the unit normal vector at the main contact point which is given in the fixed coordinate system $S_o$.

The fulfillment of equations (70) and (71) guarantees the tangency of surfaces at the main contact point. Equation (72) guarantees that the instantaneous ratio of the gear-drive will be equal to the theoretical one. Equations (70), (71) and (72) permit one to obtain the machine and tool settings for gear cutting.

In accordance with the initial conditions of synthesis, the number of equations (73) may be equal four or five. Five equations must be obtained to satisfy equalities (65), equality $\Delta_2{}^* = 0$ (see equalities (65) and expression (69). In some cases of synthesis the requirement that $H_g{}^{(1)} = o$ is omitted (it declines the observing of equality $\Delta_2^* = o$). Then the number of necessary equations is equal four.

The fulfillment of equations (73) guarantees the optimization of meshing conditions in the vicinity of the main contact point. From these equations are determined the parameters of curvatures of both surfaces $\Sigma_1$ and $\Sigma_2$. When it is desired that one of these parameters must be a variable, it is necessary to omit the equation $H_g{}^{(1)} = 0$; and to vary in the process of synthesis the value of the derivative $U_{21}'(\phi_1)$ while holding it in acceptable limits.

## An Example of a Local Synthesis of Hypoid Gear-Drive

The initial parameters are: the shortest distance between gear axes $E$ = 32 mm; the theoretical value of ratio $U_{21} = N_1 : N_2 = 0.14634$ ($N_i$ $(i = 1,2)$ is the tooth number of gear $i$); the principal curvatures of surface $\Sigma_2$ are $H_s = 0$, $H_q = 0.00624$; the principal directions on $\Sigma_2$ are given by unit vectors $\mathbf{e}_s = (0.05321, -.32736, -0.94340)$, $\mathbf{e}_q = (0.66567, -0.71584, 0.21806)$; unit vectors $\mathbf{e}_s$ and $\mathbf{e}_q$ are projected on the axes of coordinate system $S_C$; the tangent to contact point path on surface $\Sigma_2$ has to coincide with unit vector $\mathbf{e}_s$; the coordinates of mean contact point in system $S_o$ are (140.8, $-$5.15, $-$29.30); projections of unit normal vector $\mathbf{n}^{(M)}$ in the same system $S_o$ are ($-$0.74435, $-$0.61677, $-$0.256); the approach of surfaces is $\delta$ = 0.007 mm and the length of great axis of contact ellipsis is $2a$ = 18 mm.

The following conditions must hold during synthesis $(a)$ that the contact point path had to coincide at the mean contact point M with the geodetic line of surface $\Sigma_2$ (i.e., $H_g{}^{(2)} = o$); $(b)$ that the derivative is $U_{21}'(\phi_1) = d/d\phi_1 (U_{21}) = -0.0008$

On the ground of article equations the following results were obtained: $G$ = 0.29053 (see angle $G$ in Fig. 3) $H_f = -0.03417$, $H_h$ = 0.00527. After that, by known parameters of curvatures the machine and tool settings for gear cutting may be obtained.

## Conclusion

An improved local synthesis method for gearings with approximate meshing is proposed. Unlike already known methods the proposed one is based: (1) on the connections between principal curvatures of two surfaces being: $(a)$ point-contacted, $(b)$ line-contacted; (2) on observing conditions under which the contact point path on the surface is a geodetic line in the local sense.

## References

1 Baxter, M.L., "Exact Determination of Tooth Surfaces for Spiral Bevel and Hypoid Gears," AGME - paper, No. 139, Oct. 1966

2 Baxter, M. L., "Effect of Misalignment of Tooth Action of Bevel and Hypoid Gears," ASME - paper -61-MD-20

3 Baxter, J. L., Second-Order Surface Generation, *Industrial Mathematics,* Vol. 23, part 2, 1973

4 Litvin, F. L., *The Theory of Gearings,* 2nd ed., in Russian, Nauka, Moscow, U.S.S.R., 1968

5 Litvin, F. L., and Gutman, Ye. I., "Obtaining of Connections between Surface Principal Curvatures by Arbitrary Value of Derivative of Ratio Function," in Russian, Proceedings "Mechanika Mashin," Mauka, No. 45, 1974.

6 Litvin, F. L., and Gutman, Ye. I., "A Common Solution of the Problem of Analysis for Optimization Mismatch Gearings with Application to Hypoid Drives," in Russian, Proceedings "Mechanika Mashin", Nauka, No. 45, 1974.

7 Litvin, F. L., "Die Beziehungen zwischen de Krummungen der Zahnoberflachen bei raumlichen Verzahnungen," "Zeitschrift fur Angewandte Mathematik and Mechanik," ZAMM, H. II, 1969.

