

# Optimizing multi-reservoir operation rules: an improved HBMO approach

Abbas Afshar, Mahyar Shafii and Omid Bozorg Haddad

## ABSTRACT

We present an improved version of Honey Bees Mating Optimization (HBMO) algorithm to develop operating rules for multi-reservoir systems. The performance of the proposed model is tested through sensitivity analysis and comparing the result with those of a real-coded Genetic Algorithm (GA) for a 60-month period single-reservoir operation problem. The improved model is subsequently employed to derive release rule and storage balancing functions which form operating policy for a multi-reservoir system along two case examples: (i) water supply and (ii) hydropower generation. The obtained operating rule curves can be used to guide the system operators in decision-making. These rule curves provide the operator with the opportunity to systematically look at the system and to make proper decisions. The obtained results showed that the optimization technique proposed in this study is capable of solving complex multi-reservoir systems operation problems. Moreover, the proposed structure properly handled the tight constraints defining the parallel reservoirs operation in such a way that all the generated solutions were feasible after a particular set of iterations. The proposed optimization algorithm of this study can be developed more in future to solve multi-modal optimization problems, and also to define operation policies for highly complex multi-reservoir systems.

**Key words** | genetic algorithms, HBMO, multi-reservoir systems, operation rule, optimization

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## INTRODUCTION

The coordinated operation of multiple-reservoir systems is typically a complex decision-making process involving many variables, objectives and considerable risk and uncertainty. Most of the existing reservoir systems are still managed based on fixed pre-defined rules. These rules are usually presented in the form of graphs or tables and indicate the actions to be taken by the system operators as a function of a relatively small number of variables such as time of the year, state of the system and expected future hydrological conditions (Loucks & Sigvaldason 1982; Yeh 1985; Wurbs 1996).

Predefined operating rules are often evaluated using simulation models but these rules must be defined before they can be simulated. Defining these rules is a challenging task, especially those that apply to multiple reservoirs serving

multiple purposes and objectives. Optimization models can help define these rules which satisfy various constraints on system operation while minimizing future spills or deviations from various water releases, storage volumes and/or energy production targets (Oliviera & Loucks 1997).

Various mathematical optimization models have been proposed to derive the properties of efficient fixed and pre-defined operating rules. Operating policy for multi-reservoir systems must specify not only the total release from the system but also the amounts, if any, to be released from each reservoir in the system. Several rules of thumb that provide some guidelines to the operation of multi-reservoir systems have been developed and practiced during previous decades by researchers (Clark 1956; Bower *et al.* 1962; Wu 1988; Johnson *et al.* 1991).

When the complexity of system increases or when the system serves several purposes and its operation is heavily constrained, these rules cannot be applied directly and do not provide clear indications on how to operate the system efficiently. A procedure that considers all the constraints and objectives is then needed to produce guidelines for these complex systems.

Oliviera & Loucks (1997) proposed a genetic algorithm (GA)-based methodology which identifies the system release rule and the reservoir balancing functions as piecewise linear functions. System release rules typically indicate the total release to be made from the reservoir system as a function of water available in the system and time of the year. Storage volume target functions identify the amount of water which should be released from each reservoir to meet the total system release target. Having both of these functions, it is possible to define a multi-reservoir system operating policy that permits the coordinated operation of the entire system. The coordinates of their inflection points were selected as the decision variables of the optimization problem in order to define the operating policy that optimizes system performance.

The release rule as illustrated in Figure 1(a) defines the releases ( $R_t$ ) in each within-year period as a function of the existing total system storage ( $W_t$ ). Release from each reservoir, if any, can be obtained from reservoir storage balancing functions. These functions are also defined as a set of piecewise linear functions presenting the reservoirs storage volumes based on the total system storage (Figure 1(b)). They applied the proposed GA model to

derive an optimal operation rule based on a 12-month inflow time series. However, it is generally believed that operating policy derivation must benefit and consider much longer input time series to be effective. Bozorg Haddad *et al.* (2006) developed and applied honey bees mating optimization algorithm (HBMO) to derive a rule curve for operation of a single reservoir. In a most recent research, Bozorg Haddad & Mariño (2007) illustrated and tested the applicability and performance of the same algorithm in highly non-convex hydropower system design and operation.

This paper presents an improved version of the HBMO algorithm to develop optimum operating rules for a parallel, highly constrained, multi-reservoir system. We compare the performance of the proposed algorithm with the original HBMO algorithm in operating a single reservoir. A set of parametric rules have been defined for a complex parallel reservoir system and their parameters optimized with the proposed modified HBMO algorithm. The performance of the proposed model is tested through conducting appropriate sensitivity analysis and comparison with those of a real-coded-GA model for a 60-month period in water supply and hydropower reservoirs. The objective function considered in this paper minimizes the total squared deviation from target demand consisting of water supply and hydropower demands.

This article addresses two distinct variant over the previous HBMO-based models. First, this article derives optimum rules for the joint operation of parallel reservoir systems. This new structure imposes some additional constraints on the model which make the problem highly

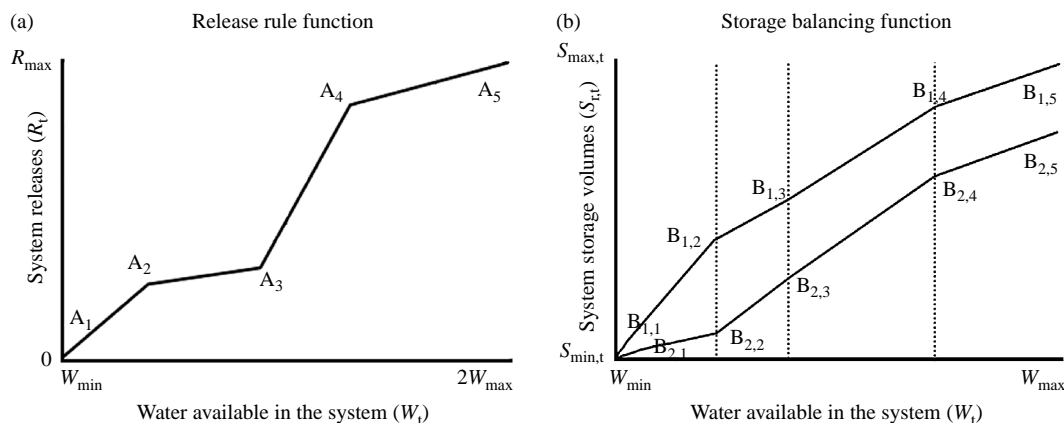


Figure 1 | System release rule and storage balancing functions.

constrained and demands special approaches for their satisfaction and feasible solution generation. Implementation of appropriate operators in such a hard constrained model demands special approaches for searching within the feasible zone. Second, the algorithm itself has been polished and clearly re-defined to be quite readable and the differences with GA algorithms have been highlighted. Some new operators are added and/or modified to make it computationally more efficient and precise for locating good near optimal solutions. In order to compare the performance of the new version with the original HBMO algorithm, the same single reservoir case example is used and the results are presented.

## IMPROVED HBMO ALGORITHM

The honey bees mating process is one of interesting biological behaviour which occurs during mating flights far from the nest. In each mating, sperm reaches the queen's spermatheca and accumulates there to form the genetic pool of the colony. She uses the stored sperm to fertilize the eggs. 'Nurse bees' secrete the nourishing, 'royal jelly' and feed it to their queen. This natural behaviour has become the basis of development of HBMO algorithm to solve mathematical problems (Abbas 2002) and also to handle practical engineering problems (Bozorg Haddad *et al.* 2006). Detailed description of how HBMO has been inspired by the natural mating process of honey bees, as well as how the components of optimization algorithm correspond to those in the natural system, are provided by Bozorg Haddad *et al.* (2006).

Table 1 shows the correspondence between different elements of optimization and natural system. This paper proposes an improved version of the original algorithm with application to optimization of multi-reservoir systems operation. In the following sections about the optimization process, technical expressions are used and the corresponding natural elements, if they exist, are provided in parenthesis.

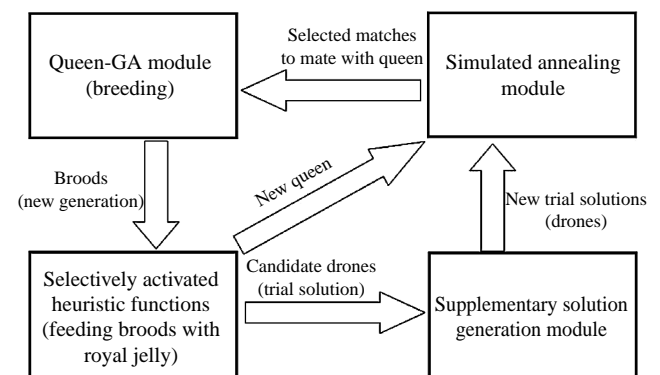
Generally speaking, the proposed HBMO algorithm may be recognized as a hybrid Simulated Annealing (SA) and Queen Genetic Algorithm (QGA) with selectively activated heuristic functions to enhance the solutions through a local search process in the improvement phase

**Table 1** | Correspondence between different components of honey bees natural mating and those in optimization algorithm (HBMO)

Natural process	Optimization algorithm
Queen	The best solution
Queen's goodness	Objective function
Queen's energy	Termination criterion of SA process
Queen's speed	Temperature in SA process
Drones	Trial solutions for crossover with the best solution
Bees' genes	Decision variables
Queen's spermatheca	Crossover pool
Broods	Reproduced offspring
Workers	Heuristic functions to improve the reproduced offspring
Hive (queen, drones, and broods)	All feasible and non-feasible solutions of the problem

of the algorithm. Figure 2 depicts the inter-relationship between different modules of the proposed algorithm, which is composed of two repetitive features: (1) reproduction and (2) improvement.

