Meson-Baryon Potentials by One-Hadron-Exchange Mechanisms with Gaussian Sources

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Meson-baryon potentials in momentum space are proposed on the basis of one-hadron-exchange mechanisms. The Gaussian source parameters and SU(3) coupling constants are determined so as to reproduce the πN and KN scatterings at low energies, fixing all the parameters predetermined in the baryon-baryon potentials proposed previously. Using the same values, we predict the $\bar{K}N$-$\pi\Lambda$-$\pi\Sigma$ coupled-channel potentials. To explain the experimental cross sections and $\Lambda(1405)$ as a quasibound state, we need an overall factor of approximately 0.7.

Subject Index: 200, 201, 232

§1. Introduction

The theory of the interaction between hadrons is one of the most longstanding problems in nuclear and particle physics. The pion theory of the nuclear force proposed by Yukawa was the first realistic theory of the nuclear force. It has been established as the theory of the long-range part of the nuclear force. Using the three-range strategy by Taketani and coworkers as a guideline, many authors have tried to explain the second (intermediate) region of the nuclear force by two-pion-exchange potentials (TPEPs) or one-boson-exchange potentials (OBEPs). In today’s words, all these theories can be regarded as the hadron-exchange models of hadron-hadron interaction. In those works, the potentials in the third (inner) region were described as phenomenological ones.

Today, hadrons are described as composite particles of quarks and gluons based on quantum chromodynamics (QCD), which is considered to be the fundamental theory of the strong interaction. In principle, the interactions between two hadrons could be explained by QCD. In fact, the lattice QCD gave some successful results for the hadron-hadron interaction. Such first-principle approach itself is very important to provide the basis of the interaction theory. However, from the physical viewpoint, the model theory in terms of hadronic degrees of freedom can be very useful. For example, even if we can derive the nucleon-nucleon ($NN$) potential through large-scale lattice QCD calculations, the model of OPEP for the long-range part of the nuclear force provides us with a simple physical picture of the nuclear force.

In QCD, only colorless or hadronic states of quarks and gluons can propagate in the hadronic vacuum. Therefore, it is very natural that not only the states but also
their dynamics in the hadronic vacuum can be described on the basis of hadronic degrees of freedom. The success of the meson-exchange models of the nuclear force supports such expectation. However, since there are many hadronic degrees of freedom, we could not determine the unique model of the nuclear force.

In the recent three decades, our knowledge of baryon-baryon (BB) interactions has considerably grown. In particular, the development of hypernuclear physics has provided very useful information for hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions. Hypernuclear spectroscopy provides distinctive information on the spin dependence of ΛN interaction. On the other hand, the amount of experimental data on meson-baryon (MB) scatterings has also considerably increased and we can use the phase shift analyses not only for πN scattering but also for KN scattering.5)–6) For ¯KN scattering, we cannot use yet detailed information on the scattering S-matrix but can use experimental elastic and reaction cross sections, ¯KN → ¯KN, ¯KN → πΛ, and ¯KN → πΣ.7)–10) In this work, we examine whether our potential model can be used to reproduce these experimental data.

Our aim is to construct a unified potential model to describe consistently all the baryon-baryon (BB), meson-baryon (MB), and meson-meson (MM) interactions, where baryons and mesons respectively denote octet baryons and pseudoscalar mesons which are stable in the strong interaction. There are some pioneering works in a similar line. Jülich group11)–17) has attempted to describe hadron-hadron interaction on the basis of hadron-exchange mechanisms. Nijmegen group18)–20) has constructed various versions of baryon-baryon potentials and has recently extended their model to meson-baryon potentials (πN and KN potentials).21) However, their models assume some inconsistent values of the same physical parameters in different channels and introduce arbitrary SU(3) breakings. At present, we do not have a unified model for the hadron-hadron interactions.

In this paper, we propose a model of meson-baryon (MB) potentials. All the parameters in our model are determined to be consistent with the potential model of baryon-baryon (BB) interactions proposed in the previous work.22) This potential model was made of only one-meson-exchange mechanisms and had no phenomenological short-range repulsions (SRRs). We have also proposed the model23) of the BB potentials, which includes SRRs. However, both models give almost the same quality of fitting to BB experimental data. This denotes that we cannot determine SRRs uniquely using only the BB sector. To avoid this ambiguity, we start this work with no SRR. Our model in this paper may be improved by introducing the SRR needed to reproduce the BB and MB data simultaneously.

In some potential models, TPEPs are included. In our model, σ- and ρ-exchanges with large widths are introduced and regarded as the correlated TPEP effectively. Our BB potential models22), 23) were determined without the uncorrelated TPEP. From the consistency, the MB potentials proposed in this work are constructed without the uncorrelated TPEP. As mentioned above, we have many degrees of freedom in constructing meson-exchange potentials. If we introduce TPEP explicitly, we should also introduce one-pion-one-kaon-exchange and two-kaon-exchange potentials in view of the SU(3) model. Our aim in this work is to construct the potential models within the one-hadron-exchange mechanisms.
In our model, we assume $SU(3)$ symmetric coupling constants and physical masses and widths of exchanged hadrons. These parameters are commonly used in all channels of $BB$, $MB$, and $MM$ potentials.

In §2, we explain our model of one-hadron-exchange potentials. Our potentials are made from the three types of diagram, which are explained in detail. In §3, the $SU(3)$ symmetric coupling constants are described. Some of them, that is, the meson-baryon-baryon coupling constants, are fixed to the same values used in our $BB$ potentials, and meson-meson-meson coupling constants and meson-baryon-baryon resonance coupling constants are new ingredients in $MB$ potentials. In §4, the Gaussian source functions are given. The source parameters for octet baryons are already determined in our $BB$ potentials. Additionally, we introduce the source parameters for pseudoscalar mesons and baryon resonances, which are free parameters in our $MB$ potential model. In §5, we describe how to calculate the $S$-matrix and scattering cross sections. Our potentials are given in the momentum space. We solve the Lippmann-Schwinger equations with relativistic kinematics. Our potentials have poles (at $E = m_{\text{bare}}$) on the real $E$-axis, given by the baryon-pole diagram ($m_{\text{bare}}$ means a bare mass of the baryon). To obtain the $T$-matrix, we renormalize the pole positions using the bare masses and bare coupling constants.

In §6, we give the results for meson-baryon potentials and compare them with experimental data. Our model can be used to describe simultaneously $BB$, $\pi N$, and $KN$ potentials using the common parameters. In §7, we discuss the roles of meson-exchange contributions in $MB$ potentials, in comparison with other models. Moreover, we apply our model to the $S = -1 (\pi \Lambda - \pi \Sigma - \bar{K}N)$ scattering and discuss its applicability. We provide a conclusion in §8.