8 *Understanding Tooth Contact Analysis,* Gleason Works, Rochester, N.Y., SD/3139, 1970, USA

## Discussion

**T. Krenzer**[1]. The authors are to be congratulated on presenting a very theoretical paper which shows a deep insight into the geometry of spiral bevel and hypoid gears. ASME is also to be commended for their recognition of this significant accomplishment. The paper proposes an optimum method for determining gear tooth meshing based on calculations at the mean point. The mathematical techniques and notation employed differ from that used at Gleason. However, the basic underlying philosophy is the same in that surfaces are defined by the relative position and motions between the generating tool and the work.

My discussion of this paper is divided into three parts. First, in order to provide a tool for evaluating meshing of spiral bevel and hypoid gears, the concept of tooth contact characteristics is introduced. Second, the authors' proposed procedure for calculating machine settings will be discussed relative to the above concept. Finally, the differences and similarities between the Gleason approach and the proposed method will be presented.

**Contact Characteristics.** To describe conditions of gear meshing, that is contact pattern and motion variations, on spiral bevel and hypoid gears, it is convenient to use a set of numbers called contact characteristics. These numbers can be ranked according to order; first, second, third, etc. We will not consider higher than third order.

Two numbers define position which is first order. They are $\Delta F$, the distance along the face of the gear from the center point of contact, $P$, to the mid-point on the tooth surface, $M$, and $\Delta h$, the distance in the profile direction form point $P$ to point $M$. $\Delta F$ and $\Delta h$ can be seen in Fig. 1 which is a sketch of a gear tooth in the tangent plane.

Three numbers define second order. They are:

$B$ = length of the contact pattern as a proportion of the face width.

$\beta$ = angle measuring the bias direction of the path of contact. Figure 2 is a sketch of a gear tooth showing B and $\beta$.

$Yo$ = amount of gear lag (retardation from its theoretical position) at the point where the motion transmission is transferred from one tooth to the next tooth. To better understand $Yo$, look at Figure 3 which is referred to as a motion diagram. Gear rotation is plotted on the abcissa and the variation in gear rotation is plotted on the ordinate. This is a typical

[1] Senior Research Staff Engineer, Gleason Works, June 19, 1980.



Copyright © 1981 by ASME

Downloaded from http://electronicpackaging.asmedigitalcollection.asme.org/mechanicaldesign/article-pdf/103/1/125/5809234/122_1.pdf by guest on 11 November 2024

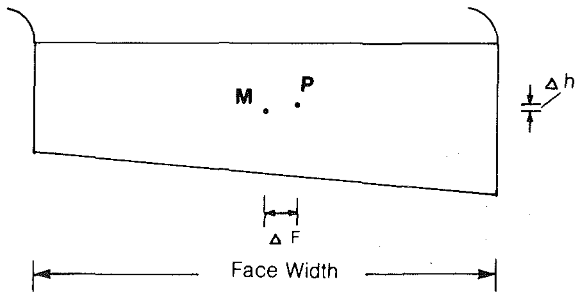



**Fig. 1**

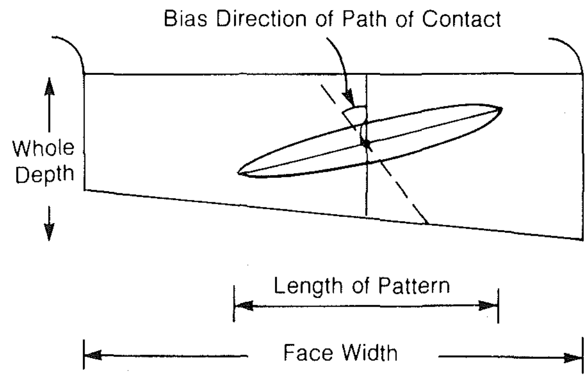
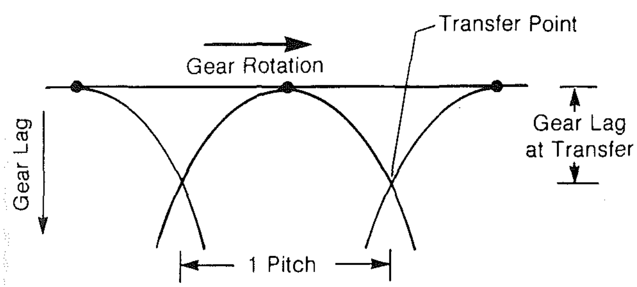



**Fig. 2**




**Motion Diagram**

**Fig. 3**

graph for a gear set where the tooth surfaces are relieved in the direction of the path of contact.