The reproduction process deals with the generation of new trial solutions in which parent selection (drones nomination for mating) and crossover (real mating) takes place. One of the parents will be the best solution (Queen), while the other (drones) is selected in a SA process. Selection is therefore a probabilistic process in which potential parents (drones) are nominated and kept for further reproduction (i.e. their sperm are deposited in Queen's spermatheca or mating pool). The selection



**Figure 2** | Inter-relationship between different modules of the improved HBMO algorithm.

process follows by crossover (real mating process). Crossover is a Queen-Bee-GA process whereby one of the parents for offspring generation is the best available solution (Queen). Crossover takes place between randomly selected genes (sperm) of one parent (drones) and those of the other (Queen) to generate offspring. This is done based on crossover probability and the reproduction process is over when the pre-defined number of offspring (i.e. broods) is generated.

The improvement process starts when the reproduction process is completed and offspring (i.e. broods) are generated. In this stage, different heuristic functions (workers) will selectively be activated to improve fitness of the generated offspring (i.e. broods' feeding). Heuristic functions are ranked according to their efficient contribution in solution improvements at each generation. Heuristic functions with a higher contribution in solution improvement will be used more extensively in the next improvement process. This feature will limit the unnecessary objective function evaluation for heuristic functions with non-significant contribution in solution improvements.

A detailed flowchart of the proposed algorithm is presented in [Figure 3](#) where the mapping between the real honey bees mating process and the mathematical representation steps are depicted. The mating process itself is translated into an SA process which is presented in [Figure 4](#). The boxes with dash lines have been added or modified to improve the original model, whereas the solid-lined boxes are identical to the original HBMO developed by previous researchers ([Bozorg Haddad et al. 2006](#)).

[Table 2](#) includes structural changes and additional steps which define the differences between the original and improved HBMO models. ID1 deals with improvement of the best solution (Queen's feeding) which has been added to the original HBMO before reproduction. Moreover, the constant speed reduction in the SA process of the original HBMO has been replaced by a linear value (i.e. ID2). In ID3 we tried to map the natural mating process of honey bees exactly in which 'Breeding' (i.e. crossover in the algorithm) and 'Brood's feeding' (i.e. mutation) takes place at two different stages. The original HBMO algorithms consider crossover and mutation simultaneously. ID4 extends the genetic operator used to define the heuristic operator's (worker's) performances in improvement stage

(broods' feeding). In the original HBMO, 6 particular genetic operators were used for improvement. In our model, the number of different operators was increased to investigate their probable positive impact on the quality of solutions. The best combination of operators was then selected based on sensitivity analysis. ID5 is associated with heuristic operators (workers) updating. The space allocated to each operator is proportional to its impact on the quality of reproduced solutions. In other words, if a working operator does not contribute to solution improvement in a given iteration, only a single slot from the population (hive) will be allocated to that operator. We keep this slot to give it the chance in the next generation for probable better performance. This strategy is also useful to decrease the population size.

As a summary of the proposed improvements, the processes of improving the best solution before reproduction (initial queen selection), speed reduction in SA process (mating flight), improvement of offspring (broods feeding) and updating of the heuristic functions (job assignment to the workers) are modified and the changes in their mathematical representations are included in the revised version of the algorithm.

More technically, and according to [Figure 3](#), the algorithm starts with random generation of a set of initial solutions. The generated solutions may or may not belong to the feasible region. Randomly generated solutions are then ranked according to their fitness in which non-feasible solutions are penalized. The fittest solution is named queen, whereas the remaining solutions are categorized as trial solutions (i.e. drones). When the best solution (Queen), trial solutions (drones) and the heuristic functions (workers) are defined, the hive is completely formed and reproduction (mating process) may now start.

The first step for reproduction is to nominate potential solutions as one parent (drones) to be combined with the best solution (Queen). The nomination process is a probabilistic one. In other words, drones are nominated to mate with the queen according to a probabilistic rule using an annealing function. Before analysis of the formulation, it is important to point out that at the start of the nomination process, the queen is initialized with (i) speed and (ii) some energy content which both decline over a set of transitions until it becomes zero when the nomination process is over

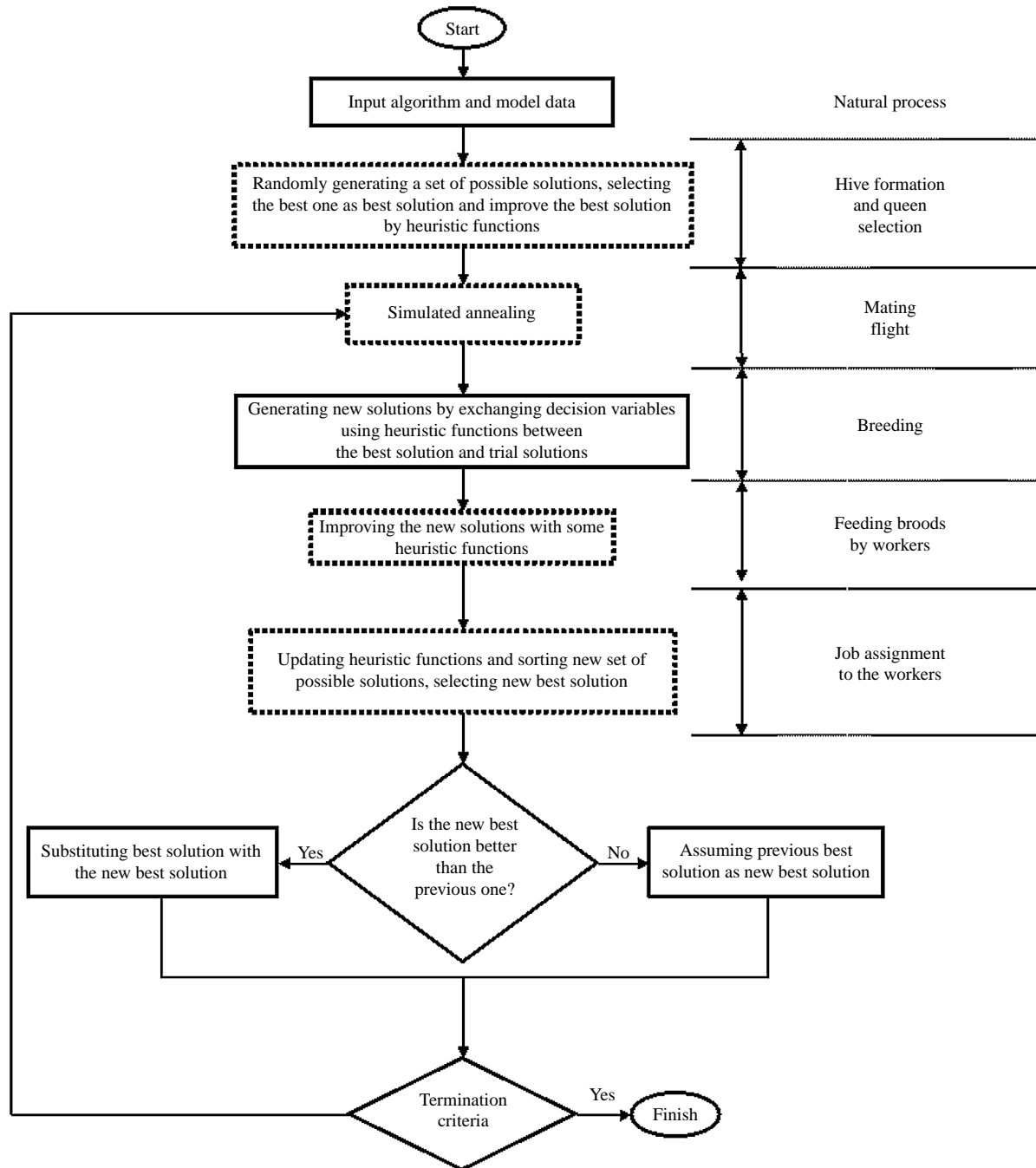


Figure 3 | Improved HBMO algorithm.

