the integrated isoscalar strength $B(\text{IS3})$ of this mode, listed in Table I.

The transition densities of the neutron mode, evaluated at $E = 4.0, 3.5, 3.0$, and $2.5$ MeV for $^{80,82,84,86}\text{Ni}$, respectively, are shown in Fig. 10. In the top panels we can see that the tail of the particle-hole transition density grows as the neutron number increases. Such a long tail can be realized for particle-hole transitions from weakly bound orbits, whose wave functions have a long tail, to continuum orbits with small kinetic energies. The long tail enhances the octupole strength because the octupole operator has a radial form factor proportional to $r^3$, giving a heavy weight at larger distances. We thus conclude that the increase of strength with increasing $N$ originates from the effect of the weak binding of neutrons.

From the particle-pair transition densities shown in the bottom panels in Fig. 10, we observe that the particle-pair transition density varies more greatly with $N$ than

| Table I. The isoscalar and electric octupole strengths $B(\text{IS3})$ and $B(\text{E3})$ of the neutron mode and the surface vibrational mode. The energy-weighted sum $S_{\text{EW}}(\text{IS3})$ of the isoscalar octupole strength in units of the classical sum rule value $S_{\text{cl}}^{\text{EW}}$ is also listed. The neutron vs proton ratio $M_n/M_p$ of the transition amplitudes as well as the peak energy is also shown for the surface vibrational mode. The values of $S_{\text{cl}}^{\text{EW}}$, the one- and two-neutron separation energies $S_{1n}$ and $S_{2n}$, the rms radius $\sqrt{\langle r^2 \rangle_n}$ and the radial expectation value $\langle r^4 \rangle_n$ of neutrons are also tabulated. |
|---------------------------------|-------|-------|-------|-------|
| neutron mode | $B(\text{IS3}) \times 10^4$ [fm$^6$] | $S_{\text{EW}}^{\text{IS3}}/S_{\text{cl}}^{\text{EW}}$ [%] | $B(\text{E3}) \times 10^4$ [fm$^6$] | $S_{\text{EW}}^{\text{E3}}/S_{\text{cl}}^{\text{EW}}$ [%] | $M_n/M_p$ | $\sqrt{\langle r^2 \rangle_n}$ [fm] | $\langle r^4 \rangle_n$ [fm$^4$] |
| $^{80}\text{Ni}$ | 1.61 | 0.7 | 3.57 | 27.7 | 2.63 | 4.28 | 466 |
| $^{82}\text{Ni}$ | 4.23 | 1.6 | 4.97 | 3.27 | 2.95 | 4.35 | 512 |
| $^{84}\text{Ni}$ | 5.85 | 1.7 | 6.74 | 3.87 | 3.27 | 4.43 | 562 |
| $^{86}\text{Ni}$ | 16.11 | 3.4 | 10.02 | 4.53 | 3.90 | 4.53 | 656 |
| surface vibrational mode | | | | | | | |
| $E_{\text{peak}}$ [MeV] | 5.56 | 4.78 | 4.16 | 3.84 |
| $B(\text{IS3}) \times 10^4$ [fm$^6$] | 3.57 | 4.97 | 6.74 | 10.02 |
| $B(\text{E3}) \times 10^4$ [fm$^6$] | 2.77 | 3.27 | 3.87 | 4.53 |
| $S_{\text{EW}}^{\text{IS3}}/S_{\text{cl}}^{\text{EW}}$ [%] | 25.0 | 26.9 | 28.6 | 33.0 |
| $S_{\text{EW}}^{\text{E3}}/S_{\text{cl}}^{\text{EW}}$ [%] | 26.9 | 28.6 | 33.0 |
| $M_n/M_p$ | 2.63 | 2.95 | 3.27 | 3.90 |
| sum rule value and separation energies | | | | | | | |
| $S_{\text{cl}}^{\text{EW}} \times 10^6$ [fm$^6$MeV] | 7.92 | 8.78 | 9.77 | 11.36 |
| $E_{1n}$ [MeV] | 2.79 | 2.54 | 1.86 | 1.32 |
| $E_{2n}$ [MeV] | 3.84 | 3.26 | 2.37 | 1.37 |
| ground state properties | | | | | | | |
| $\sqrt{\langle r^2 \rangle_n}$ [fm] | 4.28 | 4.35 | 4.43 | 4.53 |
| $\langle r^4 \rangle_n$ [fm$^4$] | 466 | 512 | 562 | 656 |
Fig. 9. The octupole strength functions obtained for $^{80-86}$Ni. See also the caption of Fig. 3.