Six numbers define third order. They are:

$\Delta B_h$ = change in contact pattern length along the path of contact. See Fig. 4.

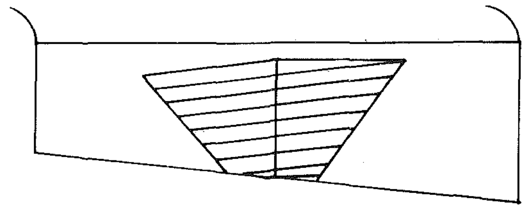
$\Delta B_F$ = change in contact pattern length along the face width. See Fig. 5.

$\frac{1}{\rho B}$ = curvature in the path of contact. See Fig. 6.

$\Delta\beta_F$ = change in direction of the path of contact along the face. See Fig. 7.

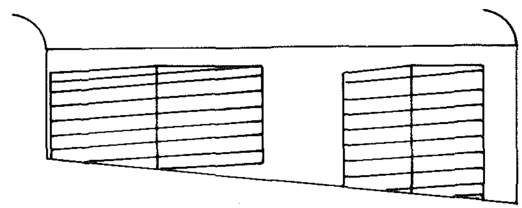
$S_m$ = S-ness in the motion curve. See Fig. 8.

$\Delta M_F$ = change in curvature of motion curve along the face. See Fig. 9.



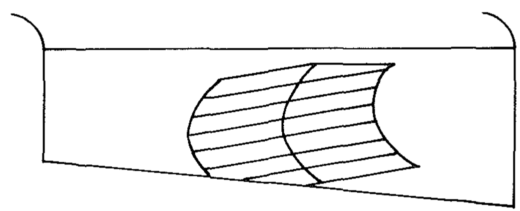
Change of Bearing Length Along the Path of Contact

**Fig. 4**



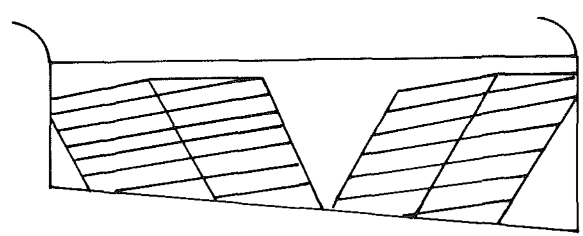
Change of Bearing Length Along the Face Width

**Fig. 5**



Curvature in the Path of Contact

**Fig. 6**



Change in Bias Along the Face

**Fig. 7**

In defining changes in contact along the face, the effect of displacements, $E$, perpendicular to the plane of the gear and pinion axes and, $P$, along the pinion axis to shift the contact pattern along the face width are considered.



Downloaded from http://electronicpackaging.asmedigitalcollection.asme.org/mechanicaldesign/article-pdf/103/1/125/5809234/122_1.pdf by guest on 11 November 2024