**§2. One-hadron-exchange potentials**

In the case of the baryon-baryon ($BB$) potential, only one-meson-exchange diagrams contribute to the one-hadron-exchange potential. In the case of meson-baryon ($MB$) potentials, three types of diagram contribute to the potentials, that is, one-meson-exchange, one-baryon-exchange, and baryon-pole diagrams, as shown in Fig. 1. These are also called $t$-, $u$-, and $s$-channel-exchange diagrams, respectively. In this section, we explain the potentials using these three types of diagram in detail.

![Fig. 1. Diagrams contributing to the meson-baryon interaction, where B and M denote baryons (baryon resonances) and mesons, respectively. (a) Meson-exchange ($t$-channel-exchange), (b) baryon-exchange ($u$-channel-exchange), and (c) baryon-pole ($s$-channel-exchange) diagrams.](image-url)
2.1. Definition of the potential

We start with the relativistic Bethe-Salpeter equation for meson-baryon scattering given by

\[ M_{fi}(q_f, q_i; P) = M_{ji}^{irr}(q_f, q_i; P) + \sum_n \int d^4q_n M_{jfn}^{irr}(q_f, q_n; P) G(q_n, P) M_{ni}(q_n, q_i; P), \]  

(2.1)

where \( M^{irr} \) is the scattering matrix given by the irreducible Feynman diagrams (one-hadron-exchange diagrams in this paper) and \( G(q, P) \) is the two-particle Green function for a meson (with a mass \( m_M \)) and a baryon (with a mass \( m_B \)) given by

\[ G(q, P) = \frac{i}{(2\pi)^4} \left[ \frac{1}{\gamma(P/2 + q) - M_B + i\delta} \right] \left[ \frac{i}{(P/2 - q)^2 - m_M^2 + i\epsilon} \right]. \]  

(2.2)

\( P \) is the total momentum of the two-particle states and \( q_c (c = i, n, f) \) are the relative momenta defined by

\[ q_c = \frac{1}{2}(p_c - k_c), \quad (c = i, n, f) \]  

with the baryon momenta \( p_c \) and the meson momenta \( k_c \) in the initial \( (c = i) \), intermediate \( (c = n) \), and final \( (c = f) \) states.

Using the projection operator to positive-energy states, we truncate the equation and obtain the approximated equation for the positive-energy states as

\[ M_{fi}^{+}(q_f, q_i; P) = (M_{ji}^{irr})^{++}(q_f, q_i; P) + \sum_n \int d^4q_n (M_{jfn}^{irr})^{++}(q_f, q_n; P) G^{+}(q_n, P) M_{ni}^{++}(q_n, q_i; P), \]  

(2.4)

where

\[ M_{cc'}^{++}(q_c, q_{c'}; P) = \bar{u}_{Bc}(p_c, s_c) M_{cc'}(q_c, q_{c'}; P) u_{Bc'}(p_{c'}, s_{c'}), \]  

(2.5)

\[ (M_{cc'}^{irr})^{++}(q_c, q_{c'}; P) = \bar{u}_{Bc}(p_c, s_c) M_{cc'}^{irr}(q_c, q_{c'}; P) u_{Bc'}(p_{c'}, s_{c'}), \]  

(2.6)

and

\[ G^{+}(q_n; P) = \frac{i}{(2\pi)^4} \left[ \frac{1}{4E(p_n)\mathcal{E}(k_n)} \right] \left[ \frac{1}{p_n^0 - E(p_n) + i\delta} \right] \left[ \frac{1}{k_n^0 - \mathcal{E}(k_n) + i\epsilon} \right], \]  

(2.7)

\[ E(p_n) = \sqrt{m_n^2 + p_n^2}, \quad \mathcal{E}(k_n) = \sqrt{m_n^2 + k_n^2}. \]  

(2.8)

To obtain a three-dimensional equation in the cm system \( P = (W, 0) \), we use the on-mass-shell approximation in intermediate states and introduce

\[ G_0(q_n; W) = \int_{-\infty}^{\infty} dq_n^0 G^{+}(q_n; W) \]

\[ = \frac{1}{(2\pi)^4} \left[ \frac{1}{4E(q_n)\mathcal{E}(q_n)} \right] \left[ \frac{1}{W - E(q_n) - \mathcal{E}(q_n) + i\delta} \right], \]  

(2.9)
and then we obtain the three-dimensional equation

$$T_{fi}(q_f, q_i; W) = V_{fi}(q_f, q_i; W) + \sum_n \int dq_n^3 V_{fn}(q_f, q_n; W) G_0(|q_n|; W) T_{ni}(q_n, q_i; W), \quad (2.10)$$

where

$$T_{cc'}(q_c, q_{c'}; W) = \bar{u}_{Bc}(q_c, s_c) M_{cc'}(q_c, q_{c'}; W) u_{Bc'}(q_{c'}, s_{c'}), \quad (2.11)$$

$$V_{cc'}(q_c, q_{c'}; W) = \bar{u}_{Bc}(q_c, s_c) M_{cc'}^{irr}(q_c, q_{c'}; W) u_{Bc'}(q_{c'}, s_{c'}). \quad (2.12)$$

Equation (2.12) is the definition of potential in this paper. We can express the potential by

$$V_{cc'}(q_c, q_{c'}; W) = F^+(q_c, q_{c'}) + (\mathbf{\sigma} \cdot \hat{q}_c) F^-(q_c, q_{c'})(\mathbf{\sigma} \cdot \hat{q}_{c'}), \quad (2.13)$$

using $F^\pm(q_c, q_{c'})$ defined by

$$F^\pm(q_c, q_{c'}) = \sqrt{E_c \pm M_c} (E_{c'} \pm M_{c'}) \left[ \pm A(s, t, u) + \frac{W_c - M_c + W_{c'} \mp M_{c'}}{2} B(s, t, u) \right], \quad (2.14)$$

$$W_c = \sqrt{M_c^2 + q_c^2} + \sqrt{m_s^2 + q_{c'}^2}. \quad (2.15)$$

The invariant amplitudes, $A(s, t, u)$ and $B(s, t, u)$, are determined on the basis of one-hadron-exchange diagrams, where $s$, $t$, and $u$ are the Mandelstam variables defined by $s = (p_i + k_i)^2 = (p_f + k_f)^2$, $t = (p_i - p_f)^2 = (k_i - k_f)^2$, and $u = (p_i - k_f)^2 = (k_i - p_f)^2$. Originally, the invariant amplitudes are defined in the on-shell scattering ($W = W_c = W_{c'}$). To use these amplitudes in the potential, we have to define the off-shell extension. We assume the on-mass-shell approximations for $s$, $t$, and $u$.

### 2.2. One-meson-exchange potentials

In our $MB$ potentials, where the meson $M$ denotes pseudoscalar meson, exchanged mesons are limited to those with natural parities ($0^+$, $1^-$, $2^+$, etc.). From the consistency to our model of baryon-baryon potentials, we introduce only two types of meson, that is, scalar mesons ($0^+$) and vector mesons ($1^-$). In fact, mesons with higher spins, such as tensor mesons ($2^+$) are not expected to play very important roles in both $BB$ and $MB$ potentials.

