Studying Gaugino Mass in Semi-Direct Gauge Mediation

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We study gaugino mass generation in the context of semi-direct gauge mediation models, where the messengers are charged under both the hidden sector and the standard model gauge groups while they do not play important roles in dynamical supersymmetry breaking. We clarify the cancellation of the leading contributions of the supersymmetry breaking effects to the gaugino mass in this class of models in terms of the macroscopic effective theory of the hidden sector dynamics. We also consider how to retrofit the model so that we obtain the non-vanishing leading contribution to the gaugino mass.

Subject Index: 112, 113, 114

§1. Introduction

If it is just hidden at and below the electroweak scale already explored experimentally, supersymmetry may be an origin of the puzzling large hierarchy between the electroweak scale and the Planck scale. It is, however, not straightforward to hide supersymmetry from observations at low energy in a phenomenologically viable way, which implies that the breaking of supersymmetry should take place in a so-called hidden sector. In fact, several mechanisms have been proposed so far which communicate the effects of supersymmetry breaking in the hidden sector to the supersymmetric standard-model. Among them, the gauge mediation1) is an attractive mechanism, where the unwanted flavor-changing neutral processes are naturally suppressed.

The pivot of gauge mediation consists of messenger fields that are charged under the standard model gauge symmetries. We can classify the models with gauge mediation into two classes: one is a class of models where the messengers are singlet under the hidden dynamics, and the other is a class of models where the messenger fields are charged under the hidden gauge dynamics. In the latter class, we may further divide the models into two types: direct and semi-direct2) (see earlier examples of the models with semi-direct gauge mediation3), 4)).

In simple situations where the supersymmetry breaking can be represented by a spurion, \(S = m + \theta^2 F\), and mediation takes place when the standard model vector-like messenger pairs are integrated out, the direct and the semi-direct mediations can be summarized as follows. The direct gauge mediation is effectively given by a superpotential term \(SQ\bar{Q}\) with standard model vector-like pairs \(Q\) and \(\bar{Q}\) of chiral
superfields as the mediators which also have hidden gauge interaction charges (whose
dynamics cause supersymmetry breaking encoded in the $S$ value). The semi-direct
mediation is given by a superpotential mass term $\mu Q \bar{Q}$ with $\mu$ a constant and a
representative term $S \psi \bar{\psi}$ with a hidden gauge interaction vector-like pair $\psi$ and $\bar{\psi}$
of standard-model singlet chiral superfields.

The important differences between the direct and semi-direct gauge mediations
are that the messengers do not play important roles in dynamical supersymmetry
breaking in the semi-direct models.\footnote{The above definition of the direct mediation is not apparently the same as the usual definition in the literature where the direct mediation models are constructed by identifying the subgroups of the global symmetries in the hidden sector with the standard model gauge groups. Indeed, most of the concrete models of direct gauge mediation satisfy both of the definitions.} As a result, in the semi-direct gauge mediation models, the rank of the gauge group in the supersymmetry breaking sector can be smaller than that in the direct gauge mediation models. This enables us to avoid the Landau pole problem in the semi-direct gauge mediation model, which is often encountered in direct gauge mediation models with low-energy supersymmetry breaking.

A difficulty in the semi-direct gauge mediation models is, however, a little hier-
archy between the gaugino masses and the sfermion masses in the supersymmetric
standard model sector. That is, in the semi-direct models, the gaugino masses vanish
to the leading order in $F$ (the gaugino screening\footnote{Note that $F/m^2 < 1$ is required for the messenger sectors not to have tachyonic modes.}), while the scalar masses emerge at the leading order in $F$. The gaugino masses are roughly suppressed by $(F/m^2)^2$ compared with the scalar masses.\footnote{For recent investigations on the semi-direct gauge mediations, see Refs. 5) and 6).} which leads to a severe “little hierarchy” problem without careful tuning between the sizes of $F$ and $m$.

In this paper, we study gaugino mass generation in strongly coupled semi-direct
gauge mediation models by using the macroscopic effective theory of the hidden strong dynamics.\footnote{For recent investigations on the semi-direct gauge mediations, see Refs. 5) and 6).} The gauge coupling constants of the supersymmetric standard model sector turn out to receive non-trivial threshold corrections when we move to the macroscopic effective theory from the microscopic theory of the hidden gauge dynamics. Such threshold corrections, which represent the gauge mediation effects from the heavy modes in the hidden dynamics, play a crucial role to determine the gaugino masses. We also consider how to retrofit the model to obtain a non-vanishing leading contribution to the gaugino mass.

The paper is organized as follows. In $\S$2, we discuss the threshold corrections to
the gauge coupling constants of the supersymmetric standard model when we move
to a macroscopic effective theory of the hidden strong dynamics. In $\S$3, we regain
the gaugino screening in terms of the macroscopic effective theory. In $\S$4, we consider a possible retrofit of the semi-direct gauge mediation model so that the leading gaugino mass contributions are not canceled. The final section is devoted to discussions.
Table I. The matter content of the $SU(N_c) \times SU(N_Q)$ model with $N_f = N_\psi + N_Q = N_c + 1$. The subgroups of the $SU(N_Q)$ ($N_Q \geq 5$) are identified with the gauge groups of the standard model. The anomalous $U(1)_A$ global symmetry can be treated as a symmetry by rotating the dynamical scale of $SU(N_c)$, i.e. $\Lambda$, with a charge given in the table.

<table>
<thead>
<tr>
<th></th>
<th>$\psi \times N_\psi$</th>
<th>$\bar{\psi} \times N_\psi$</th>
<th>$Q$</th>
<th>$\bar{Q}$</th>
<th>$A^{2N_c-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(N_c)$</td>
<td>$N_c$</td>
<td>$\bar{N}_c$</td>
<td>$N_c$</td>
<td>$\bar{N}_c$</td>
<td>1</td>
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<tr>
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<td>1</td>
<td>1</td>
<td>$N_Q$</td>
<td>$\bar{N}_Q$</td>
<td>1</td>
</tr>
<tr>
<td>$U(1)_R$</td>
<td>$1 - N_c/N_\psi$</td>
<td>$1 - N_c/N_\psi$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$U(1)_A$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$2N_Q$</td>
</tr>
</tbody>
</table>

§2. Threshold correction to the spectator gauge coupling

In this section, we consider a model with the $SU(N_c) \times SU(N_Q)$ gauge interactions, where $SU(N_c)$ is strongly interacting with a dynamical scale $\Lambda$ and is identified as the hidden gauge interaction, while $SU(N_Q)$ is a weakly coupling spectator gauge interaction whose subgroups are eventually identified as the gauge groups of the supersymmetric standard model. In the following, we show that the gauge coupling of the spectator gauge theory receives a non-trivial threshold correction when we move to a macroscopic effective theory of the hidden gauge interaction below the dynamical scale $\Lambda$. As will be shown in the next section, the threshold correction plays a crucial role to see the gaugino screening in terms of the macroscopic effective theory.

