Progress of Theoretical Physics, Vol. 124, No. 3, September 2010

Slowly Rotating Black Holes
in Einstein-Generalized Maxwell Gravity

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(Received May 2, 2010; Revised July 14, 2010)

In this paper, considering the nonlinear electromagnetic field coupled to Einstein gravity, we obtain the higher dimensional slowly rotating charged black hole solutions. By use of the fact that the temperature of the extreme black hole is zero, we find that these solutions may be interpreted as black hole solutions with inner (Cauchy) and outer (event) horizons provided that the mass parameter $m$ is greater than an extremal value $m_{\text{ext}}$, an extreme black hole if $m = m_{\text{ext}}$ and a naked singularity otherwise. Also, we find that the asymptotic behavior of the spacetime is not anti de Sitter for the special values of the nonlinearity parameter. Then, we compute the ADM mass, electrical charge, temperature, entropy, angular momentum and gyromagnetic ratio of the solutions. Calculations of the electromagnetic field, electrical charge, entropy and temperature show that they are sensitive with respect to the changing of nonlinearity parameter.

Subject Index: 453, 450

§1. Introduction

Over the last decades a lot of attention has been focused on rotating black hole solutions in presence of linear and nonlinear electromagnetic fields in the background. On one hand black holes produced at colliders may in general have an electric charge as well as other type of charges and therefore the study of charged black hole becomes of great importance. On the other hand, the question as to why the Planck and electroweak scales differ by so many orders of magnitude remains mysterious. In recent years, attempts have been made to address this hierarchy issue within the context of theories with extra spatial dimensions. These subjects motivate one to study on higher dimensional charged black hole solutions. The first higher dimensional extensions of the Schwarzschild solution have been obtained by Tangherlini$^1$ and generalized by Myers and Perry.$^2$ Also, in 3), higher dimensional charged black hole solutions have been presented. This solution is the generalization of the familiar Reissner-Nordström solution for a static and electrically charged black hole in ordinary general relativity. Recently, many authors have introduced various classes of charged rotating black hole solutions of Lovelock gravity and investigate their thermodynamics.$^4$ Also, properties of charged rotating black holes in Brans-Dicke theory are investigated by some authors.$^5$

However the Kerr-Newman solution in higher dimensions, that is the charged

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generalization of the Myers-Perry solution in the Einstein-Maxwell gravity, it still remains to be found, but there are many interesting papers focused on the slowly rotating black holes.\(^6\)–\(^8\)

From the other point of view, in the conventional methods, some generalizations of the matter sources to higher dimensions, one essential property of the these sources is lost, namely, conformal invariance. Electromagnetic field theory can be studied in a special gauge which is conformally invariant, and firstly, has been proposed by Eastwood and Singer.\(^9\) In addition, quantized Maxwell theory in a conformally invariant gauge and in flat Euclidean 4-space have been investigated by Esposito\(^{10}\) and one of the valuable works on nonlinear electrodynamics has been done in \(^{11}\). Also, there exists a generalized extension of the Maxwell action in higher dimensions, if one uses the lagrangian of the \(U(1)\) gauge field in the form\(^{12}\)–\(^{17}\)

\[
I_{CIM} = \alpha \int d^d x \sqrt{-g} (F_{\mu\nu} F^{\mu\nu})^s, \tag{1.1}
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the electromagnetic tensor field, \(A_\mu\) is the vector potential and \(\alpha\) is a constant. It is straightforward to show that the action (1.1) is invariant under conformal transformation \((g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}\) and \(A_\mu \rightarrow A_\mu\)) for \(s = d/4\) and for \(d = 4\), the action (1.1) reduces to the Maxwell action as it should be.\(^{16}\)

In the backdrop of the scenarios described so far it is therefore worthwhile to study the slowly rotating black hole solutions in a spacetime with negative cosmological constant in presence of generalized electromagnetic field (1.1).

The rest of the paper is organized as follows. We give a brief review of the field equations of Einstein gravity in the presence of generalized electromagnetic field in §2. In §3, We present slowly rotating nonlinear charged black hole solutions in Einstein gravity and then, we obtain mass, charge, temperature, entropy, angular momentum, and gyromagnetic ratio of the \((n+1)\)-dimensional black hole solutions. Then, in §4 we discuss the special case of nonlinear electromagnetic field, so-called conformally invariant Maxwell field and investigate its properties. We finish our paper with some concluding remarks.