Fig. 10. The neutron transition densities of the neutron mode obtained for $^{80-86}$Ni. The upper and lower panels show the particle-hole and particle-pair transition densities, $r^2 \rho_{iqL}(r)$ and $r^2 P_{iqL}(r)$, respectively. We calculated the transition densities in each nucleus at the excitation energy listed in the figure. The transition densities obtained neglecting the dynamical pairing effect are also plotted with the dashed curves for comparison.

the particle-hole transition density. In $^{86}$Ni and also in $^{84}$Ni this transition density does not show exponential decay in the outside of the nucleus, where $r \geq 10$ fm, but rather, it shows oscillatory behavior with its maximum amplitude at around $r \sim 15$ fm far outside the nucleus. This suggests the emission of a neutron pair from the nucleus. This is of course related to the feature that the two-neutron separation energy $E_{2n}$ decreases from $E_{2n} = 3.84$ MeV in $^{80}$Ni to the small value of $1.37$ MeV in $^{86}$Ni. The di-neutron correlation in the neutron mode becomes more significant as we approach the neutron drip line.

3.4. Model dependence

We now investigate how the neutron mode depends on the model parameters. For this purpose, we compare the above results with those obtained with another Skyrme parameter set SkM*, and those with a model adopting the Woods-Saxon potential instead of the Skyrme-HFB self-consistent mean fields. When calculating the octupole response for these models, we use the same mixed-type DDDI, but the interaction parameter $V_0$ is adjusted separately to reproduce the average pairing gap of neutrons in stable nuclei for the SLy4 case: $V_0 = -265$ and $-280$ MeV fm$^{-3}$ for SkM* and WS, respectively. The obtained average neutron pairing gap in
Fig. 11. The octupole strength functions in $^8_4$Ni, obtained with the Woods-Saxon model (top left) and the Skyrme-HFB model with SkM$^*$ (top right). The mixed-type DDDI is used in both cases. The bottom panels show the strength functions for the Skyrme parameter set SLy4 but with different pairing interactions: the mixed-type DDDI (bottom left) and the volume-type density-independent delta interaction (bottom right). See the caption of Fig. 3 for the definition of the curves.

$^8_4$Ni is 1.349, 0.797 and 0.569 MeV for the Woods-Saxon, SLy4 and SkM$^*$ models, respectively.

The octupole strength function in $^8_4$Ni obtained for the SkM$^*$ and Woods-Saxon models are shown and compared with that for SLy4 in Fig. 11. It can be seen that the neutron mode depends rather sensitively on the model. The Woods-Saxon model produces a significantly larger strength than SLy4, while the strength is smallest for SkM$^*$ among the three models. Figure 12 shows a comparison of the transition densities of the soft neutron mode of the Woods-Saxon and SLy4 models, both evaluated at $E = 3.0$ MeV. (The transition densities for SkM$^*$ are not shown here as the strength of the neutron mode itself is weak.) The Woods-Saxon model exhibits enhanced amplitudes of the transition densities, especially of the particle-pair transition density, compared with the SLy4 model.

We can relate the model dependence to differences in the single-particle levels and the Fermi energy of neutrons. The neutron Fermi energy in $^8_4$Ni is $\lambda_n = -0.722, -1.183$ and $-2.338$ MeV for the Woods-Saxon, SLy4 and SkM$^*$ models, re-
Fig. 12. A comparison of the neutron transition densities, \( r^2 \rho_{iqL}^\text{WS}(r) \) and \( r^2 \rho_{iqL}^{\text{SLy4}}(r) \), of the neutron mode in \(^{84}\text{Ni}\) obtained with the Woods-Saxon model (thick curves) vs those with the Skyrme-HFB model with SLy4 (thin curves). The mixed-type DDDI is used and the excitation energy is \( E = 3.0 \) MeV in both cases. The dashed curves in the bottom panel represent the results obtained by neglecting the dynamical pairing effect.

Fig. 13. The neutron single-particle energies for \(^{84}\text{Ni}\) in the Woods-Saxon model and the Skyrme Hartree-Fock model with SLy4 and SkM*. The dashed line indicates the neutron Fermi energy.
Fig. 14. The dependence of the neutron transition densities, $r^2 \rho_{iqL}(r)$ and $r^2 P_{iqL}(r)$, of the neutron mode on the pairing interaction. We compare results obtained with the mixed-type DDDI (thin curves) and the volume-type density-independent delta interaction (thick curves). The excitation energy is $E = 3.0$ MeV in both cases. The Skyrme parameter set SLy4 is used.

spectively. The neutron single-particle energies are shown in Fig. 13. Clearly the last neutrons are less (more) bound in the case of the Woods-Saxon model (the SkM* model). Thus, the model dependence of the low-lying octupole strength can be explained in terms of the weak-binding effect, which increases the strength of the neutron mode.