2.1. Model with $N_f = N_c + 1$

We begin with a model with $N_\psi$ flavors of the $SU(N_c)$ fundamental representation $\psi$’s which are singlets under the spectator $SU(N_Q)$ gauge group and one flavor of the bi-fundamental representation $Q$’s of $SU(N_c) \times SU(N_Q)$ (Table I). The number $N_\psi$ is chosen so that the total flavor of the $SU(N_c)$ gauge theory satisfies $N_f = N_\psi + N_Q = N_c + 1$.

At a scale below the dynamical scale $\Lambda$, the dynamics of the $SU(N_c)$ theory can be described by combining microscopic matter contents as

\begin{align}
M_\psi &= \psi \bar{\psi}, \\
N &= \bar{\psi} Q, \\
\bar{N} &= \psi \bar{Q}, \\
M_Q &= Q \bar{Q}, \\
B_s &= \epsilon \psi \cdots \bar{\psi} \bar{Q} \cdots \bar{Q}, \quad (\psi : \times N_\psi - 1, \bar{Q} : \times N_Q) \\
\bar{B}_s &= \epsilon \bar{\psi} \cdots \bar{\psi} Q \cdots Q, \quad (\bar{\psi} : \times N_\psi - 1, Q : \times N_Q) \\
B_Q &= \epsilon \bar{\psi} \cdot \bar{\psi} Q \cdots \bar{Q}, \quad (\bar{\psi} : \times N_\psi, \bar{Q} : \times N_Q - 1) \\
\bar{B}_Q &= \epsilon \psi \cdot \psi Q \cdots Q, \quad (\psi : \times N_\psi, Q : \times N_Q - 1)
\end{align}

where $\epsilon$ denotes the invariant anti-symmetric tensor of $SU(N_c)$ group and we have suppressed the indices of the gauge groups and flavors. The charges of the macroscopic fields under the spectator gauge group as well as the relevant global symmetries are given in Table II. The effective superpotential and Kähler potential of those
Table II. Macroscopic field content of the $SU(N_c) \times SU(N_Q)$ model at the scale below $\Lambda$.

<table>
<thead>
<tr>
<th>$SU(N_Q)$</th>
<th>$M_\psi$</th>
<th>$N(\times N_\psi)$</th>
<th>$\bar{N}(\times N_\psi)$</th>
<th>$M_Q$</th>
<th>$B_s(\times N_\psi)$</th>
<th>$\bar{B}<em>s(\times N</em>\psi)$</th>
<th>$B_Q$</th>
<th>$\bar{B}_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(N_Q)$</td>
<td>$1$</td>
<td>$N_Q$</td>
<td>$\bar{N}_Q$</td>
<td>$\text{adj} + 1$</td>
<td>$1$</td>
<td>$1$</td>
<td>$N_Q$</td>
<td>$\bar{N}_Q$</td>
</tr>
<tr>
<td>$U(1)_R$</td>
<td>$2 - 2N_c/N_\psi$</td>
<td>$2 - N_c/N_\psi$</td>
<td>$2 - N_c/N_\psi$</td>
<td>$2$</td>
<td>$N_c/N_\psi$</td>
<td>$N_c/N_\psi$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$U(1)_A$</td>
<td>$0$</td>
<td>$1$</td>
<td>$1$</td>
<td>$2$</td>
<td>$N_Q$</td>
<td>$N_Q$</td>
<td>$N_Q - 1$</td>
<td>$N_Q - 1$</td>
</tr>
</tbody>
</table>

Macroscopic fields are given by \(^8\)

\[
W_{\text{eff}} = \frac{1}{A^{2N_c - 1}} (BM\bar{B} - \det M),
\]

\[
K_{\text{eff}} = A^2 \left( \frac{|M|^2}{A^4} + \frac{|B|^2}{A^{2N_c}} + \frac{\bar{B}_s^2}{A^{2N_c}} + \cdots \right),
\]

where $M$, $B$, and $\bar{B}$ collectively denote $(M_\psi, N, \bar{N}, M_Q), (B_s, B_Q)$, and $(\bar{B}_s, \bar{B}_Q)$, respectively, and the ellipses denote the higher dimensional operators. Here, we have omitted $O(1)$ coefficients in the Kähler potential terms. Let us remind ourselves that $M$, $B$ and $\bar{B}$ are massless around their origins, which is consistent with the matchings of the anomalies of symmetries such as $U(1)_R - SU(N_Q)^2$ of the microscopic and the macroscopic theories.

Let us discuss the anomalies of the classical $U(1)_A$ symmetry given in Table I against the spectator $SU(N_Q)$ gauge theory in both the microscopic and the macroscopic theories. In the microscopic theory, the $U(1)_A - SU(N_Q)^2$ anomaly is given by

\[
N_c \times 1,
\]

which comes from the contribution of $Q$ and $\bar{Q}$ with the $U(1)_A$ charge 1. In contrast, the anomaly in the macroscopic theory is given by

\[
N_\psi \times 1 + N_Q \times 2 + 1 \times (N_Q - 1) = N_c + 2N_Q,
\]

where the first contribution comes from the $N_\psi$ flavors of $N$ and $\bar{N}$, the second one from $M_Q$, and the last one from $B_Q$ and $\bar{B}_Q$. As a result, we find that the anomalies in both the theories do not match with each other. There is no surprise in this disagreement because the $U(1)_A$ symmetry is anomalous to the $SU(N_c)$ gauge theory.

On the other hand, we may make $U(1)_A$ symmetry free of the anomaly against the $SU(N_c)$ gauge symmetry by rotating the dynamical scale $\Lambda$ along with the $U(1)_A$ symmetry with a charge given in Table I. Once the classical $U(1)_A$ symmetry is extended in this way, the anomalies in both the microscopic and the macroscopic theories must match with each other. In fact, the anomaly matching is realized by making the following change to the gauge kinetic function of $SU(N_Q)$:

\[
\frac{1}{g_{SU(N_Q)}^2} \rightarrow \frac{1}{g_{SU(N_Q)}^2} - \frac{1}{4\pi^2} \log \frac{A^{2N_c - 1}}{M_s^{2N_c - 1}},
\]
where $M_*$ denotes a scale at which the gauge coupling constant is defined. With the rotation of $A$ given in Table I, this term contributes to the anomaly by $-2N_Q$, and hence, by putting this contribution together with Eq. (4), we reproduce the anomaly in the microscopic theory in Eq. (3).

The additional term in Eq. (5) can be interpreted as a threshold correction from the heavy modes of the $SU(N_c)$ gauge theory with masses in a range of $O(A)$ which do not appear in Eq. (1). Note that the above threshold correction Eq. (5) does not have non-trivial dependences on the fields $M, B, \bar{B}$. If it had field dependences, the function in the logarithm would take zeros at some field values. This would imply that the $SU(N_c)$ model should possess extra massless modes at such field points, which is quite unlikely. Therefore, we conclude that the threshold correction to the spectator gauge coupling is uniquely determined by Eq. (5).

