Well storage effect isolation and straight-line method applications

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Nordic Hydrology **Vol 37 No 2 pp 121–128** © IWA Publishing 2006

Received 18 May 2005; accepted in revised form 7 October 2005

Abstract In the case of a significant well storage, the initial period of the drawdown observation is significantly influenced by the well storage. The duration of this period is so far estimated with formulae which use the transmissivity of the aquifer. But this parameter should be determined with the interpretation of the ongoing pumping test. This paper gives an alternative formula, which does not use the aquifer parameters. The formula uses only parameters which can be determined easily during a pumping test. Furthermore, the derived formula is also valid for a large diameter pumping well. In addition, a straight-line equation with well storage effect is given for the estimation of aquifer parameters. This explicitly shows the effect of pumping well storage on observation wells and becomes equivalent to the well-known Cooper and Jacob (CJ) method after large pumping duration. Moreover, simple unique formulae are offered for estimating both transmissivity and storativity parameters at large observation times. The application of the developed methodology is presented for the pumping test data, which are available in the literature.

Keywords Aquifer parameters; pumping duration; straight-line method; well storage

Introduction

Pumping wells are used for domestic and agricultural water supplies in many areas in the world. An efficient management and development of groundwater resources seems to require rapid and reasonable determination of aquifer hydraulic characteristics, namely, transmissivity and storage coefficients. In general, this objective can be achieved by means of pumping tests and their subsequent analysis. The concerned methods analyze the drawdown measurements within the wells drilled in confined aquifers which have been discussed by many researchers (Theis 1935; Cooper and Jacob 1946; Papadopulos and Cooper 1967).

These methods calculate storage coefficient and transmissivity by means of type curve matching procedures. The flow properties of an aquifer depend on its geological characteristics, which are controlled by the evolution of rock structure and by secondary geological events like weathering and rock composition alteration. Though it has been easily possible to interpret the transmissivity of the aquifer by means of Thiem (1906) equations, it is valid for a quasi-steady-state groundwater movement in porous media, but the estimation of the storage coefficient has not been possible by this equation. The storage coefficient plays a vital role for the transient behavior of groundwater flow, and also in the estimation of the groundwater discharge from an aquifer. Therefore, its precise determination is one of the prerequisites in groundwater resources assessment and management in a region.

Theis (1935) gave an equation for computing unsteady drawdowns assuming radial groundwater flow to a well in confined aquifers. This equation has been used successfully through type curve matching procedures for determination of the transmissivity and the

doi: 10.2166/nh.2006.003

storage coefficients. Cooper and Jacob (1946) suggested a straight-line graphical method based on the Theis equation for the evaluation of aquifer parameters. Later, Chow (1952) developed a graphical method that has the advantages of avoiding curve fitting and being unrestricted in its application. All of the above methods are considered for unsteady groundwater flow to infinitesimally small diameter pumping wells. They do not take into account the storage effect of the pumping well on nearby observation wells.

In low-permeability environments, the pumping wells can be large for providing a big volume as water storage. Effects of the water stored in the well are referred to as well storage, which usually creates difficulty in analyzing aquifer test data. Well storage may become important when the aquifer transmissivity and storage coefficient are small or the pumping well diameter is large (Singh 2000). Due to the significant effect of the well casing storage, conventional methods of pumping test analysis are unsuitable for large-diameter wells. The well storage influences the drawdowns especially during the early times of a pumping test, when most of the water is produced from water in the well itself. As the pumping continues, water contributes more and more to the pump discharge from the formation, and drawdown begins to follow the Theis curve at moderate and large times. Papadopulos and Cooper (1967) presented the analytical solutions allowing for unsteady groundwater flow to a fully penetrating large-diameter pumping well of constant rate from a confined aquifer. These solutions require curve plots on a double logarithmic paper as drawdown versus time. Type curves have parallel lines for different values of the storage coefficient at small time instances whereas at large times they all merge with the Theis curve.