(i.e. queen returns to the nest). The annealing function is:

$$\text{Prob}(Q, D, t) = e^{-\Delta(f)/S(t)},$$

$$\begin{cases} \text{Successful mating} & \text{Prob}(Q, D, t) \geq q_0 \\ \text{Unsuccessful mating} & \text{Otherwise} \end{cases} \quad (1)$$

where  $\text{Prob}(Q, D, t)$  is the probability of nomination of the solution (drone)  $D$  to be imposed by crossover and the best solution  $Q$  (i.e. adding the sperm of drone  $D$  to the spermatheca of queen  $Q$  at transition stage  $t$ ). In other words,  $\text{Prob}(Q, D, t)$  is the probability of a successful mating;  $\Delta(f)$  is

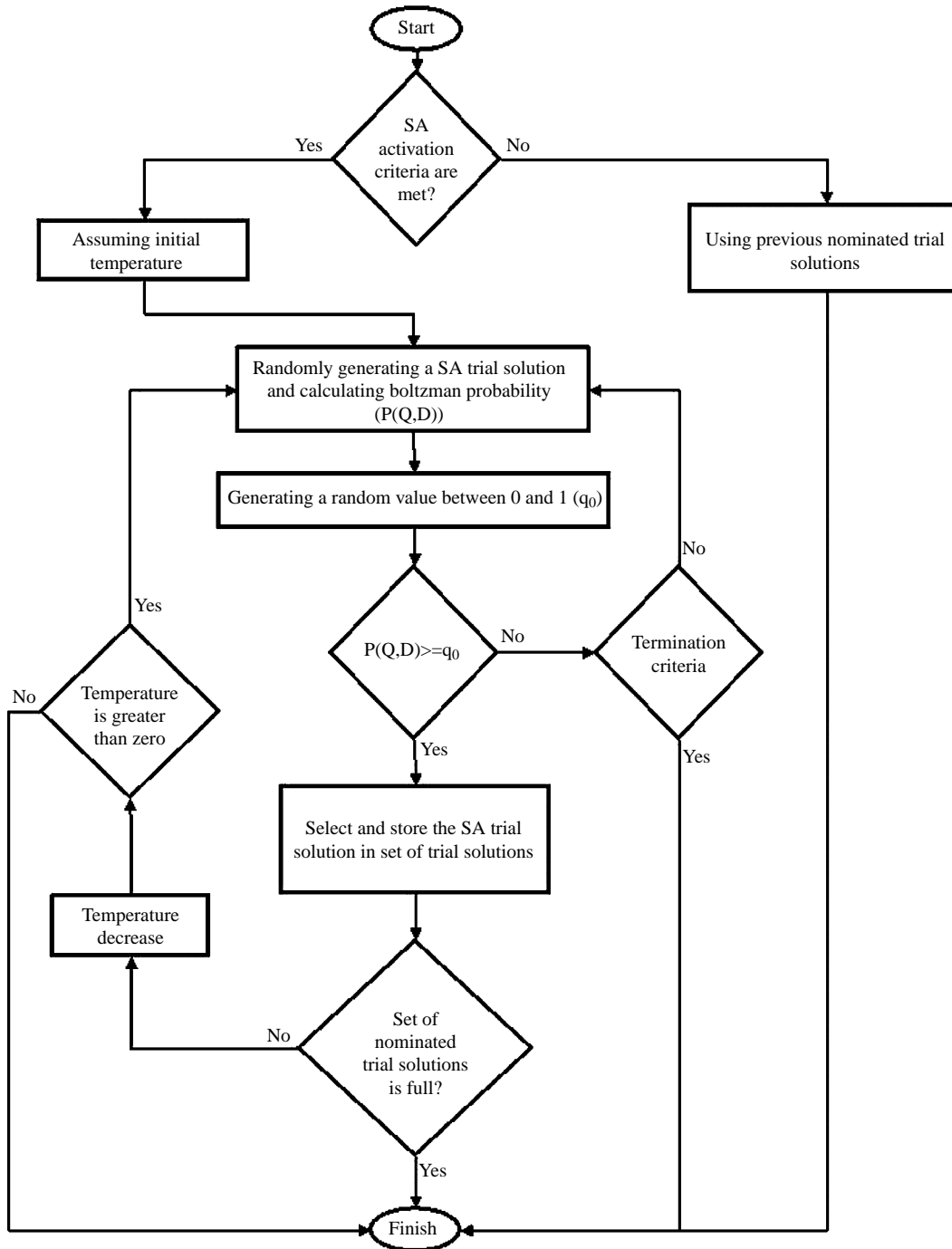


Figure 4 | SA process flowchart in the improved HBMO algorithm.

the absolute difference between the fitness of two solutions (i.e.  $|f(Q) - f(D)|$ );  $S(t)$  is the speed of the queen at transition stage  $t$ ; and  $q_0$  is a uniform random number  $\in [0,1]$ .

The nominated solutions are stored in a population (mating pool) for further crossover. The nomination process

is ended (i.e. the Queen returns back to the nest) either when the queen's energy is fully expended or the mating pool is full. The speed and energy of the queen, which were already explained as some characteristics of the queen, decay after each transition according to the following

**Table 2** | Differences between the original and improved HBMO algorithms

ID	Definition	HBMO	Improved HBMO
1	Improvement of the best solution (Queen feeding)	There is not a certain step for queen feeding after generating the population and selecting the best solution as Queen	Queen feeding has been added after its generation by using some heuristic functions as workers
2	Temperature (Queen's speed) reduction factor in SA process ( $\alpha$ )	Constant factor $\epsilon(0,1)$	Linear factor with initial value of 1 and linear reduction till zero. Technically, this reduction factor ( $\alpha$ ) is the number of successful parent nominations (mating flight and adding a drone's sperm into spermatheca) over the size of mating pool (spermatheca)
3	Crossover (breeding) and mutation (broods feeding)	These two steps are done by using the crossover and mutation functions simultaneously	Breeding would be done first using crossover functions and afterwards mutation functions are applied for broods feeding
4	Heuristic functions (workers) application	6 heuristic functions are used for local search (broods feeding)	Number and type of heuristic functions are improved. The best scheme is selected after conducting sensitivity analysis
5	Heuristic functions (workers) updating	Allocated space to heuristic functions in updating process is determined based on functions ranking. Considering multipliers of 10. Apparently, the better function quality, the more allocated space	These allocated spaces from the population (hive) are determined relatively based on the amount of improvement which is produced by heuristic functions

proposed Equations (Bozorg Hadad & Mariño 2007):

$$S(t+1) = \alpha(t) \times S(t) \quad (2)$$

$$E(t+1) = E(t) - \gamma, \quad t = 1, \dots, T \quad (3)$$

where

$$\alpha(t) = [M - m(t)]/M \quad (4)$$

in which  $\alpha(t)$  is speed reduction factor  $\in [0,1]$ ;  $M$  is the mating pool size;  $m(t)$  is the total number of solutions (drones) nominated for crossover during the first  $t$  transitions; and  $\gamma$  is the amount of energy reduction after each transition. The initial speed must be a large number, while the energy is the allowable number of queen's transitions (e.g. 4 times as much as population size) and  $\gamma = 1$ .

The next step of reproduction is the crossover whereby the genes of the best solution are swapped with those of trial solutions to generate offspring (i.e. real breeding). In this model, four types of crossover operators are used and a performance index is assigned to each operator. The performance index is updated at the end of

each computational step. The performance index assigned to each crossover operator either increases or decreases at the next generation depending on the operator's contribution on the solution enhancement.

The improvement part of the algorithm is formed by applying mutation operators (i.e. the feeding process of broods with royal jelly by the workers). Here, like the crossover operators, the rate of contribution of the operators in solution improvement is made proportional to their performance index in the previous generation. It should be noted here that in some optimization problems, a mutation operator might result in non-feasible solutions. This issue therefore must be dealt with. The mutation operator of the improved HBMO algorithm is especially formulated and implemented to avoid non-feasible solutions, which will be discussed later.

As life in the hive continues, the proposed algorithm continues and the new generation (mating flights  $g$ ) will be produced until the termination criteria (meeting the predefined number of generations  $Gen$ ) are satisfied

( $g > Gen$ ). Then, the best solution from the set of current best present solution and improved solutions are selected. If termination criteria are not satisfied, all trial solutions are discarded. This step is actually what happens in nature, since drones either die after mating or are pushed out of the hive in the winter.

Afterwards, a new set of trial solutions is generated to make the search process more and more extensive. To generate a new set of trial solutions, reproduced solutions (broods) with desirable fitness are partially used. A random process is employed to generate supplementary trial solutions (drones) needed to fulfil the requirements. A random generator borrows some of the genes from the best solution (queen). This means that the percentage of the queen's gene in these new generated solutions increases from 0 at the start to 100 at the end of the improved algorithm.

One of the important issues in the application of improved HBMO algorithm is to set and/or tune the algorithmic parameters. In order to properly establish parameter settings of the algorithm, a series of tests were carried out. Sensitivity to the size of population (hive) and mating pool (spermatheca), SA parameters and number of heuristic functions (workers) was examined through 1,000

iterations (mating flights). The problem addressed for sensitivity analysis was the same single reservoir operation problem as addressed by [Bozorg Haddad \*et al.\* \(2006\)](#) which aimed at minimization of the sum of normalized derivations of monthly reservoir releases from downstream demands, under specified constraints.

