fluids such as air and water. This conclusion would not necessarily apply to liquid metals.

**Experimental Data**

Johnson, Hartnett, and Clabaugh have obtained data with liquid metals at low Reynolds numbers, using apparatus rather similar to the exchanger in Fig. 1. These two sets of data, which were calculated on the \(\text{Nu}_{\text{probe}}\) basis for sections remote from entry, are plotted in Fig. 2.

![Graph](image)

**FIG. 2** \(\text{Nu}_{\text{probe}}\) Versus \(\text{Nu}_{\text{mix}}\) Values

English and Barrett used nickel and stainless-steel tubes of 1.3 mm ID and 1.5 mm OD, coated with a copper sheath 0.3 mm thick. The total axial conductivity of the tube was roughly 50 times that of the fluid, which was mercury. While their data appear qualitatively to show the results of having used \(T_{\text{ab}}\) rather than \(T_{\text{probe}}\) in their Nusselt numbers, a \(K\) of almost 1000, rather than 50, would be required for quantitative explanation. The \(K\) may, in effect, have been that large, for relatively massive copper bus bars were soldered to the ends of the tube, which was directly heated electrically. Not enough detail is reported to permit calculation of the effect of these bus bars on the value of \(K\).

Johnson, Hartnett, and Clabaugh used a 0.652-in-ID 0.750-in-OD mild-steel tube with an aluminum sheath 0.25 in. thick bonded to the outside. If \(K\) values of 4.8, 26, and 90 Btu/hr ft deg \(F\) are assumed, respectively, for the liquid metal, mild steel, and aluminum, the resultant \(K\) ratio is 47. Their laminar flow data would agree with the \(\text{Nu}_{\text{mix}}\) analysis for the exchanger in Fig. 1 if \(K\) were about 100.

The exchanger used by Johnson, Hartnett, and Clabaugh corresponded closely to the exchanger in Fig. 1, except for a series of circumferential slots cut through the aluminum sheath at 6-in. intervals. This was done to minimize longitudinal conduction of heat. The wall thermocouples, from which the Nusselt numbers were calculated, were midway between the slots. At low (RePr) values, these slots would have a pronounced effect on the temperature distribution, both in the tube and in the fluid; \(dT/dx\) would not be constant over the section, and the heat-transfer conditions would not correspond to long-tube conditions. Their apparatus is not, therefore, subject to the same simple analysis made for the exchanger in Fig. 1. It is not immediately obvious whether the slots would reduce or exaggerate the \((\text{Nu}_{\text{probe}} - \text{Nu}_{\text{mix}})\) discrepancy of the simpler apparatus.

**Conclusions**

The procedure for measuring mean fluid temperatures influences the temperature obtained.

Mean temperature is usually defined as the temperature which would be measured if a mixing box were inserted. If this were actually done the temperature of the fluid would be changed because the energy of the moving fluid would have been altered by an amount equal to the axial heat conduction along the fluid and along the apparatus.

The commonly used method for measuring mean temperature is to add to the inlet temperature a temperature rise corresponding to the heater input up to the section being considered. This heater-input mean temperature equals the mixing-box mean temperature.

The mean temperature, if measured from traverses with velocity and temperature probes, does not represent the actual fluid energy by an amount equal to the axial eddy diffusion of heat energy. If axial eddy-diffusion coefficients are of the same order of magnitude as radial eddy-diffusion coefficients, this discrepancy is probably negligible in practice for nonconducting fluids in long-tube heat exchangers.

Theoretical predictions of Nusselt numbers are usually made on a probe mean-temperature basis. Experimental data are usually reported on a mixing-box mean-temperature basis.

In practice, for long-tube heat exchangers at steady state, the discrepancy between the probe and the mixing-box mean temperatures would be important only for \((\text{RePr})\) values less than 30, unless the apparatus itself conducts a large amount of heat axially.

\[\text{Nu}_{\text{mix}} = \frac{hL}{k} = \frac{2hL}{k} \text{ or } \frac{4hL}{k} \text{ depending on the form of the exchanger.}\]

Two sets of experimental data for liquid metals, obtained with highly conductive apparatus somewhat similar in form to the simple heat exchanger analyzed in this paper, showed unexpectedly low Nusselt numbers at low Reynolds-number operation. It would appear that these data demonstrate qualitatively the effect of employing mixing-box mean temperatures. Neither set agrees quantitatively with the analysis of the simple exchanger of Fig. 1, and each set was obtained on apparatus which differed significantly from that simple exchanger. The experimental data, therefore, appear to demonstrate the mixing-box discrepancy, but in an inconclusive fashion.