However, many works include numerical calculations for the determination of aquifer parameters from pumping tests. Some of these methods depend on matching the observed aquifer response during the whole, early or late times of the pumping to a type curve (Wikramaratna 1985; Şen 1986, 1988, 1996; Srivastava and Guzman-Guzman 1994; Shapiro and Oki 2000; Singh 2001). Other works depend on numerical solutions of the differential equations (Rushton and Holt 1981; Patel and Mishra 1983; Rai 1985; Singh and Gupta 1986; Yeh 1987). Furthermore, Şen (1987) gave simple formulae for pumping wells with the aid of the Thiem (1906) equation in order to determine the storage coefficient of confined and unconfined aquifers. His formulae require known transmissivity beforehand, and they also depend on the pumping well radius, discharge and late time–drawdown measurements during quasi-steady-state flow conditions. Neither the Theis method nor the Cooper and Jacob method takes into account the well storage effect. The main well storage influences drawdowns even in the observation wells, especially when they are close to a large-diameter well. Therefore, they might cause biased evaluation of aquifer parameters.

This paper presents application of the straight-line method for estimation of aquifer parameters in the case of large-diameter pumping wells in confined aquifers. For this reason, a straight-line equation showing the effect of pumping well storage is given for the estimation of aquifer parameters. Furthermore, a simple pumping duration formula is suggested to avoid the well storage effect. This formula which doesn't use the aquifer parameters gives the initial duration of the drawdown observation, which is affected by well bore storage. Simple unique formulae which are valid at large pumping durations are also offered for estimating both the transmissivity and storativity parameters. These provide a reliable determination of the aquifer parameters.

Analysis

In case of axially symmetric radial groundwater flow towards a fully penetrating well in an extensive, homogeneous, isotropic and confined aquifer of uniform thickness, the analytical solutions of unsteady flow equations with well casing storage have been already obtained by different authors such as Papadopulos and Cooper (1967), Şen (1982) and Çimen (2001). Additionally, Çimen (2001) presented the following simple equation for general solution of drawdown with well storage effects:

$$s(r,t) = \frac{Q}{4\pi T}\left[\frac{1}{u_w} - \frac{1}{Sr_w^2/r_c^2}\frac{s_w(t)}{Q/4\pi T}\right]\frac{\int_u^\infty \frac{e^{-x}}{x}\,dx}{\int_{u_w}^\infty \frac{e^{-x}}{x^2}\,dx} \tag{1}$$

where $s(r,t)$ is the drawdown in the aquifer at radial distance r and at time t since pumping started with a constant discharge, Q; T and S are the transmissivity and the storage coefficient of the aquifer; r_w and r_c are the radius of the well and casing; and finally, $s_w(t) = s(r_c, t)$ is the drawdown as a function of time within the well. Furthermore, $u = u_w r_D^2$ is the dimensionless time variable valid for the aquifer; $u_w = Sr_w^2/4Tt$ is the dimensionless time variable at the well face; and finally, $r_D = r/r_w$ is the dimensionless distance variable.

In Equation (1), the term in the bracket varies with pumping time only. The integration term in the denominator also depends on the pumping time, and it can be expressed in form of the Theis well function as

$$\int_{u_w}^\infty \frac{e^{-x}}{x^2}dx = \frac{1}{u_w e^{u_w}} - \int_{u_w}^\infty \frac{e^{-x}}{x}dx. \tag{2}$$

After the substitution of this expression into Equation (1) and the completion of the necessary mathematical manipulations, the analytical form of the drawdown within the aquifer becomes

$$\frac{s(r,t)}{1 - \frac{\pi r_c^2 s_w(t)}{Qt}} = \frac{Q}{4\pi T}\frac{\int_u^\infty \frac{e^{-x}}{x}dx}{\frac{1}{e^{u_w}} - u_w\int_{u_w}^\infty \frac{e^{-x}}{x}dx}. \tag{3}$$

For $u_w \le 0.001$, which implies late times, the denominator on the right-hand side of Equation (3) approaches 1 (0.9927 for $u_w = 0.001$). In addition, the exponential integral in the numerator reduces to the Cooper and Jacob (1946) solution for $u = u_w(r/r_w)^2 \le 0.01$. By considering these limitations, the drawdown equation within the aquifer for late times of pumping can be rewritten as

$$\frac{s(r,t)}{1 - \frac{\pi r_c^2 s_w(t)}{Qt}} = \frac{Q}{4\pi T}(-0.5772 - \ln u). \tag{4}$$