2.2. Model with $N_f = N_c$

Next, let us integrate out one of the fundamental representation $\psi$’s. Here, we introduce masses to $\psi$’s and $Q$’s by

$$W_{\text{tree}} = m_i \psi_i \bar{\psi}_i + \mu QQ ,$$

with $m_i, \mu \ll A$. Then, at the scale below $A$, the effective superpotential is again given by

$$W_{\text{eff}} = \frac{1}{A^{2N_c-1}} (BM\bar{B} - \det M) + m_i M_{\psi_i} + \mu M_Q .$$

Note that even in the presence of the mass terms, the threshold correction to the spectator gauge coupling from the heavy modes in Eq. (5) is intact and does not depend on the masses in Eq. (6), since there are no invariant combinations which show no singularity in the limit of $m_i, \mu \to 0$.

In the meantime, let us assume that $m_1 \gg m_i > 1, \mu$ and integrate out the macroscopic fields which involve $\psi_1$ as constituents (see the list in Eq. (1)). The relevant equation of motion, i.e. $F$-term condition in this case, is that of the “heavy mode” $M_{\psi_1}$ which is given by

$$\frac{\partial W}{\partial M_{\psi_1}} = - \frac{M_{\psi_2} \cdots M_{\psi_{N_Q}} M_Q^{N_Q}}{A^{2N_c-1}} + m_1 = 0 .$$

Here, we are considering the vacuum around $M_{ij} = M_i \delta_{ij}$ and $B = \bar{B} = 0$ and further assuming all the diagonal components of $M_Q$ take the same value. At this vacuum, the masses of the macroscopic fields which involve $\psi_1$ and are charged under $SU(N_Q)$, i.e. $N_1$ and $B_Q$, are given by

$$M_{N_1} = \frac{M_{\psi_2} \cdots M_{\psi_{N_Q}} M_Q^{N_Q-1}}{A^{2N_c-1}} ,$$

---

* The above additional term corresponds to the “gaugino counterterm” discussed in Ref. 9).
** There can be a numerical factor in front of $A^{2N_c-1}$ in Eq. (5), which can be absorbed by $M_*$ and does not change the following analysis.
*** Here, we use the normalizations of the fields given in Eq. (7), and hence, the masses are not dimension one parameters.
After these charged fields are integrated out, the spectator gauge coupling receives a threshold correction,

\[
\Delta g^2_{SU(N_Q)} \bigg|_{N_1,B_Q,\text{heavy}} = -\frac{1}{8\pi^2} \log \frac{M_{\psi_2} \cdots M_{\psi_{N_Q}} M_{N_Q}}{A^{4N_c-2} M_*^{2-2N_c}} = -\frac{1}{8\pi^2} \log \frac{m_1}{A^{2N_c-1} M_*^{2-2N_c}},
\]

where we have used the equation of motion given in Eq. (8) in the final expression.

By putting these threshold corrections together with those from the heavy modes in Eq. (5), we obtain the total threshold correction to the spectator gauge coupling,

\[
\Delta g^2_{SU(N_Q)} \bigg|_{N_1,B_Q,\text{heavy}} = -\frac{1}{8\pi^2} \log \frac{m_1 A^{2N_c-1}}{M_*^{2N_c}} = -\frac{1}{8\pi^2} \log \frac{A^{2N_c}}{M_*^{2N_c}},
\]

where we have defined the dynamical scale of the $SU(N_c)$ gauge theory with $N_f = (N_\psi - 1) + N_Q = N_c$ flavors by

\[
A_1^{2N_c} = m_1 A^{2N_c-1}.
\]

Now, let us remind ourselves that the $N_f = N_\psi + N_Q = N_c + 1$ model with a decoupled flavor cannot be distinguished from the $N_f = (N_\psi - 1) + N_Q = N_c$ model at the energy scale well below $\Lambda$ and $m_1$. Therefore, the threshold correction derived in Eq. (11) is nothing but the one from the heavy modes in the model with the dynamical scale $A_1$ and $N_f = N_\psi + N_Q = N_c$ flavors. In this way, we can derive the threshold correction to the spectator gauge coupling from the “heavy” modes with masses of the dynamical scale $A_1$ of the $N_f = N_\psi + N_Q = N_c$ model, which is given by

\[
\Delta g^2_{SU(N_Q)} \bigg|_{\text{heavy},N_f=N_c} = -\frac{1}{8\pi^2} \log \frac{A_1^{2N_c}}{M_*^{2N_c}}.
\]

We may check the consistency of the above threshold correction by examining the anomaly matching of the classical $U(1)_A$ symmetry in the $N_f = N_c$ theory. The charge assignments of the macroscopic fields and the dynamical scale are given in Table III. In the microscopic theory, the $U(1)_A \times SU(N_Q)^2$ anomaly is given by

\[
N_c \times 1,
\]

as it appeared in the $N_f = N_c + 1$ model. On the other hand, the anomaly in the macroscopic theory is now given by

\[
N_\psi \times 1 + N_Q \times 2 = N_c + N_Q,
\]
Table III. The matter content of the $SU(N_c) \times SU(N_Q)$ model with $N_f = N_\psi + N_Q = N_c$.

<table>
<thead>
<tr>
<th></th>
<th>$\psi(\times N_\psi)$</th>
<th>$\bar{\psi}(\times N_\bar{\psi})$</th>
<th>$Q$</th>
<th>$\bar{Q}$</th>
<th>$A_{1}^{N_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(N_c)$</td>
<td>$N_c$</td>
<td>$\bar{N}_c$</td>
<td>$N_c$</td>
<td>$\bar{N}_c$</td>
<td>1</td>
</tr>
<tr>
<td>$SU(N_Q)$</td>
<td>1</td>
<td>1</td>
<td>$N_Q$</td>
<td>$\bar{N}_Q$</td>
<td>1</td>
</tr>
<tr>
<td>$U(1)_R$</td>
<td>$1 - N_c/N_\psi$</td>
<td>$1 - N_c/N_\psi$</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$U(1)_A$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>$2N_Q$</td>
</tr>
</tbody>
</table>

where the first contribution comes from $N_\psi$ flavors of $N$ and $\bar{N}$, and the second one from $M_Q$. Thus, the anomaly matching of the “quantum” $U(1)_A$ symmetry with a non-trivial rotation of $A_1$ is realized only after we add the anomaly contribution ($-N_Q$) due to the threshold correction from the heavy mode in Eq. (13).