\section*{§2. Field equations and solutions}

We consider the \((n+1)\)-dimensional \((n \geq 3)\) spacetime in which gravity is coupled to the nonlinear Maxwell field with an action

\[
S = -\frac{1}{16\pi} \int_\mathcal{M} d^{n+1}x \sqrt{-g} \left[R - 2\Lambda + (\alpha F)^s\right] - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-h} \Theta(h), \tag{2.1}
\]

where \(R\) is the Ricci scalar, \(\Lambda\) is the negative cosmological constant, \(F\) is the Maxwell invariant which is equal to \(F_{\mu\nu} F^{\mu\nu}\), \(\alpha\) is a constant which we should set it and \(s\) is a nonlinearity parameter. The last term in Eq. (2.1) is the Gibbons-Hawking surface term and is required for the variational principle to be well-defined. The factor \(\Theta\) represents the trace of the extrinsic curvature for the boundary \(\partial\mathcal{M}\) and \(h\) is the induced metric on the boundary. Varying the action (2.1) with respect to the
gravitational field $g_{\mu\nu}$ and the gauge field $A_{\mu}$, yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}, \tag{2.2}$$

$$\partial_{\mu} \left[ \sqrt{-g} (\alpha F)^{s-1} F^{\mu\nu} \right] = 0. \tag{2.3}$$

In the presence of nonlinear electrodynamics field, the energy-momentum tensor of Eq. (2.2) is

$$T_{\mu\nu} = -2 \left[ \alpha s F_{\mu\rho} F_\nu^{\rho} (\alpha F)^{s-1} - \frac{1}{4} g_{\mu\nu} (\alpha F)^s \right], \tag{2.4}$$

and for $s = 1$ and $\alpha = -1$, Eqs. (2.2)–(2.4) reduce to the standard Maxwell field coupled to Einstein gravity.

§3. The $(n + 1)$-dimensional slowly rotating nonlinear charged black holes

For small rotation, we can solve Eqs. (2.2)–(2.4) to first order in the angular momentum parameter $a$. Inspection of the $(n + 1)$-dimensional Kerr solutions shows that the only term in the metric that changes to the first order of the angular momentum parameter $a$ is $g_{t\phi}$. Therefore, for infinitesimal angular momentum we assume the metric being of the following form

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} - 2aF(r)\sin^2\theta dt d\phi + r^2(d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\Omega_{n-3}^2), \tag{3.1}$$

where $U(r)$ and $F(r)$ are functions of $r$, and $a$ is a parameter associated with its angular momentum and $d\Omega_{n-3}^2$ denotes the metric of an unit ($n-3$)-sphere. From static cases we can consider the $t$ component of the Maxwell equations in the form

$$F_{tr} = \xi h'(r), \tag{3.2}$$

where prime denotes first derivative with respect to $r$ and $\xi$ is a constant and may be fixed. In general, when we have rotational parameter there is also a vector potential in the form

$$A_\phi = ah(r) \sin^2\theta, \tag{3.3}$$

and one can show that for infinitesimal angular momentum, we have

$$F = (F_{\mu\nu} F^{\mu\nu}) = -2 \left( \frac{q}{(n-2)} h'(r) \right)^2,$$

and so the power Maxwell invariant, $(\alpha F)^s$, may be imaginary for positive $\alpha$, when $s$ is fractional. Therefore we set $\alpha = -1$, to have real solutions without loss of generality. By substituting Eq. (3.3), the Maxwell fields (3.2) and the metric (3.1)
into the field equations (2.2) and (2.3), we can obtain

\[
U(r) = 1 - \frac{2\Lambda r^2}{n(n-1)} - \frac{m}{r^{n-2}} + \Upsilon(r),
\]

\[
\Upsilon(r) = \begin{cases} 
0, & s = 0, 1/2 \\
\frac{2^{n/2} q^n \ln r}{(n-2)^{n} r^{n}}, & s = n/2 \\
\frac{r^2 (2s-1)^2}{(n-1)(2s-n)} \left( \frac{2(2s-n)^2 q^2}{r^2 (2s-1)^2(n-2)^2} \right)^s, & \text{otherwise}
\end{cases}
\]

\[
F(r) = \frac{n-2}{r^{n-2}} [m - \Gamma(r)],
\]