For both scalar and vector mesons, we consider all the nonet mesons, which are given by

$$\Phi^S_8 = \begin{bmatrix} a^0 / \sqrt{2} + f_0 / \sqrt{6} \\ a^- \\ \kappa^- \\ -a^0 / \sqrt{2} + f_0 / \sqrt{6} \\ a^+ \\ \kappa^+ \\ \kappa^0 \\ f_0 \end{bmatrix}, \quad \Phi^V_1 = \sigma, \quad (2.16)$$

$$\Phi^V_8 = \begin{bmatrix} \rho^0 / \sqrt{2} + \phi / \sqrt{6} \\ \rho^- \\ K^*^- \\ -\rho^0 / \sqrt{2} + \phi / \sqrt{6} \\ \rho^+ \\ K^{*+} \\ K^*0 \\ K^{*0} \end{bmatrix}, \quad \Phi^V_1 = \omega, \quad (2.17)$$
and singlet-octet mixing angles, which are defined by
\[
\sigma^\text{phys} = \sigma \cos \theta_S + f_0 \sin \theta_S, \quad f_0^\text{phys} = f_0 \cos \theta_S - \sigma \sin \theta_S, \quad (2.18)
\]
\[
\omega^\text{phys} = \omega \cos \theta_V + \phi \sin \theta_V, \quad \phi^\text{phys} = \phi \cos \theta_V - \omega \sin \theta_V. \quad (2.19)
\]

For baryon-baryon-meson couplings, we use those defined in our \(BB\) potentials.\(^{22}\)

For meson-meson-meson couplings, we assume derivative couplings, that is,
\[
\mathcal{L} = -\frac{g_{PPS}}{m_{\pi}} \partial_\mu \phi^P \partial^\mu \phi^S, \quad (2.20)
\]
\[
\mathcal{L} = g_{PPV} \phi^V \partial_\mu \phi^P. \quad (2.21)
\]

For vector mesons, this type of coupling is commonly used in various works. For scalar mesons, direct coupling has also been used, but the derivative coupling is more favorable from the viewpoint of chiral symmetry (low-energy theorems). Using these interaction Lagrangians, we obtain the invariant amplitudes \(A(s, t, u)\) and \(B(s, t, u)\) as follows:
\[
A(s, t, u) = -\frac{1}{m_\pi} g_{PPS} g_{BBB} \frac{(m_i^2 + m_f^2 - t)}{t - m_S^2 + i\epsilon}, \quad B(s, t, u) = 0 \quad (2.22)
\]
for the scalar meson exchange, and
\[
A(s, t, u) = \frac{g_{PPV}}{t - m_V^2 + i\epsilon} \left( g_{BBV} \frac{(m_f^2 - m_i^2)(M_i - M_f)}{m_V^2} + f_{BBV} \frac{s - u}{2M_p} \right), \quad (2.23)
\]
\[
B(s, t, u) = -\frac{2g_{PPV}}{t - m_V^2 + i\epsilon} \left( g_{BBV} + f_{BBV} \frac{M_f + M_i}{2M_p} \right) \quad (2.24)
\]
for the vector meson exchange. Using the notation by Polinder and Rijken\(^{21}\)(\(F(+) \equiv F\) and \(F(-) \equiv G\)), the partial wave potentials with \(L = L_+ = J \mp 1/2\) are given by
\[
V_{J,L}^+ = F_{L_+}^{(+)} + F_{L_{\mp}}^{(-)} \quad (2.25)
\]
and
\[
F_{L}^{(\pm)} = \frac{1}{2} \int_{-1}^{1} d\cos \theta F^{(\pm)}(q_f, q_i) f(q_f, q_i) P_L(\cos \theta), \quad (2.26)
\]
where \(f(q_f, q_i)\) is the source function defined later in §4.

2.3. One-baryon-exchange potentials

As exchanged baryons, we consider both octet baryons and baryon resonances. One difficult problem is which resonances should be introduced as exchanged baryons. Some of the baryon resonances are regarded as poles produced dynamically by the meson-baryon interaction. Such resonances should be excluded from the exchanged baryons, to avoid double counting. However, we use a very phenomenological approach. Firstly, we introduce possible baryon resonances that may produce important contributions as the exchanged one and introduce the effective coupling constants with these baryon resonances. Secondly, we perform a search for parameters...
including these coupling constants. If we obtain very small values for some of the coupling constants, we regard such resonances as dynamical ones.

For $\frac{1}{2}^+\text{-baryon exchange}$, we obtain $(\bar{M} = (M_f + M_i)/2)$

\[
A(s, t, u) = \frac{f_{\text{BBP}} f'_{\text{BBP}} / m^2_\pi}{u - M_D^2 + i\epsilon} \left( (\bar{M} + M_B) u - \bar{M}M_f M_i - \frac{M_f^2 + M_i^2}{2} M_B \right), \tag{2.27}
\]

\[
B(s, t, u) = \frac{f_{\text{BBP}} f'_{\text{BBP}} / m^2_\pi}{u - M_D^2 + i\epsilon} \left[ u + 2\bar{M}M_B + M_f M_i \right]. \tag{2.28}
\]

For $\frac{1}{2}^-\text{-baryon exchange}$, we assume the coupling structure

\[
\mathcal{L} = -\frac{f'_{\text{BBP}}}{m_\pi} \bar{\Psi}^B \gamma_\mu \Psi^{B'} \partial^\mu \phi^P,
\]

where $\Psi^B, \Psi^{B'},$ and $\phi^P$ are fields of octet baryon $B$, $\frac{1}{2}^-\text{-baryon} B'$, and pseudoscalar meson $P$. As a result, we obtain the same expressions with the above $A$ and $B$ but replace $M_B$ with $-M_{B'}$. For $3/2^-\text{-baryon exchange}$, we use the coupling given by ($\Psi^D_\mu$ is a $3/2^+$-baryon field),

\[
\mathcal{L} = \frac{f_{\text{DBP}}}{m_\pi} \bar{\Psi}^B \Psi^D_\mu \partial^\mu \phi^P,
\]

then, we obtain the expressions of $A$ and $B$ given by

\[
A(s, t, u) = \frac{f_{\text{DBP}} f'_{\text{DBP}} / m^2_\pi}{u - M_D^2 + i\epsilon} \left[ \left( \frac{t - m_f^2 - m_i^2}{2} \right) + \frac{2u - M_f^2 - M_i^2}{6} \right]
\]

\[
+ \frac{M}{6M_D^2} (M_f^2 - m^2_i - u)(M_i^2 - m_f^2 - u)
\]

\[
+ \frac{1}{12} (m_i^2 - m_f^2)(M_f - M_i)
\]

\[
+ \frac{1}{12M_D} \left( (M_f^2 - m_i^2 - u)(M_i^2 - M_f M_i - 2m_f^2) \right)
\]

\[
+ (M_i^2 - m_f^2 - u)(M_f^2 - M_i M_f - 2m_i^2) \right] \right), \tag{2.31}
\]

\[
B(s, t, u) = -\frac{f_{\text{DBP}} f'_{\text{DBP}} / m^2_\pi}{u - M_D^2 + i\epsilon} \left[ \left( \frac{t - m_f^2 - m_i^2}{2} \right) \right]
\]

\[
+ \frac{1}{6M_D^2} (M_f^2 - m^2_i - u)(M_i^2 - m_f^2 - u)
\]

\[
- \frac{1}{6} (M_D M + 4M^2 - m_f^2 - m_i^2)
\]
\[ \frac{-\bar{M}}{3M_D} \left( M_f M_i - \frac{m_f^2 M_f + m_i^2 M_i}{2M} - u \right). \tag{2.32} \]