2.3. Model with $N_f = N_c - 1$

We may further integrate out the second flavor, $\psi_2$, by taking $m_2 \gg m_{i>2}, \mu$, and consider the threshold correction in the model with $N_f = N_c - 1$ flavors. The relevant equation of motion in this case is

$$\frac{\partial W}{\partial M_{\psi_2}} = -\frac{M_{\psi_1}M_{\psi_3} \cdots M_{\psi_{N_\psi}} M_Q^{N_Q}}{A^{N_c-1}} + m_2 = 0 .$$

(16)

At this vacuum, the macroscopic field $N_2$ decouples with a mass

$$M_{N_2} = \frac{M_{\psi_1}M_{\psi_3} \cdots M_{\psi_{N_\psi}} M_Q^{N_Q-1}}{A^{N_c-1}} = \frac{m_2}{M_Q} ,$$

(17)

and contributes to the threshold correction by

$$\frac{1}{\Delta g_{SU(N_Q)}^{2}} |_{N_2} = -\frac{1}{8\pi^2} \log M_{N_2} M_* = -\frac{1}{8\pi^2} \log \frac{m_2 M_*}{M_Q} .$$

(18)

By adding this contribution to Eq. (11), we obtain

$$\frac{1}{\Delta g_{SU(N_Q)}^{2}} |_{N_1, N_2, B_Q, \text{heavy}} = -\frac{1}{8\pi^2} \log \frac{m_2 A_{1}^{2N_c}}{M_Q M_*^{2N_c-1}} .$$

(19)

As a result, by interpreting this threshold correction as the one from the heavy modes in the model of $SU(N_c)$ gauge theory with $N_f = N_c - 1$ flavors, we obtain

$$\frac{1}{\Delta g_{SU(N_Q)}^{2}} |_{\text{heavy}, N_f = N_c - 1} = -\frac{1}{8\pi^2} \log \frac{A_{2}^{2N_c+1}}{\det M_Q^{1/N_Q} M_*^{2N_c-1}} ,$$

(20)
where we have defined the dynamical scale of the model with \( N_f = N_c - 1 \) by
\[
A_2^{2N_c+1} = m_2 A_1^{2N_c} = m_1 m_2 A_2^{2N_c-1}.
\] (21)

In the above expression, we have also taken into account the \( SU(N_Q) \) symmetry by replacing \( M_Q \) with \( \det M_Q^{1/N_Q} \).

Let us note that the threshold correction from the heavy fields in the case \( N_f = N_c - 1 \) depends on \( \det M_Q \) and is singular at \( M_Q = 0 \). This singularity is consistent with the fact that the origin of the macroscopic fields are removed by the effective superpotential, \(^{10}\)
\[
W_{\text{ADS}} = \frac{A_2^{2N_c+1}}{\det M},
\] (22)

and the anomaly matchings of the global symmetries require that the global symmetries are broken spontaneously. Here, \( M \) collectively denotes the meson fields consisting of \( N_f = N_c - 1 \) flavors of \( \psi \) and \( Q \).

2.4. Model with \( N_f = N_\psi + N_Q \)

By repeating the above discussion, we obtain the threshold correction to the spectator gauge coupling from the heavy modes in the model with \( N_f = N_\psi + N_Q < N_c + 1 \) flavors when we move from the microscopic to the macroscopic theory. The resultant threshold correction is given by
\[
\frac{1}{\Delta g_{SU(N_Q)}^{2}} \bigg|_{\text{heavy}, N_f = N_\psi + N_Q} = -\frac{1}{8\pi^2} \log \frac{A_{\text{eff}}^{3N_c-N_f}}{\det M_Q^{(N_c-N_f)/N_Q} M_*^{N_c+N_f}},
\] (23)

for \( N_c + 1 > N_f > 1 \),\(^{1}\) while it is given by
\[
\frac{1}{\Delta g_{SU(N_Q)}^{2}} \bigg|_{\text{heavy}, N_f = N_c + 1} = -\frac{1}{4\pi^2} \log \frac{A_{\text{eff}}^{2N_c-1}}{M_*^{2N_c-1}},
\] (24)

for \( N_f = N_c + 1 \). Here, \( A_{\text{eff}} \) denotes the dynamical scale of the \( SU(N_c) \) gauge theory with \( N_f \) flavors.

Note that we have not used the equation of motion of \( M_Q \) in the above derivations of the threshold correction. Thus, the threshold correction derived above are not changed even if we consider more generic superpotential of \( Q \)’s,
\[
W_{\text{tree}} = m_i \bar{\psi}_i \psi_i + f(Q\bar{Q}).
\] (25)

This property of the threshold correction will play an important role to discuss a possible retrofit of the semi-direct gauge mediation model so that the gaugino-screening is overcome.

\(^{1}\) Here, we have derived this result by deforming the model with \( N_f = N_c + 1 \) flavors with mass terms to the constituent fields. We may also obtain the same result by applying the anomaly matching condition directly to the model with \( N_f = N_\psi + N_c \) flavors.
§3. Gaugino screening in macroscopic effective theory

Now, let us discuss the gaugino screening in the semi-direct gauge mediation model. The gaugino mass at the leading order in $F$ can be extracted from the spurion dependence of the gauge coupling constant after the messenger fields are integrated out,\(^{11}\) i.e.

$$m_{\text{gaugino}}|_{\text{leading}} = \frac{1}{2} g_{\text{eff}}^2 \left| g \right|^2.$$  

(26)

The gaugino screening in the semi-direct gauge mediation can be understood as follows. In the semi-direct gauge mediation models, the tree-level superpotential is given by

$$W_{\text{tree}} = S \bar{\psi} \psi + \mu \bar{Q} Q,$$  

(27)

where $\psi$ is again a fundamental representation of $SU(N_c)$, and $Q$'s are bi-fundamental representations of $SU(N_c) \times SU(N_Q)$ which play a role of messengers. The supersymmetry breaking effect is encapsulated in the spurion field $S = m + \theta^2 F$.\(^*\) In order for the gaugino masses in the supersymmetric standard model to be generated at the leading order in $F$ at the one-loop level, the effective gauge coupling constants should have non-trivial $S$ dependences after the messengers are integrated out, such as,

$$\frac{1}{g_{\text{eff}}^2} \sim \frac{c}{8\pi^2} \log \frac{S}{M_*},$$  

(28)

where $c$ is a numerical coefficient. Such an $S$ dependence, however, contradicts with the anomaly matching of the $U(1)_R$ symmetry given, for example, in Table I and an appropriate assignment to $S$. Therefore, the above dependence must be vanishing, and hence, the gaugino masses at the leading order in $F$ are vanishing.

In the following, we reanalyze this gaugino screening in terms of the macroscopic effective theory. The explicit analysis in terms of the macroscopic theory opens new possibilities to extend the semi-direct gauge mediation model to that with the gaugino mass emerging at the leading order in $F$.

3.1. Gaugino screening in macroscopic theory

Here, as an example, we consider the $N_f = N_\psi + N_Q < N_c$ model with a tree level superpotential

$$W_{\text{tree}} = S \bar{\psi} \psi + \mu \bar{Q} Q,$$  

(29)

where $S = m + F \theta^2$ also denotes the spurion and we are assuming $\sqrt{F} \ll m$. In this model, the effective superpotential of the macroscopic fields,

$$M_\psi = \psi \bar{\psi}, \quad N = \bar{\psi} Q, \quad \bar{N} = \psi \bar{Q}, \quad M_Q = Q \bar{Q},$$  

(30)

\(^*\) In this paper, we do not address the explicit model of dynamical supersymmetry breaking in the hidden sector. Instead, we assume that the supersymmetry breaking effects are effectively encapsulated in the spurion $S$. 

\footnote{In this paper, we do not address the explicit model of dynamical supersymmetry breaking in the hidden sector. Instead, we assume that the supersymmetry breaking effects are effectively encapsulated in the spurion $S$.}
is given by\(^{10}\)

\[
W_{\text{eff}} = (N_c - N_f) \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)} + SM_\psi + \mu M_Q .
\] (31)

Here, again, \(M\) denotes the \(M_\psi\), \(N\), \(\bar{N}\), and \(M_Q\) collectively.