While the influence of mean-temperature-measuring procedures can be shown in principle, it would be difficult to calculate their effects for any but the simpler systems. In long channels, however, they would appear to be significant only at low (RePr) values or for highly conductive apparatus.

**Discussion**

J. P. Hartnett and W. J. Clabaugh. The paper introduces a concept which certainly should have been considered in the analysis of the laminar-flow, liquid-metal, heat-transfer results. However, even assuming an effective \(K\) value of 47 the resulting lowest Nusselt values are still approximately 2.0 and therefore...

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3. See footnote 4 of paper.

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considerably below the so-called limiting value of 4.36 for laminar flow. It is felt by the writers that the assumed \( h \) value of 47 is already on the high side since the circumferential slots should decrease this value appreciably. Any detailed analysis along the lines proposed by the author would be extremely difficult in view of entrainment effects, exit effects, and limited data on the tube-wall temperature distribution. The better attack would be to rerun the tests with more instrumentation and perhaps with a different exchanger which would permit a more simple analysis. However, it is still felt that the low experimental data should not be completely discounted until further experimental work is accomplished, particularly in view of the many existing anomalies in liquid-metal heat transfer.

R. H. Norris,\textsuperscript{11} For problems where axial heat flow is not negligible, the attempt to select some particular definition of mean or effective temperature difference between the duct surface and the liquid metal may not be worth while.

The justification for customary use of the log-mean temperature difference in turbulent counterflow-convection problems with negligible axial conduction is that it results in a mean coefficient of heat transfer \( h_t \), which is substantially independent of the length of the duct and is equal at most points of interest to the local coefficient \( h_t \). The heat flow in this case is essentially one-dimensional, so that we can then consider the thermal-resistance distribution as simply a string of equal resistance “lumps” connected in parallel. The log-mean temperature difference then leads to simpler results than use of the temperature difference at the inlet.

But in two or three-dimensional fields of heat flow, which arise when axial heat flow is appreciable in a duct, we have a two or three-dimensional distributed thermal-resistance field which cannot adequately be replaced by any single string of equal average lumps of resistance. It can be represented only by a whole network, at least two-dimensional in nature, of resistance elements, which, at least at the ends, include some of unequal magnitudes. In this case, the local temperature rise above the fluid temperature well upstream of the inlet (if uniform transversely there) may be the most convenient form by which to represent the results. There will then be some value for the ratio of this temperature rise, above this inlet temperature, to total heat input, for each point of the system. This ratio will depend on the point chosen, on the distribution of heat input, and on the length-to-diameter ratio, as well as on the usual parameters. This ratio might perhaps be called an effective thermal resistance, although it will not represent a true local resistance element of a network for the particular spot chosen.

Regardless of what form is chosen to represent the results, however, the old concept of either a local, or a mean, heat-transfer coefficient \( h \), does not seem to have the usual convenient significance if both axial and radial heat flow are of considerable influence.

R. A. Searby,\textsuperscript{12} The author has demonstrated the importance of longitudinal heat conduction in the evaluation of heat-transfer coefficients for fluids of high conductivity under circumstances in which the Peclet number is low. It may be useful to view the problem from a slightly different and initially more general point of view, in which the heat-transfer coefficient is defined as \( h = \frac{Q}{\Delta T M} \), the conventional definition, with \( T_M \) the mixed mean, or bulk, temperature as defined by Equation [6] of the paper. Experiment will yield values different from the true value when errors are made in the evaluation of the three quantities defining the coefficient.