While the expression within the bracket on the right-hand side of this equation gives the straight-line expression, the term $\pi r_c^2 s_w(t)/Qt$ represents the storage effect of the pumping well on the observation wells. As pumping duration increases, it tends to zero. Similarly, when $r_c \to 0$ (infinitesimally small diameter well) the storage effect of the well is not appreciable. The drawdown measurements at observation wells close to the main large-diameter well are affected from the well storage only during the early times. Sometimes, the effect of the well storage may continue for longer times in observation wells due to the hydrogeological characteristics of the aquifer. The classic straight-line method does not take into account the pumping duration and well storage effect on observation wells. For this reason, Equation (4) which includes the well storage effect can be used for determining the aquifer parameters. Therefore, a graph can be obtained from the left-hand side of Equation (4) versus log t as log r or log t/r^2. This graph is expected to pass through the field data as a curve during the early and moderate times of pumping. The initial point of the straight-line plotting on the graph for late times will show the minimum pumping duration that is necessary for the main well storage effect negligence.

Determination of pumping duration and aquifer parameters

The time period of a pumping test is one of the most important characteristics in any pumping analysis such as the pumping rate, the well casing diameter and the aquifer structural properties. The aquifer parameters obtained from inadequate pumping test data cannot be satisfactory. In the case of groundwater flow towards large-diameter wells, adequate pumping duration is important especially due to the storage effects of the pumping well. Therefore, various formulations have been suggested for adequate pumping period by different researchers.

First, Papadopulos and Cooper (1967) showed that the well should be pumped for a certain duration before well storage is no longer dominant on drawdown value. They gave the following expression for the reasonable period of the Theis function as

$$t > 250\frac{r_c^2}{T}. \tag{5}$$

However, Herbert and Kitching (1981) explained that if the well diameter is large and/or the transmissivity is low, the pumping time can still be substantial before drawdown data can be matched to the Papadopulos and Cooper type curves. They determined the time required for matching data to the type curves as

$$t > 25\frac{r_c^2}{T}. \tag{6}$$

Similarly, Şen (1995) suggested test durations for determining the aquifer parameters from a large-diameter well using time–drawdown data as

$$t > \alpha\frac{r_c^2}{T} \tag{7}$$

where α is a constant having different values depending on the storage coefficients.

All the above relationships depend on transmissivity. This dependence causes difficulty in estimating the duration of the pumping test. In order to neglect the storage effect of the pumping well on the observation wells, it is assumed that the well storage expression in Equation (4) is less than 0.005. Hence, the duration of pumping can be calculated as

$$\frac{Qt}{\pi r_c^2 s_w(t)} \geq 200. \tag{8}$$

As can be seen from this equation, it is not dependent on the transmissivity value but on the measured drawdown in the well. This relationship is also valid in the case of large-diameter pumping wells. For large observation times, the transmissivity and storage coefficient formulae from Equation (4) together with the pumping duration condition, Equation (8), reduce to the Cooper and Jacob's formulae. For example, from the late time–drawdown (t–s) graph one can find that

$$T = \frac{2.303Q}{4\pi\Delta s} \tag{9}$$

and

$$S = \frac{2.25T\,t_0}{r^2} \tag{10}$$

where Δs is the difference in drawdowns over one logarithmic cycle, and t_0 is the value of time at the intercept.

In order to remove exactly the storage effect on the observation and pumping wells, the pumping duration criterion (Equation (8)) at larger observation times can be considered as

$$\frac{Qt}{\pi r_c^2 s_w(t)} \geq 1000. \tag{11}$$

Here, the drawdown measurements in both the pumping and observation wells in homogeneous confined aquifers increase uniformly. Hence, the aquifer parameters can be determined directly from the drawdowns. By neglecting the expression $\pi r_c^2 s_w(t)/Qt$ in Equation (4), formulae of the aquifer parameters together with Equation (11) can be written simply as

$$T = \frac{Q}{4\pi} \frac{\ln(t_2/t_1)}{s(r,t_2) - s(r,t_1)} \tag{12}$$

and

$$S = \frac{2.25T\,t}{r^2} e^{-4\pi Ts(r,t)/Q} \tag{13}$$

where t_1 and t_2 are the successive late times of the pumping.