We speculate that the neutron mode also depends on the effective pair interaction. Let us explicitly examine the dependence on the effective pair interaction. For this purpose we compare results obtained with the mixed-type DDDI with another calculation using the density-independent delta interaction, which is defined by Eq. (2.1) but with $\eta = 0$. (SLy4 is used as the Skyrme parameter set.) The strength $V_0 = -215$ MeV fm$^{-3}$ is adjusted in the same way as for the mixed-type DDDI, i.e., to reproduce the average neutron pairing gap in stable Ni isotopes. The density-independent delta interaction is often called the volume pairing. This pairing produces an average neutron pairing gap of $0.467$ MeV in $^{84}$Ni, slightly smaller than that in the mixed-type DDDI. As far as the octupole strength function is concerned, there is no significant dependence on the type of the pairing interaction, as
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Fig. 15. The electric dipole strength function $dB(E1)/dE$ in $^{80-86}$Ni, shown in the top panels, obtained with the Skyrme parameter set SLy4 and the mixed-type DDDI, compared with the isoscalar octupole strength function $dB(IS3)/dE$ shown in the bottom panels. The smoothing parameter is $\epsilon = 0.2$ MeV. The peak at $E = 0$ in the electric dipole strength function is associated with the spurious mode.

can be seen from the comparison in Fig. 11. There is, however, a large difference in the particle-pair transition density $P_{ppiqL}^{\text{pp}}(r)$ of neutrons, as shown in Fig. 14. The amplitude of the particle-pair transition density $P_{ppiqL}^{\text{pp}}(r)$ in the volume-pairing case is smaller than that of the mixed-type DDDI by a factor of $\sim 2$. It can also be seen that the dynamical pairing effect is significantly smaller in the volume-pairing case. These results indicate that the di-neutron correlation in the neutron mode is sensitive to the choice of the effective pairing interaction: effective pairing interactions such as the mixed-type DDDI, which has greater interaction strength at low densities, give stronger di-neutron correlation in the neutron mode. This feature is in agreement with our previous finding for the soft dipole excitation.\(^{15}\)

3.5. Comparison with soft dipole excitation

The analyses in the preceding subsections revealed that the weak-binding effect and the di-neutron correlation in the octupole neutron mode are similar to those seen in the soft dipole excitations.\(^{15}\) In this subsection we make a more explicit and quantitative comparison between the dipole and octupole cases. For this purpose we calculate the electric dipole response using the same Skyrme-HFB + continuum QRPA model. (The calculations in our previous work\(^{15}\) are based on the Woods-Saxon model and are not suitable for direct comparison with the present calculations.)

The calculated electric dipole strength function $dB(E1)/dE$ is shown in Fig. 15. In the same energy region as the octupole neutron mode (i.e. just above the one-neutron separation energy), a broad distribution of the E1 strength emerges corresponding to the soft dipole excitation. It can also be seen that the monotonic increase of the strength with $N$ is in parallel with that of the octupole neutron mode. If we evaluate the $B(E1)$ strength by integrating within an energy interval $[E_{1n}, E_{1n} + 5 \text{ MeV}]$ above the one-neutron separation energy (as performed
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Fig. 16. The same as Fig. 8 but for the soft dipole excitation. Here the angular momentum cutoff is $l_{\text{cut}} = 5, 7, 9, \ldots, 17$. The transition densities are evaluated at $E = 3.0$ MeV using the mixed-type DDDI and SLy4.

in Ref. 15)), the obtained E1 strength is $B(\text{E1}; 0_{gs} \rightarrow 1^-) = 0.83, 1.59, 2.23$ and $3.33 \, e^2 \text{fm}^2$ for $^{80,82,84,86}\text{Ni}$, respectively. The energy-weighted sum in the same energy interval is $1.9, 3.3, 4.0, 4.8\%$ of the classical (TRK) sum rule value for $^{80,82,84,86}\text{Ni}$. The E1 strength and the energy weighted sum in a smaller energy interval $[E_{1n}, E_{1n} + 1.5 \, \text{MeV}]$ are $0.11, 0.24, 0.35$ and $0.87 \, e^2 \text{fm}^2$ and $0.15, 0.30, 0.36$ and $0.68\%$ for the same isotopes. The strengths of the octupole neutron mode and the soft dipole mode are comparable in terms of the ratio of the energy-weighted strength to the classical sum rule value, i.e., $1.7\%$ for the octupole neutron mode and $4.0\%$ for the soft dipole in $^{84}\text{Ni}$ (0.4\% if the same energy interval $[E_{1n}, E_{1n} + 1.5 \, \text{MeV}]$ is used).