By using the effective superpotential, we obtain the \(F\)-term conditions of the (light) macroscopic fields,

\[
\frac{\partial W}{\partial M_\psi} = -N_\psi \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)} \frac{1}{M_\psi} + N_\psi S = 0 ,
\]

\[
\frac{\partial W}{\partial M_Q} = -N_Q \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)} \frac{1}{M_Q} + N_Q \mu = 0 ,
\] (32)

where we are again considering the vacuum with \(M_{ij} = M_i \delta_{ij}\) and further assuming that all the diagonal components of \(M_c\) and \(M_Q\) take the same values, respectively. At this vacuum, the expectation values of \(M_Q\) and \(M_\psi\) have non-trivial \(F\) dependences,\(^*)

\[
M_\psi = \left( \frac{\mu N_Q A_{\text{eff}}^{3N_c-N_f}}{S^{N_c-N_\psi}} \right)^{1/N_c} \propto \left( 1 - \frac{N_c - N_\psi}{N_c} F \theta^2 \right) ,
\]

\[
M_Q = \left( \frac{S^{N_\psi} A_{\text{eff}}^{3N_c-N_f}}{\mu N_c N_Q} \right)^{1/N_c} \propto \left( 1 + \frac{N_\psi F}{N_c} \theta^2 \right) .
\] (33)

Now let us consider the gauge mediation effects when the macroscopic fields \(N\) and \(\bar{N}\), which are charged under \(SU(N_Q)\), and the adjoint representation in \(M_Q\) are integrated out. The masses of those fields are given by

\[
M_N = \frac{1}{M_\psi M_Q} \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)} = \frac{S}{M_Q} \propto \left( 1 + \frac{N_c - N_\psi}{N_c} F \theta^2 \right) ,
\]

\[
M_{MQ} = \frac{1}{M_Q^2} \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)} = \frac{\mu}{M_Q} \propto \left( 1 - \frac{N_\psi F}{N_c} \theta^2 \right) .
\] (34)

Thus, the gaugino mass of the spectator gauge theory obtains a non-trivial contribution at the leading order in \(F\), which is obtained from the threshold correction,

\[
\frac{1}{\Delta g_{SU(N_Q)}^2} \bigg|_{N,M_Q} = -\frac{N_\psi}{8\pi^2} \log M_N M_* - \frac{N_Q}{8\pi^2} \log M_{MQ} M_* ,
\]

\[
= -\frac{1}{8\pi^2} \log \frac{\mu N_Q S^{N_\psi} M_* M_Q^{N_Q+N_\psi}}{M_Q^{N_Q+N_\psi}} ,
\]

\(^*)\) As long as we consider the leading order in \(F\), we can take the \(F\)-term condition as the relation between the superfields (see Appendix A).
From this threshold correction, we obtain the gaugino mass in the spectator gauge theory:

\[ m_{\text{gaugino}}|_{N,M_Q} = -\frac{g_{SU(N_Q)}^2 N_\psi (N_c - N_f)}{16\pi^2} \frac{F}{m} . \tag{36} \]

As we have already discussed in the previous section, however, we should also pay attention to the threshold correction from the heavy modes in Eq. (23). Since it depends on \( M_Q \), the threshold correction from the heavy modes also gives a non-trivial gaugino mass at the leading order in \( F \) through the \( F \) dependence of the vacuum expectation value of \( M_Q \):

\[ \frac{1}{\Delta g_{SU(N_Q)}^2} \bigg|_{N,M_Q,\text{heavy}, N_f = N_\psi + N_Q} = -\frac{1}{8\pi^2} \log \frac{\mu_{N_Q} S_{N_\psi} A_{\text{eff}}^{3N_c - N_f}}{M_Q^{N_c - N_f} M_s^{N_c + N_f}} , \]
\[ \sim -\frac{1}{8\pi^2} \log \left( 1 - \frac{N_\psi (N_c - N_f)}{N_c} \frac{F}{m} \theta^2 \right) . \tag{37} \]

The resultant contribution to the gaugino mass at the leading order of \( F \) from the heavy mode is given by

\[ m_{\text{gaugino}}|_{\text{heavy}, N_f = N_\psi + N_Q} = \frac{g_{SU(N_Q)}^2 N_\psi (N_c - N_f)}{16\pi^2} \frac{F}{m} . \tag{38} \]

Therefore, we find that both the contributions to the gaugino mass from the light modes Eq. (36) and the heavy modes Eq. (38) are canceled with each other, which reproduces the gaugino screening.

This cancellation can be seen more concisely in terms of the threshold correction. By combining the two contributions in Eqs. (35) and (37), we obtain the total threshold correction,

\[ \frac{1}{\Delta g_{SU(N_Q)}^2} \bigg|_{N,M_Q,\text{heavy}, N_f = N_\psi + N_Q} = -\frac{1}{8\pi^2} \log \frac{\mu_{N_Q} S_{N_\psi} A_{\text{eff}}^{3N_c - N_f}}{M_Q^{N_c} M_s^{N_c}} , \]
\[ = -\frac{1}{8\pi^2} \log \frac{\mu_{N_c}}{M_s^{N_c}} , \tag{39} \]

where we have used the vacuum expectation value of \( M_Q \) in Eq. (33). From the final expression, we easily see that the threshold correction has no \( S \) dependence, which shows the gaugino screening.

The above arguments indicate that the vacuum expectation value of \( M_Q \) plays an important role in the gaugino screening. This observation sheds light on possibilities that we may overcome the gaugino screening by deforming the tree level superpotential of \( QQ \) so that the total threshold correction depends on \( S \). In fact, as is argued in the next section, we can make the semi-direct gauge mediation get the gaugino mass at the leading order in \( F \) by using such a deformation.
3.2. Semi-direct mediation via “spurious” dynamical scale

Before closing this section, we mention another description of the above semi-direct gauge mediation. In the above analysis, we have treated $\psi$’s and $Q$’s in a similar way as the constituent fields. We could, however, have integrated out all the $\psi$’s first before moving to the macroscopic effective theory. In that case, the model just looks like the one with $N_f = N_Q$ and the effective dynamical scale,

$$A'_\text{eff}^3 N_c - N_Q = S^{N_\psi} \frac{A'_\text{eff}^3 N_c - (N_\psi + N_Q)}{\text{det} M_Q}.$$  (40)

The important difference from the model in the previous section is that the effective dynamical scale now plays a role of the spurion of supersymmetry breaking which has a non-trivial $\theta^2$ dependence.