### 2.4. Baryon-pole potentials

Seemingly, the baryon-pole diagram contributes to only one partial wave, which corresponds to the angular momentum and the parity \((J^\pi)\) of the exchanged baryon. However, in relativistic formalism, which includes the negative energy state, it can make background contributions to other partial waves. As mentioned by Polinder and Rijken, these background contributions are not very small. Our potentials include these contributions.

The invariant amplitudes \(A\) and \(B\) for baryon-pole diagrams are given by the interchange \(s \leftrightarrow u\) and \(m_i\) (initial meson) \(\leftrightarrow m_f\) (final meson) in those for baryon-exchange diagrams.

The baryon-pole potential has a pole on the real axis of total energy \(\sqrt{s} (= W)\). This pole results in the pole of the \(S\)-matrix on the complex plane of \(\sqrt{s}\). The position of the pole \((\sqrt{s} = M_R + i\Gamma/2)\) of the \(S\)-matrix should correspond to the physical mass \((M_R)\) and widths \((\Gamma)\) of the exchanged baryon (baryon resonance). Thus, we introduce the bare mass \(M_{\text{bare}}\) and the bare coupling constant \((g_{\text{bare}})\) to the baryon-pole potential and then perform the renormalization calculation as described below. Firstly, the total potential is divided into two parts

\[ V_{t,u}^{JL\pm}(q_f, q_i) = V_{t,u}^{JL\pm}(q_f, q_i) + \frac{\gamma(q_f) \gamma(q_i)}{\sqrt{s} - M_{\text{bare}}}, \tag{2.33} \]

where \(V_{t,u}^{JL\pm}\) is the nonpole part that comes from the \(t\)-channel-exchange (meson-exchange) and \(u\)-channel-exchange (baryon-exchange) diagrams, and the last term is the \(s\)-channel-exchange (baryon-pole) diagram, which can be expressed in a separable form. Secondly, we calculate the \(T\)-matrix \(T_{t,u}^{JL\pm}\) with only the nonpole part of the potential

\[ T_{t,u}^{JL\pm}(q', q) = V_{t,u}^{JL\pm}(q', q) + \int k^2 dk V_{t,u}^{JL\pm}(q', k)G_0(k; \sqrt{s})T_{t,u}^{JL\pm}(k, q) \tag{2.34} \]

and

\[ \gamma^*(q) = \gamma(q) + \int q'^2 dq' T_{t,u}^{JL\pm}(q', q)G_0(q'; \sqrt{s}) \gamma(q'), \tag{2.35} \]

\[ \Sigma = \int q^2 dq \gamma^*(q) G_0(q; \sqrt{s}) \gamma(q). \tag{2.36} \]

Finally, we obtain the full \(T\)-matrix \(T^{JL\pm}\) as

\[ T^{JL\pm} = T_{t,u}^{JL\pm} + \frac{\gamma(q_f) \gamma(q_i)}{\sqrt{s} - M_{\text{phys}}} \]

\[ M_{\text{phys}} = M_{\text{bare}} + \Sigma. \tag{2.37} \]

Note that \(M_{\text{phys}}\) and \(\gamma^*\) are \(\sqrt{s}\)-dependent. Therefore, to explicitly determine \(M_R + i\Gamma/2\) (the position of the pole of \(T^{JL\pm}\)), which corresponds to the physical mass
and width of the baryon resonance, we need the additional procedure described by Polinder and Rijken. In principle, the pole can be reproduced by fitting the experimental data of the resonance channel. We simply determine the bare mass and bare coupling constant so as to reproduce the experimental phase shifts.

§3. $SU(3)$ parameters in the interaction Lagrangians

For meson-baryon-baryon vertices, we assume the $SU(3)$-symmetric interaction Lagrangians, which are the same as those assumed in our $BB$ potentials. In the present work, we use the same values for all the parameters in the Lagrangians with those used in the $BB$ potentials. In the future, we will try to determine the values through simultaneous fitting of $BB$ and $MB$ interactions. However, in the present stage, these values are treated as a constraint from the $BB$ potentials. It is very interesting to determine whether we can find reasonable $MB$ potentials under the constraint given by our $BB$ potentials. For the meson-meson-meson vertices, we introduce the following types of interaction Lagrangian, which describe only the $SU(3)$ relations (their explicit forms are given in Eqs. (2.20) and (2.21)):

\[\mathcal{L} = g^{(8)}_{P_{PS}} \{ \text{Tr}[\Phi^8 P\Phi^8_P \Phi^8_S] + \text{Tr}[\Phi^8 P\Phi^8_P \Phi^8_S]\}/2 + g^{(1)}_{P_{PS}} \text{Tr}[\Phi^8 P\Phi^8_P \Phi^8_S],\]

\[\mathcal{L} = g^{(8)}_{P_{PV}} \{ \text{Tr}[\Phi^8 P\Phi^8_P \Phi^8_V] - \text{Tr}[\Phi^8 P\Phi^8_P \Phi^8_S]\}/2,\]

where $\Phi^8_P$, $\Phi^8_P$, $\Phi^8_V$, and $\Phi^8_V$ are given in Eqs. (2.16) and (2.17), and $\Phi^8_P$ and $\Phi^8_P$ are given by

\[\Phi^8_P = \left[\begin{array}{ccc}
\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\
\pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\
K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta_8
\end{array}\right], \quad \Phi^8_P = \eta_1, \quad (3.3)

\[\eta_1^{phys} = \eta_1 \cos \theta_P + \eta_8 \sin \theta_P, \quad \eta_8^{phys} = \eta_8 \cos \theta_P - \eta_1 \sin \theta_S. \quad (3.4)\]