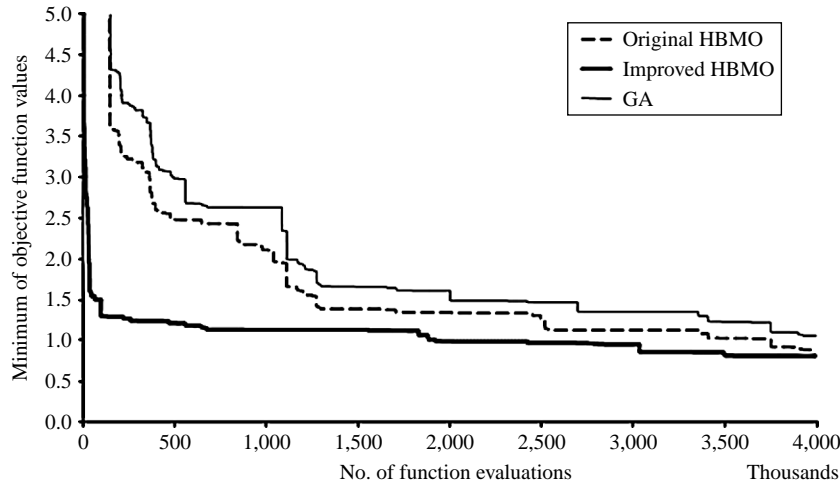
[Table 3](#) lists the schemes considered for the sensitivity analysis and the results obtained for different parameters settings. [Figure 5](#) presents the results of the best run from 10 different test runs by the improved and the original HBMO. Convergence trend to a near optimal solution in [Figure 5](#) clearly reveals the superiority of the improved algorithm over the original algorithm of [Bozorg Haddad \*et al.\* \(2006\)](#). Minimum values of the objective function for a long run, with 4 million function evaluations (i.e. approximately 2,000 mating flights) are obtained as 0.807 and 0.88 for improved and initial HBMO models, respectively. The results show almost 8.3% improvement over original HBMO with the same number of function evaluations or mating flights.

In order to compare the improved HBMO model with other evolutionary algorithms, an alternative Elitist-GA approach ([Shafiei \*et al.\* 2007](#)) was used with real-value representation. This GA algorithm was, however, changed

**Table 3** | Sensitivity analysis results and parameters setting

Description	Schemes	Obtained objective function value	Selected scheme
Hue size and spermatheca size in terms of percentage of hive size	200–20%	1.20	Hive size: 250 spermatheca size: 20% hive size
	250–10%	1.17	
	250–20%	1.01	
	250–30%	1.13	
	300–20%	1.23	
	400–20%	1.18	
Number of heuristic functions (Workers)	500–20%	1.15	6 heuristic functions
	3	1.61	
	6	1.17	
	8	1.69	
	15	1.35	
Number of simulated annealing iterations	64	1.85	Number of SA iterations equal to spermatheca size
	Equal to spermatheca size	0.93	
	Two times as much as spermatheca size	0.98	
	Four times as much as spermatheca size	0.95	





**Figure 5** | Convergence of the best objective function values through 10 different runs applying real-coded-GA and the original and improved HBMO algorithms to solve the single-reservoir operation optimization problem.

to a ‘real-coded GA’ specifically developed and tested for the same problem as HBMO. The tuneable parameters for the GA model were all selected through sensitivity analyses. The final GA structure, used in this part, includes tournament selection, single-point-cut crossover and uniform mutation (Michalewicz 1994) with probabilities of 90% and 5%, respectively. The same single reservoir problem was solved employing both the improved HBMO and real-coded-GA models. The best results through 10 independent runs versus the number of function evaluations are also depicted in Figure 5.

As it is clear, for the problem under consideration, the improved HBMO algorithm also performs better than the real-coded-GA with the same number of function evaluations. Realizing this improvement, it was decided to employ the improved HBMO to develop rule curves for the operation of parallel multi-reservoir systems.

## MODEL STRUCTURE AND FORMULATION

In the HBMO algorithm each honey bee represents a set of decision variables that defines a solution to the problem. In this research, the system release rule and storage balancing functions, which define operating policy, were assumed to be piecewise linear functions (Figure 1(a) and (b)). The coordinates of their inflection points are the decision variables of the optimization problem which is represented

by a honey bee genome. The problem is to find the best solution that defines the operating policy that minimizes the deviations from water supply and hydropower targets.

The problem is constrained by physical constraints (i.e. mass balance, minimum and maximum storage and releases, etc.) and operational rule constraints (i.e. reservoir storage balancing functions and release rule graphs). The release rule graph for each period consists of  $NR$  points which form  $NR - 1$  connected linear segments. To ensure feasibility and to restrict solution space of the optimization problem, the following limitations were applied:

$$W_{t,1} = W_{\min} \quad (5)$$

$$W_{t,N} = 2W_{\max} \quad (6)$$

$$R_{t,\min} = 0 \quad (7)$$

$$R_{t,\max} = R_{\max} \quad (8)$$

where  $(W_{t,1}, R_{t,\min})$  and  $(W_{t,N}, R_{t,\max})$  are the coordinates of the first and the last points on the release rule graph at the end of period  $t$ , respectively (A1 and A5 in Figure 1(a)),  $W_{\min}$  and  $W_{\max}$  are the minimum and maximum allowable active system storage volumes and  $R_{\max}$  is the maximum reasonable release. It should be noted that the constraint given by Equation (6) is due to the fact that spill is not separately considered in the formulation of the paper, Hence, it is likely that the storage value in the system exceeds  $W_{\max}$  at the beginning of each period, which will be compensated at the end of the period with a higher release

value. The number of decision variables equals  $2(NR - 2)$  per period to define the release rule. In this study,  $NR$  was set to 5.

The reservoir storage balancing function for each reservoir in each period consists of  $NB$  points as illustrated in Figure 1(b). What is shown in Figure 1(b) is a set of balancing functions for a two-reservoir system. Each non-decreasing line corresponds to one of the reservoirs. These graphs define the reservoir storage values as a function of total water in the system, which should be fixed by an operator at the end of each period. In other words, the operator permits a set of releases from different reservoirs in such a way that their storage levels are fixed to the values given by the graphs. The following definitions and/or assumptions were used to structure the model:

$(W_t^{\min}, S_r^{\min})$ : the coordinate of the first point for reservoir  $r$  at period  $t$

$(W_t^{\max}, S_r^{\max})$ : the coordinate of the last point for reservoir  $r$  at period  $t$

$$\sum_{r=1}^R S_{r,t} = W_t \quad (9)$$

$$0 < \frac{S_{r,t}}{W_t} < 1 \quad (\forall r, t) \quad (10)$$

where  $W_t^{\min}$ ,  $W_t^{\max}$  are the minimum and maximum allowable active system storage volumes;  $S_r^{\min}$ ,  $S_r^{\max}$  are the corresponding minimum and maximum active storage volumes for reservoir  $r$ ; and  $R$  is the number of reservoirs. Thus, the number of coordinate points equals  $(NB - 2) \times (R - 1) + (NB - 2)$  or  $(NB - 2) \times R$  for balancing functions in each period. In this research,  $NB$  was set to 5. Adding the number of decision variables from the release rule for all time periods (i.e.  $t \times 2(NR - 2)$ ) to those for storage balancing functions (i.e.  $t \times (NB - 2) \times R$ ), the total number of decision variables for defining operating policy equals  $t \times [2(NR - 2) + (NB - 2) \times R]$  for a system with  $R$  reservoirs and  $t$  periods.

In addition, the release rule and balancing functions have to be non-decreasing functions of total system storage to ensure that the release values do not exceed the water available in the system. Release and storage values of each reservoir are limited between a minimum and a maximum value. Moreover, considering mass balance constraint,

the other set of limitations are required as follows:

$$S(r, t) = S(r, t + 1) - Q(r, t) + R(r, t); \quad \forall t \quad (11)$$

$$R_{\min}(r, t) \leq R(r, t) \leq R_{\max}(r, t); \quad \forall t \quad (12)$$

$$S_r^{\min} \leq S(r, t) \leq S_r^{\max}; \quad \forall t \quad (13)$$

where  $S(r, t)$  is the storage vector at the beginning of period  $t$ ;  $Q(r, t)$  is the inflow vector during period  $t$ ;  $R(r, t)$  is the release vector in period  $t$ ;  $R_{\min}(r, t)$  and  $R_{\max}(r, t)$  are minimum and maximum release values during period  $t$ ; and  $S_r^{\min}$  and  $S_r^{\max}$  are minimum and maximum active storage values for reservoir  $r$ , respectively. Therefore, operation of parallel water supply reservoirs is structured to minimize the objective function:

$$\text{Min} \sum_{t=1}^T \left( \frac{R_t - D_t}{D_{\max}} \right)^2 \quad (14)$$

in which  $R_t$  and  $D_t$  define the system release and demand at period  $t$ , respectively.  $D_{\max}$  defines the maximum demand during the operating horizon which is used to normalize the objective function.