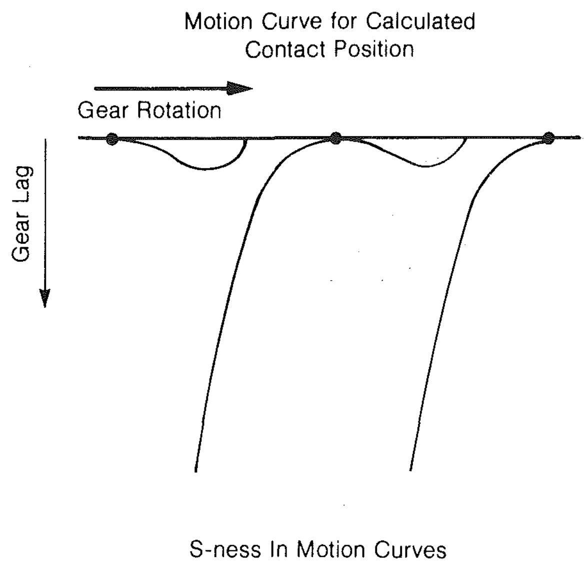



Fig. 8

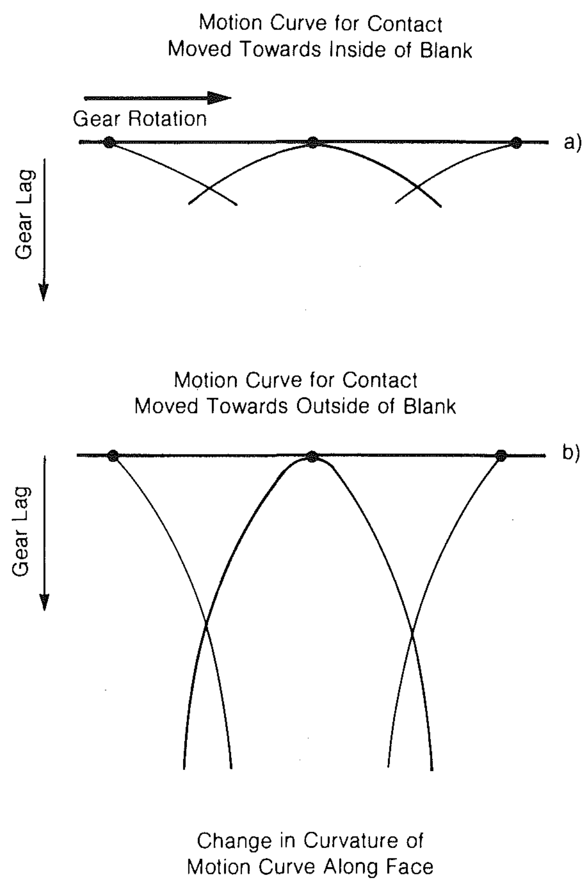



Fig. 9

**Discussion of Paper Relative to Contact Characteristics.** The paper recognizes the importance of defining tooth surfaces that hold specified first and second order characteristics. The authors go one step further in recognizing that curvature in the path of contact is a contact characteristic that should be controlled. They impose the condition that the triple scalar product of the contact normal at the mean point, the relative velocity between mating surfaces and the relative acceleration should equal zero. This insures no curvature in the path of contact which is in general a desirable contact characteristic.

Gleason has long recognized that contact characteristics beyond the second order must be considered when defining a desired contact condition, curvature of the path of contact is only one of the six third order characteristics. We believe that all third order is important and should be considered.

**Procedures Presently in Use.** We have successfully employed two methods to achieve desired third order contact: first TCA is used to evaluate machine setups which hold second order. The gear engineer then systematically changes the machine setups using the remaining available freedoms to give what he considers the optimum contact considering third order.

Our latest Formate* and Helixform* tilt programs calculate third order surfaces and third order characteristics. A weighted function is set up to define the relative importance of the third order characteristics and an optimizing technique is used in calculating a machine setup that minimizes this function.

**Conclusion.** In conclusion we agree with the authors on the need for machine setting calculations that include contact characteristics other than first and second order. Their procedure considers one such characteristic. We feel that this is not sufficient and have defined all the third order characteristics. Further, procedures have been set up to control these characteristics in the calculation of machine settings.

*Formate and Helixform are registered trademarks of the Gleason Works.

## Discussion

**R. L. Huston[1].** The authors propose a new method for determining cutting machine parameters to optimize the meshing kinematics. Because of the limitations of cutting machines and the restrictions of fabrication processes, the commonly used spiral bevel and hypoid gears have "approximate meshing" that is, the angular velocity ratio varies during the relative rotation of the gear and pinion. The essence of the authors' method, which is itself an approximation in the local sense and in the neglect of *3rd* order terms, is to make the contact path coincide with a geodesic curve.

The reviewer finds this concept and method to be novel and intuitively satisfying. The development and exposition of it through differential geometry formulae however, is a bit abstract and it is probably not in a form which is immediately accessible to designers and engineers. Also, the style and notation may be difficult for some to follow. The reviewer believes the concept should be explained and developed further by the authors and/or other theoreticians with the objective of obtaining more explicit design criteria.

Finally, the paper should serve as a stimulus for further theoretical investigations of spiral bevel and hypoid gear meshing kinematics, and as such, it should be of interest to theoreticians and kinematicians, as well as designers.

**Professor of Mechanics, Department of Mechanical Engineering, University of Cincinnati, Cincinnati, Ohio.**



Downloaded from http://electronicpackaging.asmedigitalcollection.asme.org/mechanicaldesign/article-pdf/103/1/125/5809234/122_1.pdf by guest on 11 November 2024

## Discussion

**V. Simon**[1]. These authors are to be congratulated for an excellent contribution to the geometry and kinematics of hypoid gears.