In the above expressions, we assumed symmetric ($F/(F+D) = 0$) and antisymmetric ($F/(F+D) = 1$) couplings for scalar and vector meson couplings, respectively. As a result, we have only three coupling constants that are optimized to explain the experimental meson-baryon scattering data. The singlet-octet mixing angles $\theta_S$, $\theta_V$, and $\theta_P$ were predetermined in the model of the $BB$ potentials. For the interaction Lagrangians including decuplet baryons ($\Delta$ and $\Sigma^*$), we assume the $SU(3)$ relation given by ($f_{\Delta} = f_{\Delta^{++} p\pi^+}$)

\[f_{\Sigma^{*+} \Delta^+} = \frac{1}{\sqrt{2}} f_{\Delta}, \quad f_{\Sigma^{*+} \Sigma^0} = -\frac{1}{\sqrt{6}} f_{\Delta}, \quad f_{\Sigma^{*+} p\bar{K}^0} = \frac{1}{\sqrt{3}} f_{\Delta}. \quad (3.5)\]

Therefore, the coupling constants including $\Sigma^*$ are determined uniquely using the $\Delta N \pi$ coupling constant. We introduce the Roper resonance and $S_{11}$ resonance to describe the $\pi N$ scattering. The coupling constants including these resonances are determined by fitting the phenomenological $\pi N$ phase shifts.
§4. Gaussian source functions

In our model, hadrons (baryons and mesons) are regarded as sources of meson fields with the Gaussian shape. Each hadron of the $SU(3)$ multiplets has a common quark-gluon structure, but its mass breaks the $SU(3)$ symmetry almost linearly to the number of $s$-quarks in the hadron. Therefore, we can expect that its size as a many-body system also breaks the $SU(3)$ symmetry. From this viewpoint, we introduce the $SU(3)$ breaking in the size parameters, as shown below.

Each hadron ($h$) has a fixed size that is expressed as a source parameter, $A_h$. For $MB$ potentials, we have three types of diagram as shown in Fig. 1. We define the source functions as

$$ f(q_f, q_i) = \exp(-q_f^2 - q_i^2)/A^2) $$

for the meson-exchange diagrams (Fig. 1(a)), and

$$ f(q_f, q_i) = \exp(-q_f^2 + q_i^2)/2A^2) $$

for the baryon-exchange and baryon-pole diagrams (Figs. 1(b) and (c)), where $A = A_1^2 A_2^2/(A_1^2 + A_2^2)$. $A_1$ and $A_2$ are defined by

$$ A_1 = (A_{B_1} + A_{M_2})/2 + \Delta A_{M_2}, \quad A_2 = (A_{M_2} + A_{M_4})/2 + \Delta A_{M_4}, \quad (4.3) $$

$$ A_1 = (A_{B_1} + A_{B_2})/2 + \Delta A_{M_4}, \quad A_2 = (A_{B_3} + A_{B_4})/2 + \Delta A_{M_2}, \quad (4.4) $$

$$ A_1 = (A_{B_1} + A_{B_2})/2 + \Delta A_{M_2}, \quad A_2 = (A_{B_2} + A_{B_4})/2 + \Delta A_{M_4}, \quad (4.5) $$

for the diagrams shown in Figs. 1(a), (b), (c), respectively. In the above expressions, $B_i$ and $M_i$ are baryons and mesons (the suffixes 1 and 2 (3 and 4) that denote those in the initial (final) states), and $\tilde{B}$ and $m$ are an exchanged baryon (baryon resonance) and a meson, respectively. The source parameters for baryons satisfy the $SU(3)$ symmetry with breaking linearly to the strangeness $S(B)$,

$$ A_B = A_N + \Delta A \cdot S(B), \quad \text{for} \quad B = N, \Lambda, \Sigma^{0,\pm}, \Xi^{0,-}, \quad (4.6) $$

$$ A_{B^*} = A_\Delta + \Delta A^* \cdot S(B^*), \quad \text{for} \quad B^* = \Delta, \Sigma^*, \quad (4.7) $$

and, in addition, $A_{N^*}$ and $A_{S_{11}}$ are free parameters determined by fitting to experimental data. For mesons, we again assume the linear breaking of the $SU(3)$ symmetry

$$ A_M = A_\pi + \Delta A^* \cdot |S(M)| \quad \text{for} \quad M = \pi, K, \bar{K}. \quad (4.8) $$

We have a total of six free parameters, namely, $A_\Delta$, $\Delta A^*$, $A_{N^*}$, $A_{S_{11}}$, $A_\pi$, and $\Delta A^*$. All other source parameters have been predetermined in the $BB$ potentials. (In practical calculations, we have fixed $\Delta A^*$ to zero.)

§5. Calculation of the coupled-channel scattering problems

Using the potentials defined in the momentum space, we have to solve the coupled-channel scattering problems. To fit to the experimental phase shifts and
scattering lengths and cross sections, we determine the \( S \)-matrix by solving the Lippmann-Schwinger (LS) equation. We assume the relativistic kinematics and on-mass-shell relation in the three-dimensional LS equation. The effects of the Coulomb force in cross sections are estimated by adding the Coulomb amplitudes to the amplitudes estimated without Coulomb force.

5.1. Coupled-channel \( S \)-matrix

To calculate the \( S \)-matrix, we solve the Lippmann-Schwinger equation

\[
T_{AB}(q', q) = V_{AB}(q', q) + \sum_C \int_0^\infty \frac{k^2 dk}{(2\pi)^3} V_{AC}(q', k) \frac{1}{4E(k)\tilde{E}(k)\sqrt{s - E(k) - \tilde{E}(k) + i\epsilon}} T_{CB}(k, q). \tag{5.1}
\]

To obtain \( T_{AB} \), we solve firstly the integral equation given by

\[
R_{AB}(q', q) = V_{AB}(q', q) + \mathcal{P} \sum_C \int_0^\infty \frac{k^2 dk}{(2\pi)^3} V_{AC}(q', k) \frac{1}{4E(k)\tilde{E}(k)\sqrt{s - E(k) - \tilde{E}(k)}} R_{CB}(k, q); \tag{5.2}
\]

then, we solve the algebraic equation given by

\[
T_{AB} = R_{AB} - i\frac{1}{32\pi^2\sqrt{s}} \sum_{C(\text{open})} q_C R_{AC} T_{CB}, \tag{5.3}
\]

where, \( \sum_{C(\text{open})} \) denotes the summation only by the open channels \( C \). The \( S \)-matrix is given by

\[
S_{AB} = \delta_{AB} - i\frac{\sqrt{q_{AB}}}{16\pi^2\sqrt{s}} T_{AB}. \tag{5.4}
\]

To determine the scattering length (\( L = 0 \)) and scattering volume (\( L = 1 \)), we use the three-point extrapolation method for

\[
a = \lim_{q \to 0^+} \text{Im} \left( \frac{1}{q^{2L+1}} \frac{S(q) - 1}{S(q) + 1} \right). \tag{5.5}
\]

In our calculations, we employ the particle (charge) bases and use physical masses for all hadrons in the initial and final states. However, the experimental analyses of scattering lengths (volumes) and phase shifts have been conventionally given in the isospin bases. For these quantities, we calculate the \( S \)-matrix using the common masses for the isospin doublets (triplets) and determine the \( S \)-matrix in the isospin base by suitable linear combinations of those defined in the particle base.