The model is subject to the constraints defined by Equations (5)–(13). The improved HBMO algorithm starts by randomly generating the initial set of solutions. At this step two issues are of concern. First, each generated solution must define a set of non-decreasing linear graphs for the release rule and balancing functions. If the generated solutions fail to satisfy these constraints, they will be rejected and the process continues until an appropriate solution is obtained. Second, if the accepted solutions fail to satisfy mass balance (i.e. non-feasible solutions) they will be penalized in the evaluation process. As we are dealing with a minimization problem, in order to penalize the solutions, a dynamic penalty value is added to the fitness of non-feasible solutions to reduce the chance of it to be selected for next generations. Accordingly, the penalty value, used in this paper, is made proportional to the squared values of the constraints violations. Thus, as the value of non-feasibility increases, the trial solution dynamically receives a higher penalty which reduces its chances of being selected.

Real mating between the queen and selected drones (generated trial solutions) takes place in the next step where Queen-GA operators are activated. Upon the offspring

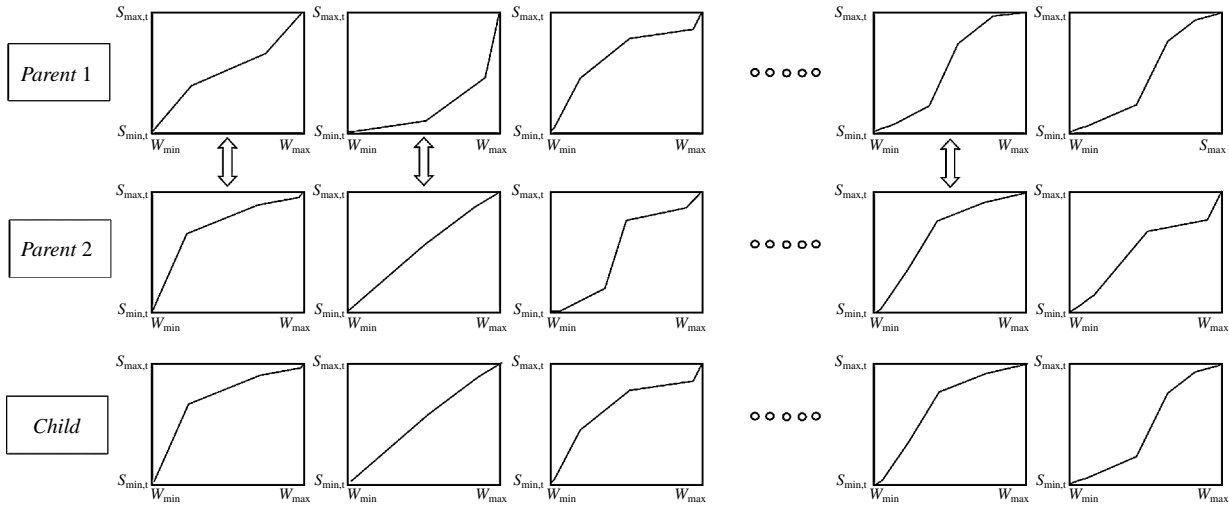


Figure 6 | Schematic application of crossover operator.

generation, the selected heuristic functions (workers) are activated to improve the fitness of the offspring in a local search process. Figure 6 shows how a crossover operator was applied to the vectors of decision variables. As an example, decision variables defining operating rule for time periods  $t_1$  to  $t_4$  from one solution is replaced by the values of decision variables for the same period from another randomly selected solution.

When local search is applied by the heuristic functions, the reproduced solutions must still satisfy the non-decreasing constraints for release rule and balancing functions. Therefore, upon applying the heuristic functions, the coordinates of the deflection points on the release rule curve or balancing function have to be bonded between

the coordinates of previous and the next point on the curve (Figure 7).

Technically, mutation is to change the coordinates of the points of a particular rule curve (assume point B in Figure 7) in a feasible manner. We therefore restrict ourselves to selecting a new position for B such that its new coordinates be located within a rectangular bonded with points A and C as its corner points. So, we can add/subtract  $D_x$  to/from the horizontal component of B coordinates that is always less than  $\text{Max}(D_x)$ , as shown in Figure 7. The same method is used to determine a new value for the vertical component of B coordinates. In this way, the non-decreasing shape constraint of the graph is satisfied.

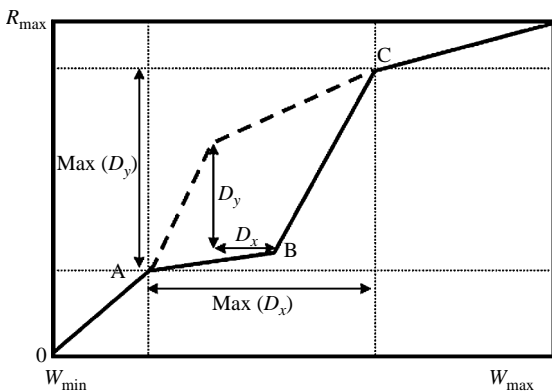


Figure 7 | Application of mutation operator.

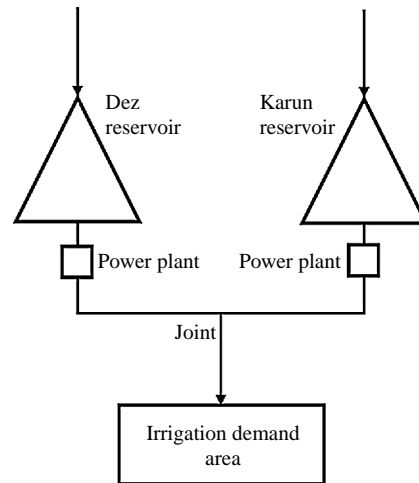


Figure 8 | Structure of the multi-reservoir system.

**Table 4** | Inflow sequences and monthly demands for water supply and hydropower case examples

Reservoir	Year	Inflows in operation periods (MCM)												Yearly (MCM)
		1	2	3	4	5	6	7	8	9	10	11	12	
Karun	1	304	320	347	374	480	979	823	215	360	323	285	288	5,097
	2	296	316	295	395	368	639	450	650	523	424	238	231	4,824
	3	310	339	524	465	556	754	1,053	363	246	226	335	325	5,497
	4	283	309	402	342	500	545	920	418	277	258	327	321	4,902
	5	284	308	246	305	412	510	1,073	389	253	210	354	347	4,692
Dez	1	125	189	329	363	485	642	1,463	755	825	507	326	239	6,248
	2	198	301	523	577	723	953	441	510	282	173	111	82	4,872
	3	68	103	178	197	281	390	1,021	912	504	310	199	146	4,309
	4	121	184	319	352	504	654	761	728	403	247	159	117	4,549
	5	97	147	255	282	402	564	1,051	940	519	319	205	150	4,931
Joint demand values (MCM)		932	882	838	812	928	1,085	1,234	1,304	1,355	1,264	1,172	1,067	12,873

Having applied the GA operators and heuristic functions, the improved solutions are compared with the queen to select the new queen, if applicable. Once again new drones (trial solutions) are generated and queen starts her new mating flight. The algorithm continues till the stop criteria are met.

## MODEL APPLICATION AND RESULTS

The improved HBMO algorithm was applied to develop operating rule curves for a two-parallel reservoir system in the south of Iran. Dez and Karun reservoirs with 2,360 and 1,284 MCM net capacities form two parallel reservoirs which are used for hydropower generation as well as satisfying irrigation demand (Figure 8). Minimum and maximum operational storage for Dez and Karun are set to (453, 2,813) and (1,518, 2,802) MCM, respectively.

Two different objectives (i.e. hydropower generation and irrigation water supply) were used separately in two different case studies. Case (A) considers a 60-month inflow sequence with the objective of minimizing the measure of total irrigation deficit. Case (B) considers hydropower generation and attempts to minimize the sum of deviations from the total installed capacity of the system.

Table 4 presents the 60-month inflow sequence to Dez and Karun reservoirs along with total monthly demands. The second year of the data provided in Table 4 is used for

case (B). Table 5 presents the supplementary data which are necessary to model the hydropower case study.

### Case (A): multi-reservoir operation rule curves for water supply

To demonstrate the capability of the proposed version of HBMO algorithm in developing multi-reservoir rule curve, a 60-month inflow sequence was considered. The general structure of the system is presented in Figure 8. Monthly inflow sequences to each reservoir along with monthly demand are provided in Table 4. It is intended to develop a joint operating policy which satisfies the demand with minimum deficit. The multi-reservoir operation for water supply, defined by Equations (5)–(14), was applied to two reservoir system shown in Figure 8. The best and average of the objective function values obtained in 5 runs and 1.5 million function evaluations were 2.08 and 2.15, respectively (Table 7, Water Supply row, shows different objective function values for multiple runs).