The analysis of the paper, and that given here, deal with a pipe heated uniformly over part of its length, as illustrated in Fig. 1 of the paper. Only longitudinal conduction is considered, under the implication that the radial temperature distribution, while not negligible, retains a symmetry which enables the specification of the longitudinal heat flow in terms of only one temperature, that of the inner wall for the pipe and of the mixed mean for the fluid. With these assumptions the following equations are obtained from energy balances on sections of the wall and of the fluid, respectively:

\[
k_w A_w \frac{d^2 T_w}{dx^2} - \pi D h_w (T_w - T_M) = W \quad \ldots \ldots [16]
\]

\[
k_f A_f \frac{d^2 T_M}{dx^2} - A_f \rho c_e \frac{dT_M}{dx} + \pi D h_w (T_w - T_M) = 0 \quad \ldots \ldots [17]
\]

In the unheated sections of the tube the heat generation per unit length \( W \) is zero.

It is difficult to be specific enough about conditions at the points of inlet and outlet fluid-temperature measurement to prescribe the boundary condition without some question about physical reality. If the tube is presumed to extend far upstream of the heated section then there \( T_M = T_w = T_{inlet} \), with \( x = -\infty \). At the outlet section, distant \( d \) downstream of the heated section, the action of the mixing pot may give \( T_M = T_w = T_{outlet} \). Even with such simplifications, however, the solution has not been worked out completely.

A large simplification is obtained by neglecting the conduction in the tube wall entirely, so that a single equation is obtained from Equations [16] and [17] of this discussion. With

\[
P = \frac{u D}{x} \quad \text{and} \quad Q = \frac{W}{\pi k_f}
\]

the following solution is obtained for this case and boundary conditions of \( T_f = T_b \) at \( x = -\infty \), and \( dT_f/dx = 0 \) at \( x = l \), the end of the heated section

\[
T_f = T_b + \frac{4}{P} \int_{P}^{1 - e^{-P}} 1 - e^{-P} \left( \frac{P - x}{D} \right)
\]

Fig. 3 of this discussion shows schematically this type of temperature distribution. The wall temperature is above the fluid temperature by the amount \( W/(\pi k_f D) \) and is equal to the fluid temperature in the unheated sections of the exchanger.

Fig. 3 shows also the interpretation upon which the results obtained in footnote 4 of the paper were obtained. There the measured inlet and outlet temperatures were assumed to exist at the ends of the heated section, and the mixed mean temperature of the fluid was assumed to change linearly between these points, as it would if there were no longitudinal heat conduction. The wall temperature was assumed to vary linearly, parallel to the fluid temperature, and for illustration, a dashed line is placed through the wall temperatures on this basis in Fig. 3. In the comparison of heat-transfer coefficients that follows, the differences between the analytically predicted wall-temperature distribution and this estimated distribution, as shown by the dashed line, are assumed to be small.

The heat-transfer coefficient evaluated on the basis of the temperature distribution between the dashed lines is now designated as \( h_k \) and the true value as \( h_t \). Then

\[
W \left( \frac{1}{h_k} - \frac{1}{h_w} \right) = T_w - T_b - \frac{4Q x}{P D} + \frac{4Q}{P^2} \left[ 1 - e^{-4P} \left( \frac{1 - x}{D} \right) \right] + T_w + T_b + \frac{4Q x}{P D}
\]
or
\[
\frac{W}{D\pi h_t - 1} = \frac{4Q}{\pi^2} \left[ 1 - e^{-4\left(\frac{1-x}{D}\right)} \right]
\]

Now, except for a few diameters near the downstream end of the heated section
\[
e^{-4\left(\frac{1-x}{D}\right)} \approx 0
\]
and the apparent coefficient is found to be related to the true coefficient as
\[
h_a = \frac{h_t}{1 + \frac{4kD}{L} \left(\frac{\alpha}{uD}\right)^2}
\]

This is the relation obtained by the author, except that in

Equation [13] of the paper the conduction of the fluid is augmented by that which would take place in the wall, and the factor \(K\) denotes the ratio of the augmented conduction to that which would take place in the fluid alone. This is certainly an approximation to the situation that would be given by a solution of Equations [16] and [17] of this discussion, but it appears that this would overestimate the effect, since conduction in the wall in a sense to the situation that would be given by a solution of Equations [16] and [17].