5.2. Coulomb effects

For the cross sections in the \( \bar{K}N(S = -1) \) sector, the experimental data are given in particle base and include Coulomb effects. Therefore, to compare them with our calculations, we should consider the Coulomb effects. In this paper, we use a simple
approximation to calculate the cross sections as follows. Firstly, we calculate the scattering amplitude \( f_s \) with only the strong interaction (without Coulomb force). Secondly, we determine the cross sections using the sum of the amplitudes, \( f = f_s^c + f_{\text{pureCoulomb}} \), where \( f_s^c \) is defined by \( f_s \) in which the partial wave amplitudes are multiplied by \( \exp(i\sigma_L + i\sigma'_L) \) (\( \sigma_L \) and \( \sigma'_L \) are Coulomb phase shifts in the initial and final states, respectively). Such an approximation can be justified except at very low energies (see Fig. 10).

§6. Results

Our potential model is determined so as to simultaneously reproduce the \( \pi N \) and \( KN \) scattering lengths and phase shifts in the \( S \)- and \( P \)-waves at low energies. Our model satisfies the unitarity in the two-particle channels. Thus, its applicability is limited to the energy region where the inelasticities caused by three-particle channels are small. We choose the energy region \( T_{\text{lab}} \leq 600(500) \text{ MeV} \) for \( \pi N(KN) \) scattering. In the present \( MB \) model, we always fixed the values of all the parameters that have been determined in the \( BB \) potentials.

On the other hand, we do not take into account the \( S = -1 \) data (\( \bar{K}N, \pi\Lambda \), and \( \pi\Sigma \) data) in determining our potential model for two reasons. Firstly, the \( S = -1 \) data have not yet been established sufficiently. For example, the data of the scattering length \( a_{K^-p} \) are contradictory. Secondly, we can check the applicability of our model. In practice, without any additional parameters, we can almost uniquely determine the \( S = -1 \) \( S \)-wave interaction (\( \pi\Lambda, \pi\Sigma \), and \( \bar{K}N \) coupled-channel potentials).

6.1. Values of parameters in our potential model

We have 11 free parameters to fit the \( MB \) data, that is, 6 coupling constants and 5 Gaussian source parameters. In addition, we have 8 parameters for the bare masses and bare coupling constants in the baryon-pole diagrams. As baryon resonances, we consider \( \Delta(1254) \), \( S_{11}(1567) \), \( N^*(1440) \), and \( \Sigma^*(1385) \). The parameters determined so as to reproduce \( \pi N \) and \( KN \) scattering data are given in Tables I and II. We obtain very small coupling constants for \( \pi \pi \)-scalar meson coupling. These come from the isovector nature of the \( S \)-wave \( \pi N \) interaction, which excludes the isoscalar-type attractive interaction. In other models, for example, that by Polinder and Rijken, and that by Jülich group, some additional ingredient (Pomeron or \( \sigma' \)) is assumed to cancel out the strong attraction from scalar-meson-exchange.

6.2. Results for the \( \pi N \) scattering

The phenomenological properties of the \( \pi N \) low-energy scattering have been established very quantitatively. We can use the precise phase shifts and the low-energy parameters (scattering lengths and volumes). In Table III, we show the \( \pi N \) scattering lengths and scattering volumes. Despite of the very small experimental error bars, our results are located within the error bars in four partial waves among six partial waves. For \( \pi N \) phase shifts in \( S \)- and \( P \)-waves, we obtain the results shown in Fig. 2. We find a reasonable fit to the experimental phase shifts, except some
Table I. Values of parameters in our potential model.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\pi\pi\pi}$</td>
<td>-0.0921</td>
</tr>
<tr>
<td>$g_{\pi K\kappa}$</td>
<td>0.0241</td>
</tr>
<tr>
<td>$g_{\pi\rho\rho}$</td>
<td>1.7634</td>
</tr>
<tr>
<td>$f_{\Delta N\pi}$</td>
<td>4.0270</td>
</tr>
<tr>
<td>$f_{S_{11}N\pi}$</td>
<td>0.1196</td>
</tr>
<tr>
<td>$f_{N^*\pi\pi}$</td>
<td>0.0829</td>
</tr>
<tr>
<td>$\Lambda_{\Delta N\pi}(fm^{-1})$</td>
<td>4.1205</td>
</tr>
<tr>
<td>$\Lambda_{S_{11}N\pi}(fm^{-1})$</td>
<td>4.3011</td>
</tr>
<tr>
<td>$\Lambda_{N^*\pi\pi}(fm^{-1})$</td>
<td>4.4878</td>
</tr>
<tr>
<td>$\Lambda_{\pi\pi\pi}(fm^{-1})$</td>
<td>6.6059</td>
</tr>
<tr>
<td>$\Lambda_{\pi K\kappa}(fm^{-1})$</td>
<td>3.9363</td>
</tr>
</tbody>
</table>

Table II. Bare masses and bare coupling constants in baryon-pole diagrams. $M_{phys}$ are in MeV/$c^2$. $f_{phys}$ are given in Table I ($f_{NN\pi}/\sqrt{4\pi}=0.2722$).

<table>
<thead>
<tr>
<th>exchanged particle</th>
<th>$M_{phys}$</th>
<th>$M_{bare}/M_{phys}$</th>
<th>$f_{bare}/f_{phys}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>939</td>
<td>1.313</td>
<td>0.4548</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>1254</td>
<td>1.100</td>
<td>0.3374</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>1567</td>
<td>1.630</td>
<td>9.4781</td>
</tr>
<tr>
<td>$N^*$</td>
<td>1440</td>
<td>1.327</td>
<td>3.4588</td>
</tr>
</tbody>
</table>

Table III. $\pi N$ scattering lengths (fm) and volumes (fm$^3$).

<table>
<thead>
<tr>
<th>partial wave</th>
<th>exp</th>
<th>calc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>0.2473 ± 0.0043</td>
<td>0.2470</td>
</tr>
<tr>
<td>$S_{31}$</td>
<td>-0.1444 ± 0.0057</td>
<td>-0.1378</td>
</tr>
<tr>
<td>$P_{11}$</td>
<td>-0.2368 ± 0.0058</td>
<td>-0.2356</td>
</tr>
<tr>
<td>$P_{31}$</td>
<td>-0.1316 ± 0.0058</td>
<td>-0.1333</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>-0.0877 ± 0.0058</td>
<td>-0.0994</td>
</tr>
<tr>
<td>$P_{33}$</td>
<td>0.6257 ± 0.0058</td>
<td>0.6254</td>
</tr>
</tbody>
</table>

disagreements in the high-energy region ($T_{lab} > 300$ MeV). In Fig. 3, the contributions from various diagrams are shown. We find that the background contributions from $N$-pole and $\Delta$-pole diagrams are important and that the $\Delta$ contributions are somewhat larger than those in other models because of the large coupling constant $f_{\Delta N\pi}$. The meson-exchange contributions will be discussed in the next section.