Applications to field data and discussions

The first confined aquifer test example is taken from the 24-hour field data reported by Kresic (1997). In the aquifer domain, there are three observation wells with distances $r = 5.5$ m, $r = 40.5$ m and $r = 118$ m in addition to the main well. The pumping rate is 0.008 m^3/s, and the diameter of the well screen is 0.35 m. This diameter is not exactly reflected in the effect of large-diameter well. But then again this well is accepted as a large-diameter well, and the time–drawdown data is considered for the application of the straight-line methodology and the formulae presented in this paper. In order to show the difference between the Cooper and Jacob method and the methodology of this paper according to Equation (4) the time–drawdown curves for three observation wells (OW1, OW2 and OW3) are plotted on semi-logarithmic paper (see Figure 1). Differences between these curves prior to 10 000 s are apparent, and the well storage effect appears exactly in each observation well. The closer the observation well (OW1) to the main well, the bigger is the well storage effect.

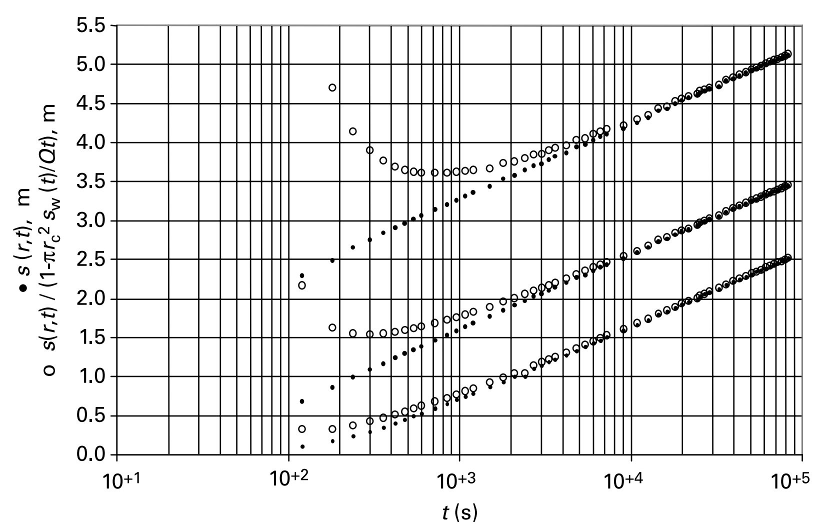

Figure 1 Time–drawdown graph for Kresic's data

For reliable estimation of the aquifer parameters, the minimum pumping duration which does not appear in the well storage effect is determined approximately as 23 400 s according to Equation (8). For this duration, the drawdown within the pumping well (s_w) was obtained as 9.151 m. Hence, the pump duration criterion given by Equation (8) is calculated as 212.6. For the data after 23 400 s, the slope of the straight-line on semilogarithmic paper is obtained as 0.95. Kresic (1997) obtained 0.92, according to the Cooper and Jacob method. This difference affects the values of the transmissivity and storage coefficient. The parameter values obtained from the classical methods by Kresic (1997) and the straight-line methodology of this paper are presented in Table 1.

On the other hand, the necessary pumping duration for well storage isolation from Equation (5) is obtained as $t = 4970$ s with $T = 1.54 \times 10^{-3}$ m^2/s. As can be seen from Figure 1, this duration is shorter for reliable evaluation of the aquifer parameters, because the well storage effect is not yet completely eliminated. Furthermore, the necessary pumping duration from Equation (6) is comparatively very short, but the necessary pumping duration from Equation (7) is rather large.

As can be seen in Table 1, Equations (12) and (13) are applied to Kresic's data again. The pumping duration criterion (see Equations (8) and (11)) is 712.3 for $t = 82\,800$ s and $s_w = 9.666$ m. Although the pumping duration criterion is less than 1000, the estimations of the aquifer parameters are sufficient.