**Table 5** | Supplementary data for the hydropower case example

Reservoir	Power plant		River-reservoir		
	Efficiency	Plant factor	Tail water elev. (masl)	$S_{min}$ (MCM)	$S_{max}$ (MCM)
Karun	0.9	0.25	365	1,518	2,802
Dez	0.9	0.55	172	453	2,813

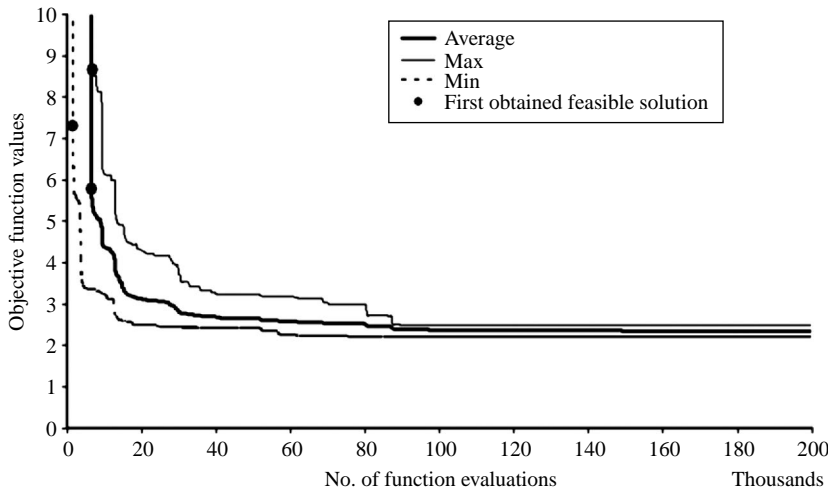


Figure 9 | The best, average and worst objective function values of 5 trial runs versus number of function evaluations (water supply case example).

Figure 9 illustrates the convergence of the best, average and the worst run from 5 different runs to near optimal solutions. The points designated by dark circles denote the first feasible solution obtained during different runs. As seen in Figure 9, objective function values properly converge to near optimum state, which could be a sign of successful application of the algorithm for the addressed problem.

Furthermore, the algorithm could lead to the first feasible solutions at the beginning of the runs, and it could keep feasibility of the solutions till the end of the iterations. This could also be a sign that the algorithm could successfully satisfy different constraints of the problem.

Figures 10 and 11 show the best obtained release rules and balancing functions through 5 runs, respectively.

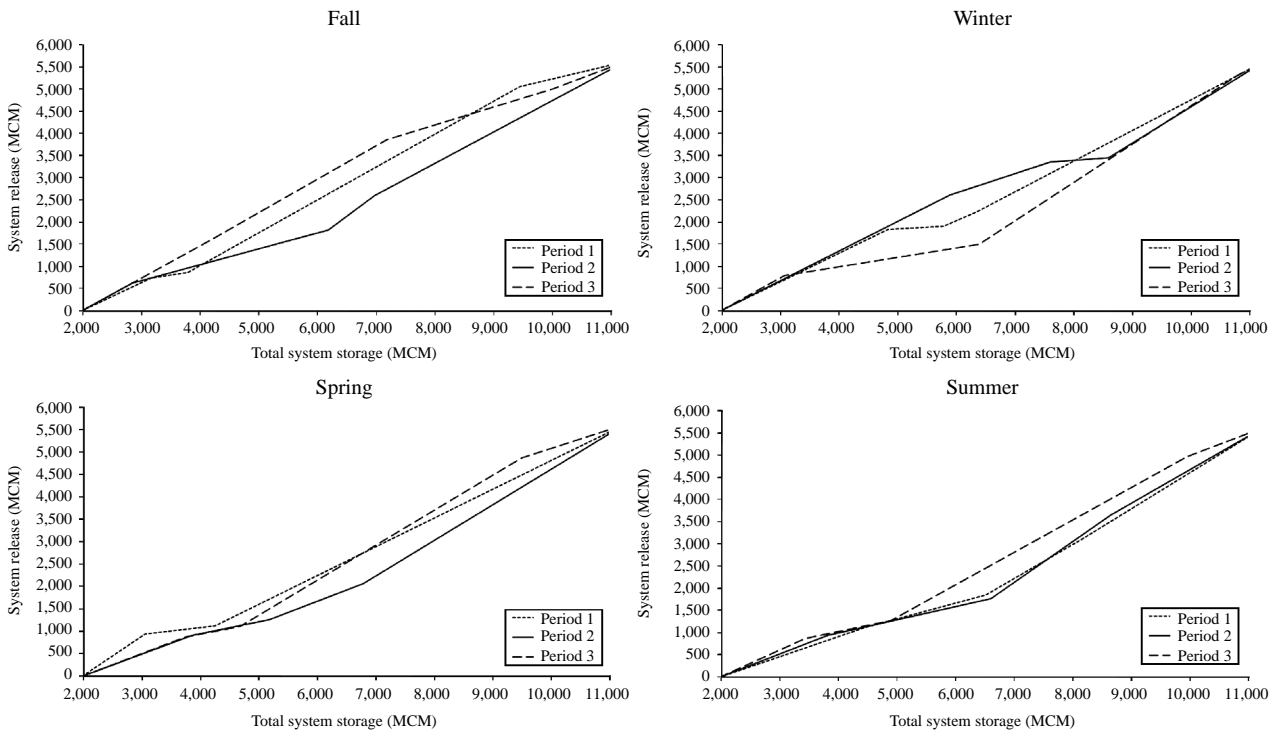


Figure 10 | System release rules (water supply case example).

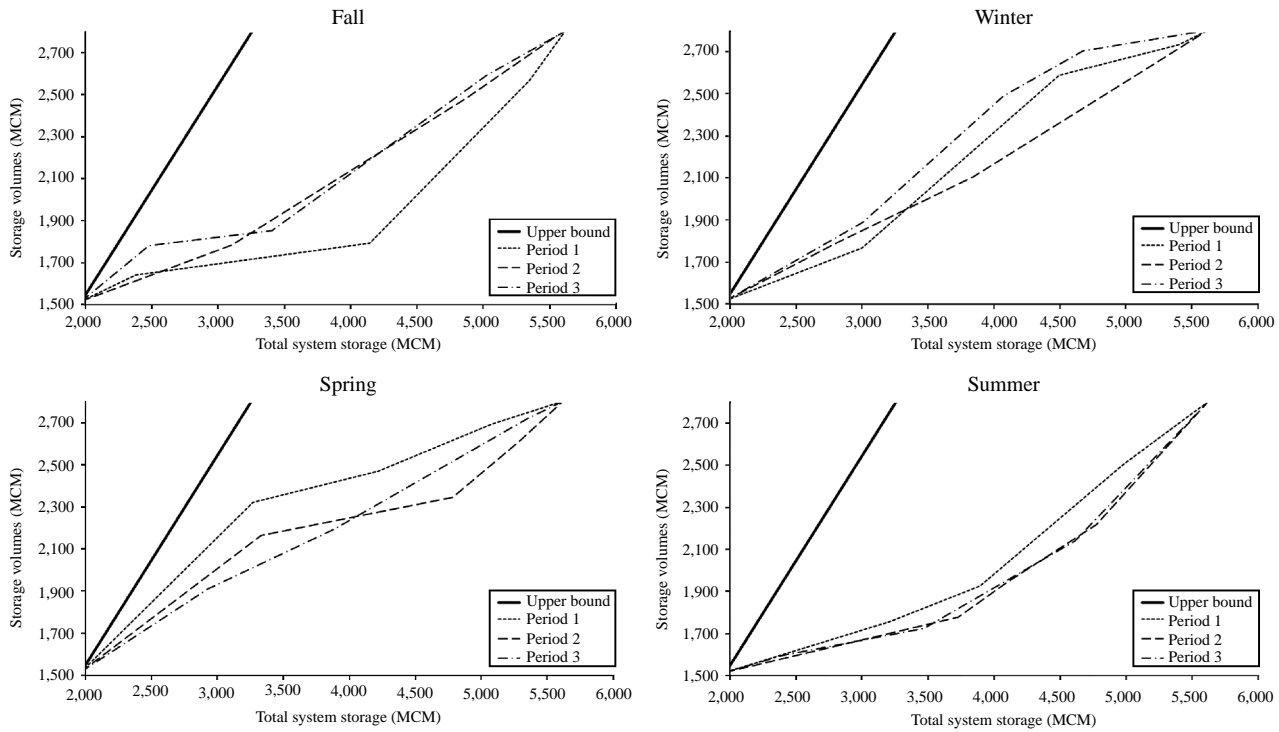


Figure 11 | Storage balancing functions for Dez reservoir (water supply case example).

In order to better visualize the graphs, they are illustrated in seasonal groups, e.g. the first 3 graphs (corresponding to Fall’s months) have been shown in the upper left panel, etc.

The releases from the system along with system demand during operation periods are presented in Figure 12, where it can be seen that yearly average deficit of 2,800 MCM is

expected to occur. Note that the system release follows the same trend as the demand. The operation therefore seems quite reasonable and takes into consideration the assigned objective function (Equation (14)).