In this description, we may again check the gaugino screening by adding the gauge mediation effect through the effective superpotential

$$W_{\text{tree}} = (N_c - N_Q) \left( A'_\text{eff}^3 N_c - N_Q \frac{1}{\text{det} M_Q} \right) + \mu M_Q,$$  (41)

and the one from the heavy modes of the model with $N_f = N_Q$,

$$\frac{1}{\Delta g_{\text{SU}(N_Q)}^2} \bigg|_{\text{heavy}, N_f = N_Q} = -\frac{1}{8\pi^2} \log \frac{A'_\text{eff}^3 N_c - N_Q}{\text{det} M_Q^{(N_c - N_f)/N_Q} M_{N_c + N_Q}^N}.$$  (42)

§4. Retrofitted semi-direct mediation

In the previous section, we discussed the gaugino screening in terms of the effective macroscopic theory. There, we saw that the vacuum expectation value of $M_Q$ plays an important role for the screening. In this section, we show that we can retrofit the model of the semi-direct gauge mediation so that it has the gaugino mass at the leading order in $F$ by deforming the superpotential of $Q\bar{Q}$.

As an example of such a modification, let us consider to deform the model with $N_f = N_\psi + N_Q$ by adding a quartic terms of $Q$’s:

$$W_{\text{tree}} = S\psi \bar{\psi} + \mu \text{tr} Q\bar{Q} + \kappa (\text{tr} Q\bar{Q})^2,$$  (43)

where $\kappa$ is a numerical coefficient and we have written “tr” explicitly, which has been suppressed in the preceding sections, so that the flavor structure of the quartic term is clarified. With the above deformation, the supersymmetric expressions of the masses of the $N$ and $M_Q$ in Eq. (34) are not changed,* while it affects the $F$-term condition of $M_Q$:

$$\frac{\partial W}{\partial M_Q} = -N_Q \left( A'_\text{eff}^3 N_f \frac{1}{\text{det} M} \right) \frac{1}{M_Q} + N_Q\mu + 2N_Q\kappa \text{tr} M_Q = 0.$$  (44)

* If we deform the model by $\text{tr} M_Q M_Q$ instead of $(\text{tr} M_Q)^2$, the masses of the adjoint component of $M_Q$ are changed. Our analysis can be extended to a model with such a deformation straightforwardly.
Therefore, the threshold correction to the $SU(N_Q)$ gauge coupling in the deformed model is obtained by changing the $\mu$ in Eq. (39) to $\mu + 2 \kappa \text{tr} M_Q$, i.e.

$$\Delta g_{SU(N_Q)}^{2} \bigg|_{\mu,N,M_{\text{heavy}},N_f=N_{\psi}+N_Q} = -\frac{1}{8\pi^2} \log \left( \frac{\mu + 2 \kappa \text{tr} M_Q}{M_{Nc}^{Nc}} \right).$$

Contrary to the previous result in Eq. (39), the threshold correction in the deformed model has a non-trivial $F$ dependence through the vacuum expectation value of $M_Q$.\(^{*})\)

As a result, we obtain a gaugino mass at the leading order in $F$, which is given by

$$m_{\text{gaugino}} = -N_{\psi} \frac{g^2_{SU(N_Q)} \kappa F}{8\pi^2} \langle \text{tr} M_Q \rangle,$$

where we have used

$$\text{tr} M_Q \simeq \langle \text{tr} M_Q \rangle \left( 1 + \frac{N_{\psi}}{N_c} \frac{F}{\theta^2} \right),$$

with

$$\langle \text{tr} M_Q \rangle \simeq N_Q \left( \frac{m_{\psi} \Lambda_{Nc-N_Q}}{\mu Nc} \right)^{1/Nc}.$$\(^{(49)}\)

Here\(^{\ast\ast})\) we have kept only the leading order contribution in $\kappa$ by assuming $\mu > \kappa \langle \text{tr} M_Q \rangle$. Let us remind ourselves that the sfermion masses are also generated at the thresholds at the masses of $N, M_Q$ and heavy modes, and they are roughly given by

$$m_{\text{scalar}} = \eta \frac{g^2_{SU(N_Q)} F}{16\pi^2} m,$$

where $\eta$ is a numerical coefficient of the order of unity which is non-vanishing even in the limit of $\kappa \to 0$.\(^{\ast\ast\ast})\) Thus, by comparing Eqs. (47) and (50), we find that the gaugino mass can be comparable to the scalar masses for

$$\mu \sim \kappa \langle \text{tr} M_Q \rangle \sim \kappa \Lambda_{\text{eff}}^2 \left( \frac{\mu}{\Lambda_{\text{eff}}} \right)^{N_Q/Nc-1} \left( \frac{m}{\Lambda_{\text{eff}}} \right)^{N_{\psi}/Nc}.$$\(^{(51)}\)

\(^{\ast})\) Note that the threshold correction in Eq. (45) has a singularity at the field value

$$\mu + 2 \kappa \text{tr} M_Q = 0.$$\(^{(46)}\)

This singularity corresponds to the massless adjoint fields $M_Q$ at this field value.

\(^{\ast\ast})\) In our model, the R-symmetry is explicitly broken by the quartic term in Eq. (43), and hence, the result of the vanishing gaugino mass in Ref. 12) does not apply to our model.

\(^{\ast\ast\ast})\) The coefficient $\eta$ is non-calculable since it includes the contributions from the heavy modes of the strong dynamics.
Finally, we comment on a possible origin of the quartic deformation term \( \kappa (\text{tr} \, Q \bar{Q})^2 \). From the gaugino mass obtained in Eq. (47), we need to have a rather large coupling constant \( \kappa = O(\mu / \langle \text{tr} \, M_Q \rangle) \), so that the gaugino masses are not too suppressed compared with the scalar masses. Such a large coupling can be realized, for example, by introducing an extra singlet field \( X \) which couples with \( \text{tr} \, Q \bar{Q} \) as

\[
W = -\frac{1}{2} m_X X^2 + k \, X \, \text{tr} \, Q \bar{Q}. \tag{52}
\]

After integrating out \( X \), we obtain the desired quartic term with \( \kappa = k^2/(2m_X) \). Thus, the coupling of the order of \( \kappa = O(\mu / \langle \text{tr} \, M_Q \rangle) \) can be realized for \( m_X = O(\langle \text{tr} \, M_Q \rangle / \mu) \) and \( k = O(1) \).

§5. Discussion

We have studied gaugino mass generation in the semi-direct gauge mediation models. We found that the gaugino mass screening can be understood as a cancellation between the gaugino mass contributions from the heavy modes and the light modes of the hidden gauge dynamics. In such a cancellation, the vacuum expectation value of the macroscopic messenger field plays an important role. We also proposed how to retrofit the semi-direct gauge mediation model so that the gaugino mass emerges at the leading order in the spurion SUSY-breaking scale \( F \).

We have not addressed explicit models of dynamical supersymmetry breaking in the hidden sector. In order to construct a realistic semi-direct gauge mediation model, we must identify this sector. However, in doing this, we immediately encounter one problem for the following reason. That is to say, in general, supersymmetry is restored due to the quartic term in the tree level superpotential when we take the dynamical model as the supersymmetry breaking sector.\(^\ast\) Therefore, it is necessary to find the model whose supersymmetry is dynamically broken even in the presence of the quartic term. Apparently, we may consider metastable supersymmetry breaking models, for instance.

It is also interesting to consider the extension of our analysis to the \( N_f > N_c + 1 \) case. One complexity in this case is that we have to estimate the effect of the remaining hidden gauge interaction.