6.3. Results for the $KN$ scattering

The $KN$ potential is rather simple in comparison with the $\pi N$ potential. Since there is no baryon with a positive strangeness, the baryon-pole diagram does not contribute to the $KN$ interaction. On the other hand, additional mesons ($a_0$, $\omega$, and $\phi$) have contributions in meson-exchange diagrams.

Results for $KN$ scattering lengths and volumes are given in Table IV. We find a good fit to the experimental values. Results for $KN$ phase shifts are shown in Fig. 4. Our model successfully reproduces the repulsive nature in the $S$-wave interaction. In the $P_{01}$-wave (with $I=0$ and $2J=1$), we can explain the attractive potential at
Fig. 2. $\pi N$ phase shifts in $S$- and $P$-waves. Solid lines denote the results with the present model. The data with error bars are the results of single-energy phase shift analysis by SAID.

Fig. 3. Contributions of various diagrams to the $\pi N$ potentials. The ordinates are given by $V_{JL}^J / (4\pi)$ (dimensionless). Contributions are given by solid ($f_0$), dashed ($\sigma$), short-dashed ($\rho$), dotted ($N$-ex), and double-dashed ($\Delta$-ex) lines. The dash-double-dotted lines are those from the background contributions from $N$- and $\Delta$-pole diagrams. The pole parts of pole diagrams ($S_{11}$-pole in the $S_{11}$-wave, $N$-, $N^*$-poles in the $P_{11}$-wave, and $\Delta$-pole in the $P_{33}$-wave) are omitted.
Fig. 4. $K\bar{N}$ phase shifts in $S$- and $P$-waves. Solid lines are the results with the present model. The data with error bars are the results of single-energy phase shift analysis by SAID.

Table IV. $K\bar{N}$ scattering lengths (fm) and volumes ($fm^3$).

<table>
<thead>
<tr>
<th>partial wave</th>
<th>exp</th>
<th>calc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{01}$</td>
<td>$0.00 \pm 0.02$</td>
<td>$-0.075$</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>$-0.33 \pm 0.02$</td>
<td>$-0.353$</td>
</tr>
<tr>
<td>$P_{01}$</td>
<td>$0.08 \pm 0.02$</td>
<td>$0.148$</td>
</tr>
<tr>
<td>$P_{11}$</td>
<td>$-0.16 \pm 0.02$</td>
<td>$-0.098$</td>
</tr>
<tr>
<td>$P_{03}$</td>
<td>$-0.13 \pm 0.02$</td>
<td>$-0.006$</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>$0.07 \pm 0.02$</td>
<td>$0.030$</td>
</tr>
</tbody>
</table>

low energies ($T_{lab} < 150$ MeV), but cannot reproduce the behavior at higher energies ($T_{lab} > 200$ MeV), where the attraction is too weak. This comes from the strongly repulsive contribution from the $\omega$-exchange, as discussed below. In the $P_{03}$-wave, we cannot fit the experimental dip structure at around $T_{lab} = 250$ MeV. However, this dip disappears in the energy-dependent phase shift analysis.$^6$

The contributions of various diagrams to $K\bar{N}$ potentials are given in Fig. 5. We find very strong repulsions from $\omega$-exchange in all partial waves. In the $P_{01}$- and $P_{13}$-waves, $\phi$-, $\Sigma^*$- and $\Lambda$-exchange contributions provide considerably strong attractions, but are insufficient to provide the attractions required at $T_{lab} > 200$ MeV.

§7. Discussion

7.1. Role of scalar-meson-exchange contributions

In this subsection and the next, we briefly discuss the meson-exchange contributions in our potentials, in comparison with other models, especially those given by Polinder and Rijken (the NSC $\pi N$ and $K\bar{N}$ models).

The scalar-meson-exchange contributions in the $\pi N$ and $K\bar{N}$ potentials are shown in Fig. 6. We find that in our model, all scalar mesons have very small con-
Fig. 5. Contributions of various diagrams to the KN potentials. The ordinates are given by \( V_{JL}^{\pm}/(4\pi) \) (dimensionless). Contributions are given by solid (\( f_0 \)), dashed (\( \sigma \)), short-dashed (\( \rho \)), dotted (\( A \)-ex), dash-dotted (\( \Sigma \)-ex), short-dash-dotted (\( a_0 \)), double-dashed (\( \Sigma^* \)-ex), dash-double-dotted (\( \omega \)), and quadruple-dashed (\( \phi \)) lines.

Fig. 6. Scalar-meson-exchange contributions in \( \pi N \) (S-wave) (in left panel) and \( \pi N \) (S01-wave) (in right one) potentials. In the figures, sigma, \( f_0 \), \( a_0 \) and Pomeron denote \( \sigma \)-, \( f_0 \)-, \( a_0 \)- and Pomeron-exchange contributions, and (P&R) and (ours) correspond to those in the model by Polinder and Rijken and in our model, respectively (the Pomeron-exchange is not included in our model).

In the NSC models, the Pomeron exchange, which has a scalar-isoscalar-type contribution, cancel the \( \sigma \)-exchange contributions. The physical width of \( \sigma \) meson is very large (\( \sim 600 \text{ MeV} \)). This large width may require a large \( \pi \pi \sigma \) coupling constant. However, such a large coupling constant completely contradicts the small \( \sigma \)-exchange contribution obtained in our model. This problem can be solved by constructing the model of \( \pi \pi \) interaction, which will be treated in our future model. In the Julich model, a repulsive interaction by artificial \( \sigma' \)- or \( \sigma_{\text{rep}} \)-exchange with a heavy mass (\( \sim 1200 \text{ MeV} \)) is introduced so as to cancel the attractive \( \sigma \)-exchange contribution.
7.2. Role of vector-meson-exchange contributions

In $\pi N$ potential, only $\rho$ meson contributes among vector mesons. The left panel of Fig. 7 shows the $\rho$ contributions in the $S_{11}$-, $P_{11}$-, and $P_{13}$-waves. You need a factor of $-3$ to obtain those in the isospin $I = 3/2$ (the $S_{31}$-, $P_{31}$-, and $P_{33}$-waves, respectively). In the $S_{11}$-wave, the $\rho$ contribution in our potential is weaker than that in Polinder and Rijken’s (PR) model, but in the $P_{11}$-wave, the relation is opposite. This comes from the difference in the ratio $(f/g)_{NN\rho}$. In our potential, the ratio is fixed to 5.95, as determined using the BB potential model.\(^{22}\) In the case of the PR model, a very small value of the ratio, 2.12, is used. This is inconsistent with the BB models. The middle and right panels show the vector-meson-exchange contributions in the $KN$ $S$-wave potentials. In our potentials, the $\omega$ and $\phi$ contributions are much stronger than those in the PR model. This comes from the large $NN\omega$ and $NN\phi$ coupling constants predetermined in the BB model. The strong repulsive contribution from $\omega$ gives the repulsive nature of the $KN$ interaction. The same contribution gives a strong attraction in the $\bar{K}N$ interaction as discussed below.