Another independent set of data given by Lohman (1972) and treated by Kashef (1986) and Singh (2001) is considered. Each test is recorded at two observation wells, which are located at 121.92 m and 243.84 m distance from the pumping well (Table 2). The constant discharge rate is 1.888 m^3/min, and the diameter of the pumping well is assumed as infinitesimally small. The data is evaluated with Equations (12) and (13). The parameter results are given in Table 3, and it shows that these equations can provide additional

Table 1 Comparison of the estimated aquifer parameters for the data taken from Kresic (1997)

Method	T (m^2/s)	S
Kresic (1997)		
Theis		
OW1	1.38×10^{-3}	8.7×10^{-5}
OW2	1.51×10^{-3}	4.7×10^{-5}
OW3	1.50×10^{-3}	5.2×10^{-5}
Cooper and Jacob		
OW1 ($t-s$)	1.59×10^{-3}	–
OW2 ($t-s$)	1.59×10^{-3}	5.2×10^{-5}
OW3 ($t-s$)	1.59×10^{-3}	6.0×10^{-5}
Composite (t/r^2-s)	1.49×10^{-3}	5.0×10^{-5}
This study		
Cooper and Jacob*		
OW1 ($t-s$)	1.54×10^{-3}	3.67×10^{-5}
OW2 ($t-s$)	1.54×10^{-3}	4.14×10^{-5}
OW3 ($t-s$)	1.54×10^{-3}	4.65×10^{-5}
Equations (12) and (13)		
PW	1.49×10^{-3}	1.37×10^{-6}
OW1	1.49×10^{-3}	5.70×10^{-5}
OW2	1.49×10^{-3}	5.25×10^{-5}
OW3	1.49×10^{-3}	5.48×10^{-5}

*Values of the transmissivity and the storage coefficient calculated from Equations (9) and (10) beginning from $t = 23400$ s.

Table 2 The final portion of the data taken from Lohman (1972)

Time since pumping started, *t* (min)	OW1 $r_1 = 121.92$ m s_1 (m)	OW2 $r_2 = 243.84$ m s_2 (m)
210	0.856	0.619
240	0.878	0.643

Table 3 Comparison of the estimated aquifer parameters for Lohman's data

Method	*T* (m^2/min)	*S*
Lohman (1972)		
Theis		
OW1	0.8779	1.978×10^{-4}
OW2	0.8779	1.978×10^{-4}
Cooper and Jacob		
OW1	0.8736	1.850×10^{-4}
OW2	0.8658	1.883×10^{-4}
Kashef (1986)		
Theis		
OW1	0.8032	2.2×10^{-4}
OW2	0.8907	2.0×10^{-4}
Cooper and Jacob		
OW1	0.8532	2.0×10^{-4}
OW2	0.8860	2.0×10^{-4}
Singh (2001)		
Temporal derivative of drawdowns		
OW1	0.8605	1.963×10^{-4}
OW2	0.8592	2.058×10^{-4}
This paper		
Equations (12) and (13)		
OW1	0.9119	1.606×10^{-4}
OW2	0.8359	2.121×10^{-4}

information about the aquifer parameters. The results for the further observation well (OW2) appear close to the other methods. The main cause of deviation in the parameter values calculated from OW1 data is not to take the storage effect of the pumping well into consideration by the authors above.

Conclusions

In this paper, a straight-line equation with well storage effect is given for the estimation of aquifer parameters. This equation is applied to available pumping test data from the literature, and it is shown that the pumping well storage affected the drawdowns of observation wells. This effect is explicitly shown in Figure 1. Furthermore, a formula for the minimum pumping duration is given for well storage effect isolation $(Qt/\pi r_c^2 s_w(t) \geq 200)$. The formula depends on the time–drawdown value of the pumping well which can be determined during a pumping test. It is seen that this pumping criterion affects the values of the aquifer parameters obtained by using the Cooper–Jacob method. In addition to the straight-line equation, simple unique formulae are suggested to estimate both the transmissivity and storage coefficient of the aquifer. It is seen that these formulae can provide additional information about the parameter estimations for large observation times $(Qt/\pi r_c^2 s_w(t) \geq 1000)$.

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