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\(^\ast\) For example, at the singularity in Eq. (46), the effective masses of \( Q \)'s are vanishing, and hence, the supersymmetry can be restored around this point.
Appendix A

Adiabatic Supersymmetry Breaking

In this appendix, we consider how the supersymmetry breaking effect of the spurion $S = \langle S \rangle + F_S \theta^2$ affects the $F$-term VEVs of the other superfields (we collectively name them $M$), which are vanishing in the limit of $F_S = 0$. We consider that the superpotential and the Kähler potential are given by $W(M, S)$ and $K(M, S)$, respectively. In this case, the scalar potential is given by

$$-V = K_{M\bar{M}} |F_M|^2 + (K_{SM} F^*_M F_S + \text{c.c.}) + K_{S\bar{S}} |F_S|^2 + (W_M F_M + W_S F_S + \text{c.c.})$$

$$= K_{M\bar{M}} |F_M + \frac{1}{K_{M\bar{M}}} (K_{SM} F_S + W^*_M)|^2 + K_{S\bar{S}} |F_S|^2$$

$$- \frac{1}{K_{M\bar{M}}} |K_{SM} F_S + W^*_M|^2 + (W_S F_S + \text{c.c.}).$$

(A.1)

Thus, we see the minimal of $F_M$ at

$$F_M = - \frac{1}{K_{M\bar{M}}} (K_{SM} F_S + W^*_M).$$

(A.2)

Now, let us assume that the $W_M = 0$ at the vacuum in the limit of $F_S = 0$, and calculate the $F_M$ at the leading order of $F_S$ by solving $\partial V/\partial M = 0$. At the leading order of $F_S$, the relevant terms in $\partial V/\partial M$ are

$$\frac{\partial V}{\partial M} = \frac{1}{K_{M\bar{M}}} (K_{SM} F_S + W^*_M) W_{MM} - W_{MS} F_S$$

$$= - F_M W_{MM} - W_{MS} F_S,$$

(A.3)

and hence, we obtain the leading contribution to the $F$-term as

$$F_M = - \frac{W_{MS}}{W_{MM}} F_S.$$  

(A.4)

Here, all the scalar VEVs on the right-hand side are those in the limit of $F_S = 0$.

We compare the result with the supersymmetric VEV which is obtained by the $F$-term condition, i.e. $W_M = 0$. We assume that the equation of the $F$-term condition can be extended to the one between superfields as long as the SUSY breaking is adiabatic, that is, the SUSY breaking effect of the spurion $S = \langle S \rangle + F_S \theta^2$ is turned on smoothly from the limit of $F_S = 0$. Under this assumption, the $F$-terms in the $F$-term condition satisfy

$$W_{MM} F_M + W_{SM} F_S = 0.$$  

(A.5)

By comparing this result with Eq. (A.4), we finally find out that we can reproduce the leading $F$-term obtained by using the potential analysis. Therefore, as long as we are considering the leading effect, we can make a shortcut to obtain the $F$-term VEV by using the $F$-term condition.
Table IV. The matter content of the $SU(N_c) \times U(1)$ model with $N_f = N_c + 1$.

<table>
<thead>
<tr>
<th>$SU(N_c)$</th>
<th>$\psi(\times N_\psi)$</th>
<th>$\bar{\psi}(\times N_\psi)$</th>
<th>$Q$</th>
<th>$\bar{Q}$</th>
<th>$A_2^{N_Nc-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(1)$</td>
<td>$N_c$</td>
<td>$\bar{N}_c$</td>
<td>$N_c$</td>
<td>$\bar{N}_c$</td>
<td>1</td>
</tr>
<tr>
<td>$U(1)_R$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$U(1)_A$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Appendix B

Threshold Correction to Spectator $U(1)$ Gauge Coupling

In §2, we considered the threshold correction to the spectator $SU(N_Q)$ gauge coupling from the heavy modes in the hidden $SU(N_c)$ gauge dynamics. In this appendix, we consider the model with a $U(1)$ spectator gauge group, which requires careful attention to discuss the decoupling procedures by adding mass terms to the $\psi$’s. For simplicity, we consider a model with $N_Q = 1$ with tree-level superpotential,

$$W = m_i \bar{\psi}_i \bar{\psi}_i + f(Q \bar{Q}),$$

where only $Q$’s are charged under the $U(1)$ gauge group with unit charge while $\psi$’s are singlets.

B.1. Model with $N_f = N_c + 1$

We begin with the model with $N_f = N_c + 1$. In this model, the effective macroscopic theory can be described by

$$M_\psi = \bar{\psi} \psi,$$
$$N = \bar{\psi} Q,$$
$$\bar{N} = \bar{\psi} Q,$$
$$M_Q = Q \bar{Q},$$
$$B_Q = \epsilon \psi \cdots \psi Q, \quad (\psi \times N_c - 1, \; Q: \times 1)$$
$$\bar{B}_Q = \epsilon \bar{\psi} \cdots \bar{\psi} \bar{Q}, \quad (\bar{\psi} \times N_c - 1, \; \bar{Q}: \times 1)$$
$$B_s = \epsilon \psi \cdots \psi, \quad (\psi \times N_c, \; Q: \times 0)$$
$$\bar{B}_s = \epsilon \bar{\psi} \cdots \bar{\psi}, \quad (\bar{\psi} \times N_c, \; \bar{Q}: \times 0)$$

where the definitions of the baryons are different from those in §2, since the $U(1)$ charges of the baryons are simply the sums of those of their constituents. The charge assignments of the relevant symmetries are given in Table IV. With these macroscopic fields, the effective superpotential is given by Eq. (7) for $W_{\text{tree}} = 0$. The important difference from the $SU(N_Q)$ model is that the threshold correction
from the heavy modes. In the present model, the threshold correction to the gauge coupling constant of $U(1)$ from the heavy modes, which satisfy the anomaly matching conditions of the global symmetries in Table IV, is given by

\[
\frac{1}{\Delta g_{U(1)}^2} \bigg|_{N_f = N_c + 1} = \frac{-N_c}{8\pi^2} \log \frac{A_{2N_c-1}^2}{M_{2N_c-1}^*}.
\]  

(B.3)

B.2. Model with $N_f = N_c$

In order to obtain the threshold correction to the $U(1)$ gauge coupling in the $N_f = N_c$ model, let us consider to make $\psi_1$ heavy by switching on the mass term in $\tilde{W}_{\text{tree}}$. Note that the threshold correction from the heavy modes does not change even in the presence of the mass term of $\psi_1$. In this case, the relevant equation of motion of the heavy mode, $M_{\psi_1}$, is given in Eq. (8), by assuming that only the diagonal components obtain non-vanishing values. Around this point, the heavy fields which are charged under $U(1)$ are $N_1 = \psi_1 Q, \bar{N}_1, B_{Qi} = \epsilon \psi_1 \cdots \psi_{i-1} \psi_{i+1} \cdots Q,$ and $\bar{B}_{Qi}$, and they decouple at

\[
M_{N_1} = \frac{M_{\psi_2} \cdots M_{\psi_N}}{A_{2N_c-1}},
\]

\[
M_{B_{Qi}} = \frac{M_{\psi_i}}{A_{2N_c-1}}. \quad (i = 2, \cdots, N_c)
\]  