7.3. Results for the $\pi \Lambda-\pi \Sigma-\bar{K}N$ scattering

In this work, strangeness $S = -1$ data were not included in the parameter fitting. However, our model can be used to almost uniquely determine the $S$-wave potentials in the $S = -1$ sector. Strictly speaking, we need the bare masses for $\Lambda$ and $\Sigma$ and the bare coupling constants for $YY'\pi$ ($Y,Y' = \Lambda, \Sigma$) in baryon-pole diagrams. Here, we assume the same values as those for $N$ in Table II (in practice, these contributions are not large).

Using $\pi \Lambda-\pi \Sigma-\bar{K}N$ coupled channel potentials predicted from the present model, we obtain a very attractive potential and introduce an overall factor $f (V \rightarrow fV)$ of around 0.7 to reproduce $\Lambda(1405)$ as a quasibound state. Figure 8 shows the results for $f = 0.75$, 0.70, and 0.65. The histogram in Fig. 8 (left panel) denotes the $\pi \Sigma$ mass spectrum in the isospin $I = 0$ state determined phenomenologically. The calculated curves are given by

$$\frac{\sqrt{s}^2}{q_{\pi \Sigma}} |S_{\pi \Sigma-\pi \Sigma}(I = 0) - 1|^2 = q_{\pi \Sigma} \left| \frac{T_{\pi \Sigma-\pi \Sigma}(I = 0)}{16\pi^2} \right|^2. \tag{7.1}$$
Fig. 8. Left panel: $\pi\Sigma(I = 0)$ mass spectrum in $S$-waves. Numbers of the ordinate are written for curves calculated using Eq. (7.1) in unit of fm. Right panel: $\pi\Sigma(I = 0)$ $S$-wave phase shifts. Solid, dotted, and dash-dotted lines indicate the results with $f = 0.65$, 0.70, and 0.75, respectively. The lower two lines in the right panel indicate the results without the $\bar{K}N$ channel (for $f = 1.0$ and 0.7).

The histogram expresses (the weighted events)/(10 MeV). Thus, only the comparison of shapes between the curves and the histogram is meaningful. We obtain the best fit for $f = 0.7$. In Fig. 8 (right panel), $\pi\Sigma(I = 0)$ $S$-wave phase shifts are calculated. We find a resonance pole at around $\sqrt{s} = 1405$ MeV for $f = 0.7$. In the figure, the lower two lines denote the results without the $\bar{K}N$ channel for $f = 1.0$ and 0.7. We cannot obtain any pole without the $\bar{K}N$ channel. It is found that the diagonal $\pi\Sigma$ interaction is not so strongly attractive to produce a pole and the pole at around $\sqrt{s} = 1405$ MeV comes from the attractive $\bar{K}N$ interaction.

In Table V, the results of the reaction ratios (branching ratios) at the $K^-p$ threshold are listed. The experimental data are determined from the decay properties of kaonic hydrogen. The reaction ratios $R_e$, $R_c$, and $R_n$ are defined by

$$R_e = \frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-)}{\Gamma(K^-p \rightarrow \pi^-\Sigma^+)};$$

$$R_c = \frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^+)}{\Gamma(K^-p \rightarrow \pi^0\Sigma^0, \pi^0\Lambda)};$$

$$R_n = \frac{\Gamma(K^-p \rightarrow \pi^0\Lambda)}{\Gamma(K^-p \rightarrow \pi^0\Lambda, \pi^0\Sigma^0)};$$

\[ (7.2) \]
\[ (7.3) \]
\[ (7.4) \]

Table V. Reaction ratios, $R_e$, $R_c$, $R_n$ at the $K^-p$ threshold and $K^-p$ scattering length, $a_{K^-p}$. The experimental data are taken from Refs. 25) and 26).
Fig. 9. $K^-p$ scattering length. The numbers in the figure denote values of the overall factor $f$. The rectangles with solid and dashed lines correspond to the phenomenological values derived from DEAR$^{27)}$ and KEK$^{28), 29)}$ data, respectively.

respectively. The empirical values of $R_e$ and $R_n$ support the case with $f = 0.75$. However, these quantities are very sensitive to the potential used. Thus, we need a more careful treatment (the introduction of the overall factor $f$ is a clearly oversimplified treatment).

In Fig. 9, we show the results of the $K^-p$ scattering length. Two experimental data by DEAR$^{27)}$ and KEK$^{28), 29)}$ are indicated by rectangles in the figure. We find that the DEAR (KEK) data support the result with $f = 0.75$ (0.70). Finally, we show the results for $K^-p\to K^-p, K^0n, \pi^{\pm,0}\Sigma^{\pm,0}, \pi^0\Lambda$ integrated cross sections at low energies in Fig. 10. The integrated cross sections are defined by

$$\sigma = \int_{\cos \theta = -1}^{\cos \theta = 0.966} \frac{d\sigma}{d\Omega} d\Omega, \quad (7.5)$$

which are assumed in the experimental analyses.$^7)$ We find that our results are reasonable but cannot successfully explain the experimental data by introducing only the overall factor $f$. In fact, our model overestimates the charge-exchange reaction $K^-p\to K^0n$.

§8. Conclusion

We proposed a model of meson-baryon potentials based on the one-hadron-exchange mechanisms. In practice, we constructed $\pi N$ and $KN$ potentials so as to reproduce the low-energy scattering data, keeping the values of parameters predetermined in the baryon-baryon potential model proposed in the previous work. We obtained a good agreement with the data.

Moreover, we applied our model to the $\pi\Lambda-\pi\Sigma-\bar{K}N$ coupled-channel scattering. We find a much attractive potential in the $I = 0$ $S$-wave and need an overall factor of about 0.7 to describe $\Lambda(1405)$ as a quasibound state. This strong attraction comes from the strong $\omega$-exchange contribution. To resolve this problem, we have to modify the baryon-baryon-meson coupling constants, which are fixed in this work.
Fig. 10. Integrated $\bar{K}N$ reaction cross sections. Solid, dashed, and dash-dotted lines indicate the results with the factor $f = 0.65, 0.70$, and $0.75$, respectively. In left-upper panel, the double dashed line indicates the result for pure Coulomb scattering. The experimental data are taken from Refs. 7)–10).

Therefore, we have to perform the combined analysis of baryon-baryon and meson-baryon potentials to construct a unified model of hadron-hadron interactions. In our $BB$ potential model, no phenomenological short-range repulsive cores have been introduced and their effects have been replaced effectively by the meson-exchange contributions. In future models, the phenomenological repulsive cores will be introduced explicitly, similarly to the Funabashi-Gifu model.

References