

# Closure to “Discussion of ‘Analysis of a System of Linear Delay Differential Equations’ ” (2007, ASME J. Dyn. Syst., Meas., Control, 129, pp. 121–122)

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We would like to thank Prof. Zafer for his careful reading of the article [1], which proposes the use of Lambert functions to solve delay differential equations (DDEs), and for pointing out errors in that article. First, we would like to note that the results presented in [1] for the first-order scalar delay differential equation using the Lambert function, on pp. 215–219, are correct. The errors pointed out by Prof. Zafer pertain to the system of delay differential equations in matrix-vector form and to the results presented for the chatter example, which are given on pp. 219–221. The generalization to a system of DDEs in [1] is only correct in the special case when certain matrices commute (e.g., when  $\mathbf{AB}=\mathbf{BA}$ ). In general they do not, and specifically do not in the machining chatter example presented in [1]. Thus, we thank Prof. Zafer for his correction.

Recently, new results have been obtained that generalize the matrix Lambert function approach in [1] for general linear time invariant systems of DDEs where  $\mathbf{A}$  and  $\mathbf{B}$  do not necessarily commute. Those results are reported in [2,3].

## References

- [1] Asl, F. M., and Ulsoy, A. G., 2003, “Analysis of a System of Linear Delay Differential Equations,” ASME J. Dyn. Syst., Meas., Control, **125**(2), pp. 215–223.
- [2] Yi, S., and Ulsoy, A. G., 2006, “Solution of a System of Linear Delay Differential Equations Using the Matrix Lambert Function,” American Control Conference, Minneapolis, MN, June 14–16.
- [3] Yi, S., Nelson, P. W., and Ulsoy, A. G., “Delay Differential Equations via the Matrix Lambert W-Function and Bifurcation Analysis: Application to Machine Tool Chatter,” Math. Biosci. Eng. (to be published).