(B.4)

Hence, the threshold corrections from these modes are given by

\[
\frac{1}{\Delta g_{U(1)}^2} \bigg|_{N_1, B_{Qi} (i = 2 \cdots N_c)} = -\frac{1}{8\pi^2} \log M_{N_1} M^*_1 - \frac{1}{8\pi^2} \log M_{B_{Q_2}} \cdots M_{B_{Q_{Nc}}} (M_{2N_c-3}^*)^{N_c-1} - \frac{1}{8\pi^2} \log \frac{(M_{\psi_2} \cdots M_{\psi_{N_c}})^2}{A_{N_c(2N_c-1)}M_{2N_c+5N_c-4}^2} = -\frac{1}{8\pi^2} \log \frac{m_1^2}{M_{Q}^2 A_{(N_c-2)(2N_c-1)}^4 M_{2N_c+5N_c-4}^*}.
\]  

(B.5)

where we have used the equation of motion Eq. (8), $M_{\psi_2} \cdots M_{\psi_{N_c}} = m_1 A_{2N_c-1}/M_Q$, in the final expression.

Therefore, putting the heavy modes and $N, B_Q$ contributions together, we obtain the total threshold correction, which can be identified with the correction to the $N_f = N_c$ model:

\[
\frac{1}{\Delta g_{U(1)}^2} \bigg|_{\text{heavy}, N_f = N_c} = \frac{1}{\Delta g_{U(1)}^2} \bigg|_{N_1, B_{Qi} (i = 2 \cdots N_c), \text{heavy}, N_f = N_c + 1} = -\frac{1}{8\pi^2} \log \frac{(m_1 A_{2N_c-1})^2}{M_{Q}^2 M_{2N_c-4}^*} = -\frac{1}{4\pi^2} \log \frac{A_{2N_c}^2}{M_{Q} M_{2N_c-2}^*},
\]  

(B.6)

where we have defined the dynamical scale of the $N_f = N_c$ model by $A_{1}^{2N_c} = m_1 A_{2N_c-1}^2$. 

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B.3. Model with $N_f = N_c - 1$

Now let us move on to the model with $N_f = N_c - 1$ by integrating out $\psi_2$ with a mass $m_2$. In this case, $N_2$ and $B_{Q1}$ decouple in addition to $N_1$ and $B_{Qi}$ discussed above at

$$
M_{N_2} = \frac{M_{\psi_1} M_{\psi_3} \cdots M_{\psi_{N_\psi}}}{A^{2N_c-1}} = \frac{m_2}{M_Q},
$$

$$
M_{B_{Q1}} = \frac{M_{\psi_1}}{A^{2N_c-1}} = \frac{m_2}{M_Q \det M_{N_c-2}},
$$

where $M_{N_c-2}$ denotes the mesons that consist of $\bar{\psi}_i \psi_j$, $(i, j = 3, \cdots, N_c)$. Here, we have used the equation of motion of the heavy mode $M_{\psi_2}$,

$$
\frac{\partial W}{\partial M_{\psi_2}} = -\frac{M_{\psi_1} M_{\psi_3} \cdots M_{\psi_{N_\psi}} M_Q}{A^{2N_c-1}} + m_2 = 0.
$$

As a result of the decoupling of $M_{N_2}$ and $M_{B_{Q1}}$, we obtain the threshold correction as

$$
\frac{1}{\Delta g^2_{U(1)}} \bigg|_{N_2, B_{Q1}} = -\frac{1}{8\pi^2} \log M_{N_2} M_* - \frac{1}{8\pi^2} \log M_{B_{Q1}} M_*^{2N_c-3}
$$

$$
= -\frac{1}{8\pi^2} \log \frac{m_2^2 M_*^{2N_c-2}}{M_Q \det M_{N_c-2}}.
$$

Thus, we obtain the total threshold correction from the heavy modes in the $N_f = N_c - 1$ model,

$$
\left. \frac{1}{\Delta g^2_{U(1)}} \right|_{\text{heavy, } N_f = N_c - 1} = \left. \frac{1}{\Delta g^2_{U(1)}} \right|_{N_2, B_{Q1}, \text{heavy, } N_f = N_c}
$$

$$
= -\frac{1}{8\pi^2} \log \frac{M_Q^4 \det M_{N_c-2} M_*^{2N_c-2}}{m_2^2 A_1^{4N_c}}
$$

$$
= -\frac{1}{8\pi^2} \log \frac{A_2^{4N_c+2}}{M_Q^4 \det M_{N_c-2} M_*^{2N_c-2}}
$$

$$
= -\frac{1}{8\pi^2} \log \left( \frac{A_2^{2N_c+1}}{M_Q^3 M_*^{2N_c-3}} \right) \left( \frac{W_{\text{ADS, } N_c-1}}{M_*^3} \right).
$$

Here, we have defined the dynamical scale of the $N_f = N_c - 1$ model by $A_2^{2N_c+1} = m_2 A_1^{2N_c}$, and denoted the Affleck-Dine-Seiberg superpotential\textsuperscript{10} that consists of $M_Q$ and the remaining $M_\psi$ as

$$
W_{\text{ADS, } N_c-1} = \frac{A_2^{2N_c+1}}{M_Q \det M_{N_c-2}}.
$$
B.4. Model with $N_f = N_\psi + 1$

By repeating the above procedure, we end up with the threshold correction,

$$\frac{1}{\Delta g^2_{U(1)}} \bigg|_{\text{heavy}, N_f = N_\psi + N_Q} = -\frac{1}{8\pi^2} \log \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{M_Q^{N_c-N_f+2} M_*^{N_c+N_f-4}} \right) \left( \frac{W_{\text{ADS}, N_f}}{M_*^3} \right),$$

(B.12)

with

$$W_{\text{ADS}, N_f} = (N_c - N_f) \left( \frac{A_{\text{eff}}^{3N_c-N_f}}{M_Q \det M_{N_f-1}} \right) \frac{1}{N_c-N_f},$$

(B.13)

for $N_c > N_f > 1$, while it is given by

$$\frac{1}{\Delta g^2_{U(1)}} \bigg|_{\text{heavy}, N_f = N_c} = -\frac{1}{4\pi^2} \log \left( \frac{A_{\text{eff}}^{2N_c}}{M_Q M_*^{2N_c-2}} \right),$$

(B.14)

for $N_f = N_c$ and

$$\frac{1}{\Delta g^2_{U(1)}} \bigg|_{\text{heavy}, N_f = N_c+1} = -\frac{N_c}{8\pi^2} \log \left( \frac{A_{\text{eff}}^{2N_c-1}}{M_*^{2N_c-2}} \right),$$

(B.15)

for $N_f = N_c + 1$. Here, $A_{\text{eff}}$ denotes the dynamical scale of the $SU(N_c)$ gauge theory with the $N_f$ flavors. By means of the above threshold corrections, we can check the gaugino screening for $f(M_Q) = \mu M_Q$.

References

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