In Fig. 16, we show the transition densities of the soft dipole mode. The transition densities are evaluated at the same excitation energy $E = 3.0$ MeV and in the same energy interval $[E_{1n}, E_{1n} + 1.5 \, \text{MeV}]$ as those for the octupole neutron mode. Comparing the transition densities of the octupole (Fig. 8) and dipole (Fig. 16) modes, both are similar in that the particle-pair transition density $P_{iqL}^{pp}(r)$ is most dominant, and also that the dynamical pairing effect and coherent contributions of the high-$l$ orbits play a significant role in enhancing the particle-pair transition den-
sity. If we compare the absolute magnitudes of the transition densities, it can be seen that the magnitudes of the particle-pair and particle-hole transition densities of the octupole neutron mode are smaller than those of the soft dipole mode by a factor of \( \sim 2 \) at around \( r = 10 \text{ fm} \), where the particle-pair amplitude is largest. Apart from this difference, the significance of the di-neutron correlation is comparable in both cases.

The most noticeable difference between the octupole and dipole responses is that in the octupole response the neutron mode overlaps with the surface vibrational mode present in the same energy region, and the strength of the former is overwhelmed by the latter. In contrast, the soft dipole excitation in the dipole response is well separated from the other mode of excitation, the giant dipole resonance. This may make it more difficult to experimentally identify the octupole neutron mode.

3.6. Surface vibrational mode

We now briefly mention the octupole surface vibrational mode, corresponding to the sharp peaks around \( E \approx 5.6 - 3.8 \text{ MeV} \) in \( ^{80-86}\text{Ni} \). The peak energies as well as the isoscalar and electric octupole strengths associated with the surface vibrational mode are listed in Table I. The most noticeable feature is that the isoscalar strength increases steeply by approximately a factor of three with increasing \( N \) from 80 to 86. We can also see that the neutron vs proton ratio \( M_n/M_p \) of the transition amplitudes increases more steeply than the nominal ratio \( N/Z \). The double ratio \( (M_n/M_p)/(N/Z) \) increases from 1.41 to 1.88 with increasing \( N \). These results indicate that the enhanced collectivity of the surface vibrational mode is mainly due to neutron contributions. Note that the ratio of the energy-weighted sum of the isoscalar strength to the classical sum rule value \( S_{\text{EW}} \) remains constant at around 25–33%. This is partly because the increase of the strength is compensated by the decrease of the excitation energy (see Table I), and partly because \( S_{\text{EW}} \) itself increases with \( N \) due to the increasing radial expectation value \( \langle r^4 \rangle_n \). This suggests that the increases of the isoscalar and neutron strengths of the surface vibrational mode can be regarded as a softening caused by the weak binding of neutrons.

The enhanced collectivity of the octupole vibrational mode in neutron-rich nuclei close to the drip line is pointed out in Ref. 27), in which, however, a doubly closed shell nucleus \( ^{60}\text{Ca} \) was analyzed using the Skyrme-HF plus continuum RPA without the pair correlation. Our results suggest that enhanced collectivity is generally observed in nuclei near the neutron drip line. A large deviation of the \( M_n/M_p \) ratio from the nominal ratio \( N/Z \) was obtained in a Skyrme-BCS+ QRPA calculation\(^{23}\) for the neutron-rich oxygen isotope \( ^{24}\text{O} \), but with much smaller deviation in less neutron-rich isotopes \( ^{18-22}\text{O} \). We refer also to Ref. 59), in which the enhanced collectivity due to the weak-binding effects is discussed in the case of the low-lying quadrupole vibrational mode.

§4. Conclusions

We have investigated the low-lying octupole excitations of the neutron-rich Ni isotopes beyond the \( N = 50 \) shell closure using the continuum QRPA based on the
Skyrme HFB mean fields. In addition to the surface vibrational mode of the $1-h\omega$ character, a broad strength distribution of predominantly the neutron character appears just above the neutron separation energy. This broad neutron mode exhibits the following distinctive features. (i) The amplitude for neutron-pair transition density is significantly larger than that of the usual particle-hole transition density of neutrons. The neutron pair correlation is therefore the most essential aspect in characterizing the neutron mode. (ii) The large neutron-pair transition density originates from a coherent contribution of the high-$l$ orbits of the neutron, which indicates the presence of the spatial correlation among neutrons involved in this mode. (iii) The transition densities of the neutron mode display a long tail extending to outside of the nucleus. The strength of this mode increases monotonically with increasing $N$ from 52 ($^{80}\text{Ni}$) to 54, 56 \ldots. The long neutron tail and the increase of the strength with $N$ indicate that the mode originates from the weak binding of neutrons. From these features, we conclude that the spatial di-neutron correlation also appears in the octupole neutron mode in nuclei near the neutron drip line, similarly to in the soft dipole mode. This supports our expectation that soft modes having the di-neutron character may emerge generically in medium-mass neutron-rich nuclei near the drip line. It will be interesting to continue this investigation more systematically by studying modes with other multipolarities including the quadrupole mode. Such an analysis is in progress and will be reported elsewhere.

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