The papers treat the machine setting calculations for manufacture the formate or helixform gear and the generated pinion in case of line contact of their teeth, and the synthesis method for mismatched hypoid gear pair with point contact of the teeth surfaces. The presented method includes the determination of the machine setting parameters through local synthesis which provides the conjugation of the teeth surfaces at the mean contact point and the optimization of the same parameters to decrease the maximum displacement error of the driven gear. Such a calculation ensures the direction of the path of contact only in the vicinity of the mean contact point and defines the instantaneous contact elipse for dry contact (omitting lubrication on the basis of the teeth surface curvatures. However, due to the actual operating conditions, a more complete optimization of the separation and the meshing of teeth surfaces, and consequently a more complete optimization of the corresponding machine setting parameters is needed. Such an optimization is presented in [A.1], based on the following conditions:

1. At the initially selected point of contact $M$ (Fig. A1) the velocity ratio of the gear pair has the predicted value.
2. In the position of the gear and the pinion, with $M$ as the contact point, the separation of the teeth surfaces in the point $N$ is the desired $\Delta n$ ($N$ would be a point of the instantaneous contact line of the teeth surfaces in case of nonmismatched gear pair).
3. The path of contact should pass through an arbitrarily selected point $L$ and the error in displacement of the driven gear against rotation of the driver pinion at the contact in the point $L$ should not exceed the allowed value.
4. The angle of the tangential cone of the pinion root surface, in the middle of the tooth width, has the predicted value.

The mathematical interpretation of these conditions offers a systems of 22 simultaneous equations with 22 unknowns. Among the unknowns are 7 of the machine setting parameters for pinion manufacture.

The importance of such a complex optimization was proved by the results of the full thermoelastohydrodynamic analysis of the lubrication of hypoid gears. By applying the method presented in [A.2], the influence of the separation of the teeth surfaces in the point $N$ ($\Delta n$) on the load carrying capacity of the oil film was calculated. Fig. A2 shows the obtained results.

It can be seen that the sensitivity of the load carrying capacity to the separation of the teeth surfaces is considerable. Therefore, the proper choice of the amount of mismatch is very important for maintaining optimal operating characteristics of hypoid gears, and a complete optimization of the corresponding machine setting parameters is necessary.

## References


A1 Simon, V., "Optimization of the Geometry and Kinematics of the Hypoid Gears," *Fifth World Congress on Theory of Machines and Mechanisms*, Montreal, Sept. 1979.

A2 Simon, V., "Elastohydrodynamic Lubrication of Hypoid Gears," *Third International Power Transmission and Gearing Conference*, San Francisco, Aug. 1980.

A3 Simon, V., "Tooth Contact Analysis for Modified Hypoid Gears," *Fourth World Congress on Theory of Machines and Mechanisms*, Newcastle upon Tyne, Sept. 1975.

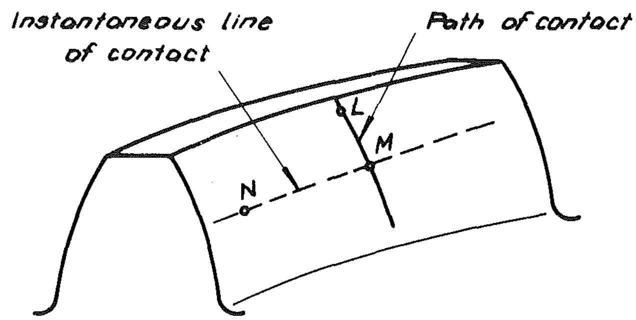




**Fig. A1 Position of *M*, *N*, and *L* characteristic points on the gear tooth**

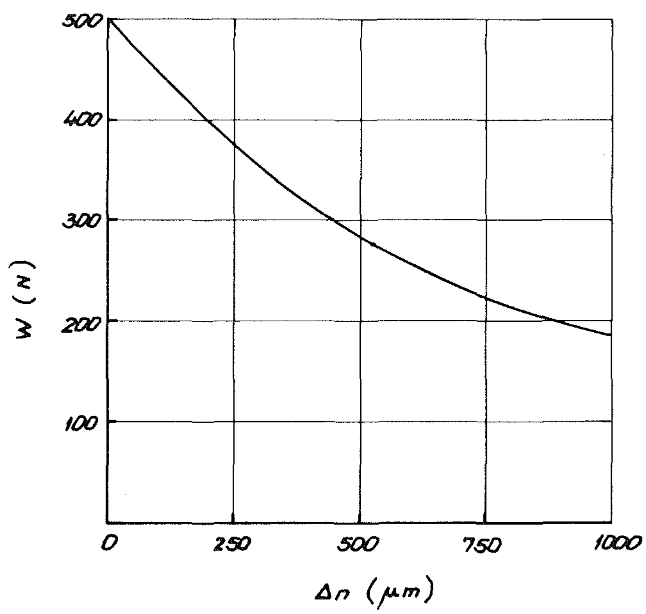



**Fig. A2 Performance curve for various teeth surface separations**


[1] Faculty of Technology, University of Novi Sad, Novi Sad, Yugoslavia





Downloaded from http://electronicpackaging.asmedigitalcollection.asme.org/mechanicaldesign/article-pdf/103/1/125/5809234/122_1.pdf by guest on 11 November 2024