Table 6 lists information on system releases and reservoirs storage volumes. It is highly important to notice

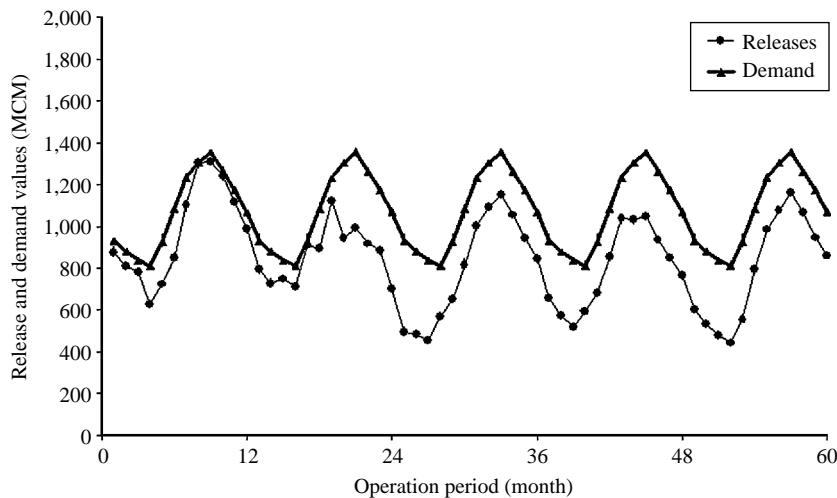
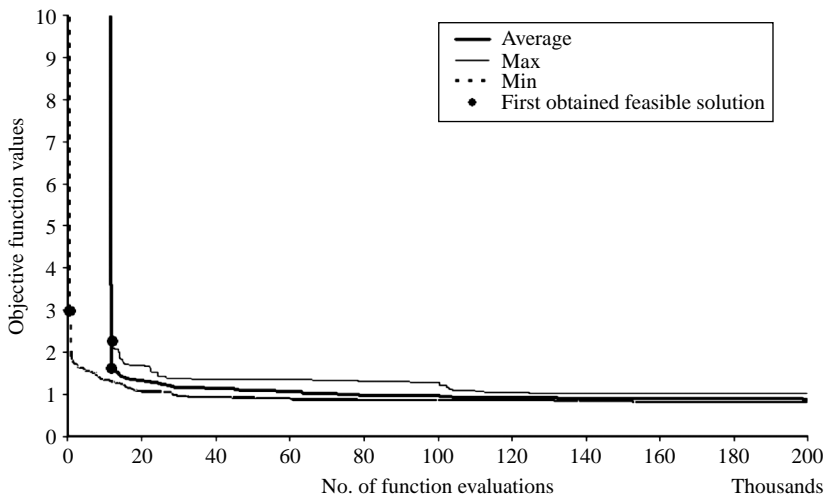


Figure 12 | System releases (water supply case example).

**Table 6** | Final releases and reservoir storages (water supply case example)

Description	Year	Initial storage volume	Number of operation period											
			1	2	3	4	5	6	7	8	9	10	11	12
Karun releases (MCM)	1	–	763	203	405	447	366	792	429	64	391	481	632	472
	2	–	440	259	263	410	223	414	158	872	463	743	451	317
	3	–	323	338	410	527	452	627	660	402	67	567	676	420
	4	–	303	302	310	399	391	330	599	515	177	635	573	384
	5	–	295	306	145	415	305	379	659	415	63	543	705	360
Dez releases (MCM)	1	–	112	609	376	182	357	58	670	1,239	919	758	485	518
	2	–	354	468	486	303	687	482	964	73	529	174	434	386
	3	–	173	146	45	40	198	187	345	689	1,082	488	266	425
	4	–	352	271	207	194	290	523	437	516	872	301	277	381
	5	–	305	225	333	26	248	418	326	662	1,097	523	241	120
Karun storage volumes (MCM)	1	2,224	1,765	1,883	1,825	1,752	1,866	2,053	2,447	2,597	2,566	2,407	2,060	1,876
	2	–	1,732	1,789	1,821	1,806	1,951	2,176	2,469	2,246	2,307	1,987	1,774	1,688
	3	–	1,675	1,676	1,790	1,729	1,833	1,960	2,353	2,315	2,493	2,151	1,811	1,717
	4	–	1,697	1,705	1,797	1,739	1,847	2,063	2,384	2,286	2,387	2,010	1,764	1,701
	5	–	1,690	1,692	1,792	1,683	1,790	1,920	2,334	2,308	2,498	2,165	1,815	1,721
Dez storage volumes (MCM)	1	1,575	1,587	1,167	1,120	1,301	1,430	2,015	2,807	2,323	2,229	1,978	1,819	1,540
	2	–	1,384	1,216	1,253	1,527	1,563	2,035	1,511	1,948	1,701	1,701	1,378	1,073
	3	–	1,675	1,676	1,790	1,729	1,833	1,960	2,353	2,315	2,493	2,151	1,811	1,717
	4	–	1,068	981	1,094	1,252	1,466	1,596	1,921	2,133	1,664	1,610	1,492	1,228
	5	–	1,019	940	862	1,118	1,271	1,417	2,143	2,421	1,843	1,639	1,604	1,333



**Figure 13** | The best, average and worst objective function values of 5 trial runs versus number of function evaluations (hydropower case example).

**Table 7** | Objective function values (water supply and hydropower case example)

Description		Number of run				
		1	2	3	4	5
Objective	Water supply	2.21	2.08	2.12	2.17	2.19
Function values	Hydropower	0.84	0.8	0.9	0.806	0.811

that Figure 12 illustrates the total release from the system, whereas Table 6 provides separate releases from different reservoirs as well as corresponding reservoirs storage values for different periods.

**Case (B): multi-reservoir operation rule curves for hydropower production**

The second problem addresses the operation of a two-reservoir system to develop an operating policy which minimizes a measure of deviation from the installed capacity. Therefore, the objective of the study is to make the power generation as close to the installed capacities as

possible. In mathematical presentation it may be defined as:

$$\text{Min} \sum_{t=1}^T \left( 1 - \frac{P_t}{\text{PPC}} \right) \tag{15}$$

where  $P_t$  is power generated by the system during month  $t$  in MW (decision variables); PPC is the system installed capacity in MW; and  $T$  is the total number of periods considered (12).

The above system is subject to the following constraints:

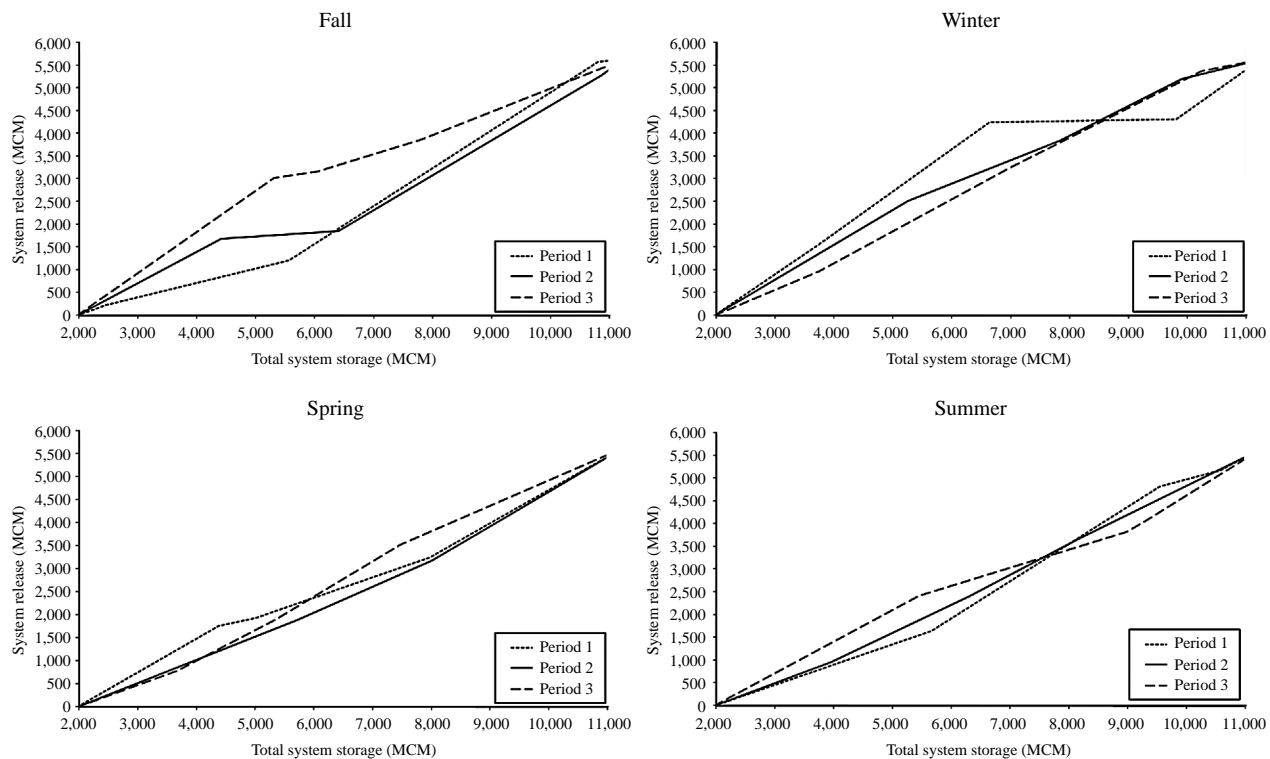
$$P(r, t) = g \times e(r, t) \times \text{RP}(r, t) / \text{PF}(r) / \text{Mul}(t) \times (\bar{H}(r, t) - \text{TW}(r)) / 1,000 \tag{16}$$