Finally, it appears that in the exchanger used to obtain the results presented in the paper (reference, footnote 4), the general effect may have been quite small, for if the radial slots which separated the aluminum sections are accounted for, conduction would occur only through the steel wall of the tube, making \(K \approx 2\). Also, such longitudinal conduction as did occur in the aluminum sections probably had little effect on the wall-temperature distribution because of the high “isothermal” effectiveness of the individual sections. That is, for the low Peclet numbers concerned, the local heat-transfer coefficient would vary only over a small portion of each 6-in-long heated section.

Fig. 3

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**Author's Closure**

Mr. Norris cautions appropriately against oversimplification of multidimensional problems, a point which the author should like to emphasize. The adoption of a mean temperature in itself implies that the temperature field being studied can be usefully considered as of one or zero dimensions. All real cases are at least three-dimensional. Unless the problem has considerable symmetry and simplicity, such a reduction in dimensions results in an inability to reconstruct the temperature field of the original in terms of the mean temperature, and the concept of a heat-transfer coefficient loses usefulness, as Mr. Norris suggests.

The idea of a mean temperature, and of a heat-transfer coefficient, is useful, however, for such a simple system as that shown in Fig. 1. The entry region, where \((T_w - T_{mean})\) cannot be represented by a single number, is well in the interior of the region \(A-A-B-B\). The influence of axial conduction, which can be large if the Peclet number is low or if the conductivity of the apparatus is large, makes it necessary to be consistent in the use of a mean temperature, as discussed in the paper. With such consistency, experimental results can be compared, without ambiguity, to theoretical predictions. This simplicity, however, relies heavily on section \(A-A-B-B\)'s being in a region where \(\partial T/\partial r\) is constant and not a function of radius or axial position.

Messrs. Hartnett and Clabaugh's comments are well taken. A detailed analysis of the effect of the repetitive circumferential slots of the apparatus described in their paper (footnote 4) would be difficult, even with laminar flow, for the radial symmetry would only reduce the problem to that of two-dimensional heat convection and conduction. As they suggest, the low experimental results cannot be considered to be explained until the system from which they were obtained can be analyzed, or until data are obtained from a simpler system which can be predicted.

Professor Seban presents an alternate analysis, in which conduction in the tube wall is assumed to be negligible. As he points out, the analysis reduces to Equation [13] at regions remote from entry, except that \(K\) assumes the special value of 1. His assumption that axial heat conduction within the fluid can be expressed in terms of the gradient of a mixed mean temperature amounts to a reduction in dimensionality in the problem, and makes the analysis approximate in axially nonlinear regions, such as entry regions. With respect to the suggestion that Equation [13] overestimates the effect of conduction in regions which are axially linear, the author does not understand how thick walls would increase the heat-transfer area and thus reduce wall temperatures below the measured values. With respect to the circumferential slots of the apparatus described by Johnson, Hartnett, and Clabaugh (footnote 4) the author is uncertain that the effect is in the direction indicated by Professor Seban. Although his assumption seems intuitively reasonable, it needs the confirmation of analysis of the multidimensional heat flow around such slots and within the moving fluid.

In a quite recent paper, Hall and Crofts mention that, in a figure-eight exchanger, a few measurements of the Nusselt number for laminar flow "showed a continued decrease as the Reynolds number decreased." They suggested that this might be due to axial conduction. In a recent letter to the author, Mr. Hall said that the effect may be studied in more detail in an axially nonlinear regions, such as entry regions, except that \(K\) assumes the special value of 1. His assumption that axial heat conduction within the fluid can be expressed in terms of the gradient of a mixed mean temperature amounts to a reduction in dimensionality in the problem, and makes the analysis approximate in axially nonlinear regions, such as entry regions. With respect to the suggestion that Equation [13] overestimates the effect of conduction in regions which are axially linear, the author does not understand how thick walls would increase the heat-transfer area and thus reduce wall temperatures below the measured values. With respect to the circumferential slots of the apparatus described by Johnson, Hartnett, and Clabaugh (footnote 4) the author is uncertain that the effect is in the direction indicated by Professor Seban. Although his assumption seems intuitively reasonable, it needs the confirmation of analysis of the multidimensional heat flow around such slots and within the moving fluid.