$$\bar{H}(r, t) = (H(r, t) + H(r, t + 1)) / 2 \tag{17}$$

$$H(r, t) = h_0 + h_1 \cdot S(r, t) + h_2 \cdot (S(r, t))^2 + h_3 \cdot (S(r, t))^3 \tag{18}$$

$$P_{\min}(r, t) \leq P(r, t) \leq \text{PPC}(r) \tag{19}$$

where  $P(r, t)$ ,  $e(r, t)$ ,  $\text{RP}(r, t)$ ,  $H(r, t)$ , and  $S(r, t)$  are produced power, efficiency, outflow discharge, head and storage of the reservoir  $r$  at period  $t$ , respectively.  $\text{PPC}(r)$  and  $\text{PF}(r)$  are



**Figure 14** | System release rules (hydropower case example).



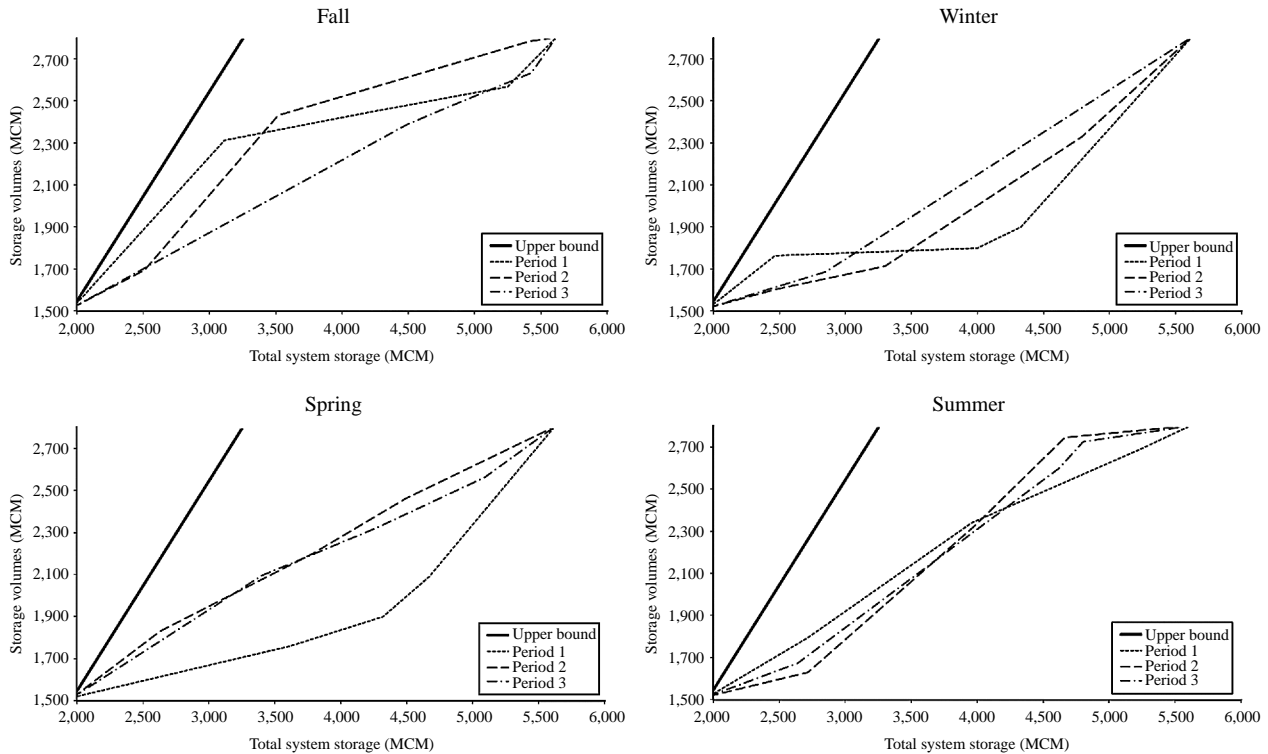


Figure 15 | Storage balancing functions for Dez reservoir (hydropower case example).

installed capacity and plant factor of reservoir  $r$ , respectively,  $Mul(t)$  is the coefficient transforming MCM to cubic metres per second,  $h_0$  to  $h_3$  are constants to obtain reservoir head from its storage and  $P_{min}(r, t)$  is the minimum produced power of reservoir  $r$  at period  $t$ .

The model defined by Equation (15) and subjected to the constraints defined by Equations (5)–(11) and (16)–(19) was applied to the same two-reservoir system shown in Figure 8. Having solved this problem, the best value for the objective function was determined as 0.8 after 5,000 mating flights, which is equal to 1.5 million function evaluations.

Figure 13 illustrates the convergence of the best, average and the worst run in 5 different runs to near optimal solutions. The points designated by the dark circles denote the first feasible solution obtained during different runs. Here, it can also be observed that the model could properly deal with the problem’s constraints and converge to the optimum state.

The objective function values and some other operation parameters after 1.5 million function evaluations are presented in Table 7, Hydropower row. Figures 14 and 15 present the optimum release rules and system balancing

Table 8 | System releases, reservoirs storage volumes and generated powers (hydropower case example)

Description	Number of operation period												
	Initial storage	1	2	3	4	5	6	7	8	9	10	11	12
System release (MCM)	–	297	631	811	799	824	727	1,620	750	912	794	757	817
Karun storage volume (MCM)	1,950	2,014	1,965	1,827	1,773	1,681	1,873	1,873	2,092	2,216	2,250	2,092	1,911
Dez storage volume (MCM)	735	868	902	1,046	1,272	1,630	2,305	1,576	1,765	1,534	1,303	1,053	730
Generated power (Karun, MW)	–	556	878	1,000	996	993	987	998	1,000	1,000	1,000	1,000	1,000
Generated power (Dez, MW)	–	79	343	513	520	520	520	520	520	520	520	520	520
Generated power (System, MW)	–	635	1,222	1,513	1,516	1,513	1,507	1,518	1,520	1,520	1,520	1,520	1,520

functions for different seasons. To be more informative, releases from the reservoirs, reservoir storage volumes and power produced at different periods are presented in Table 8. It can be seen that the total joint demand is successfully satisfied in most of the months. This means that the operating policies are appropriate to define releases and storage volumes of the system.

## CONCLUDING REMARKS

The research reported in this paper focused on the development of an improved version of HBMO algorithm to effectively derive operating rule parameters for parallel multi-reservoir systems. The first part of the study contains the required sensitivity analysis to set the parameters of the optimization algorithm, and furthermore, to apply the improved HBMO to solve a benchmark single-reservoir operation problem in comparison to the original HBMO and a real-coded-GA. It was observed that, with the same number of function evaluations, the improved HBMO algorithm enhanced the problem objective function by more than 8%. This relatively higher performance of the proposed algorithm of this paper, compared to the other two methods, encouraged the authors to apply it to two complex operation problems for a multi-reservoir system.

The obtained results and acceptable operating policies, i.e. (1) convergence of the objective function values towards an optimum state for both water supply and hydropower case examples and (2) agreement between distributions of system releases and demands for water supply example (with acceptable monthly deficits) showed that the optimization technique proposed in this study is capable of solving complex multi-reservoir systems operation problems. Moreover, the proposed structure properly handled the tight constraints defining the parallel reservoirs operation in such a way that all the generated solutions were feasible after a particular set of iterations.

The obtained release rule and storage balancing functions can be used to guide the system operators in their decision-making process. The set of rule curves provide the operator with the opportunity to systematically look at the system and to further make proper decisions. Plus, the obtained rule curve structure, composed of different graphs

dealing with system releases and reservoir storage values, would provide a higher level of flexibility in use than the classical operating policies. However, considering the variety of operating policies which could be applied to different and simple-to-complex reservoir systems, it seems reasonable to conduct research to compare different rule curves for multi-reservoir systems operation. This study is now ongoing by the authors.

Furthermore, the proposed optimization algorithm of this study might be extended, employing the concept of multi-population optimization. In other words, multi-colony HBMO algorithm can be developed in future to solve multi-modal optimization problems, as well as to estimate more efficient operation policies for highly complex multi-purpose multi-reservoir systems.

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