\[
\Gamma(r) = \begin{cases} 
\frac{2(n-1)(s-1)}{(2s-n)} r^{n-2}, & s = 0, 1/2 \\
\left[ \frac{n}{2} + (n - 2) \ln r \right] r^{n-2} + \frac{2^{n/2} q^n \ln r}{(n-2)^n}, & s = n/2 \\
2 \frac{(n-1)(s-1)}{(2s-n)} r^{n-2} + \frac{(2s-1)^2}{(n-1)(2s-n)} \left( \frac{2(2s-n)^2 q^2}{r^2 (2s-1)^2(n-2)^2} \right)^s r^{(2s-n)/(2s-1)}, & \text{otherwise}
\end{cases}
\]

\[
h(r) = \begin{cases} 
\text{constant}, & s = 0, 1/2 \\
q \ln r, & s = n/2 \\
q r^{(2s-n)/(2s-1)}, & \text{otherwise}
\end{cases}
\]

In the above expressions, \(q\) and \(m\) appear as integration constants and are related to the electrical charge and ADM (Arnowitt-Deser-Misner) mass of the black hole, respectively. In addition, these solutions reduce to slowly rotating 4-dimensional Kerr-Newman as \(s = 1, n = 3, \Lambda = 0\). Also, in the linear case \((s = 1)\) and also as \(\Lambda = 0\), the solutions reduce to the higher dimensional asymptotically flat slowly rotating charged black hole solutions (see e.g. 8)) and we should set \(\xi = -1/(n-2)\) for consistency. Likewise, the presented solutions reduce to the solutions of obtained in 12) and 13) for \(\Lambda = a = 0\) (and \(s = (n+1)/4\) for 12)). Also, this case \((s = (n+1)/4)\) corresponds with a class of \(F(R)\) gravity in the absence of any matter source.\(^{17}\) In four dimensions and in the absence of \(\Lambda\), the static solution \((a = 0)\) is the special case of spherically symmetric general solutions presented in 11).

In addition, for \(s = n/2\) the charge term in metric function is logarithmic, and the electromagnetic field is proportional to \(r^{-1}\), and in the other word this special solution is near to BTZ solution in higher dimensions\(^{18}\) and approximately, reduces to original static BTZ solution for 3-dimension \((n = 2)\).\(^{19}\)

Here, we want to investigate the effects of the nonlinearity on the asymptotic behavior of the solutions.

a) \(\Lambda < 0\):

It is worthwhile to mention that for \(0 < s < \frac{1}{2}\), the asymptotic dominant term of Eq. (3.4) is fourth term and the solutions of the slowly rotating nonlinear charged black hole are not asymptotically AdS for negative \(\Lambda\), but for the cases \(s < 0\) or \(s > \frac{1}{2}\) (include of \(s = \frac{5}{2}\)), the asymptotic behavior of solutions are the same as linear AdS case. In spite of the fact that the physical implications of this effect (different asymptotic behavior) are not exploited yet, but more investigations about it can, independently, consider for future works. Equations (3.2)–(3.6) show that the
electromagnetic field is zero for the cases $s = 0, \frac{1}{2}$, and the metric functions do not possess a charge term and they correspond to uncharged asymptotically AdS.

b) $\Lambda = 0$:

For vanishing cosmological constant, one can show that the presented solutions are asymptotically flat for all values of nonlinearity parameter unless $-\frac{1}{n-3} < s < \frac{1}{2}$.

In addition, we want to investigate the effects of nonlinearity on the electromagnetic field. Figure 1 shows that, when we decrease the nonlinearity parameter $s$, not only $F_{tr}$ diverges more rapidly near the origin but also for large values of $r$, it goes to zero more rapidly.

At this point, it is worthwhile to investigate the causal structure and physical properties of these solutions. One can show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverge at $r = 0$ and it is finite for $r \neq 0$ and goes to zero as $r \to \infty$. So we find that there is an essential singularity at $r = 0$. Now, we look for the existence of horizons. The horizons, if any exist, are given by the zeros of the function $U(r) = (g_{rr})^{-1}$. Let us denote the largest positive root of $U(r) = 0$ by $r_+$.

Moreover we can obtain some information about causal structure by considering the temperature of the black hole. By using the definition of hawking temperature on the outer horizon $r_+$ which may be obtained through the definition of surface gravity

$$T_+ = \frac{1}{2\pi} \sqrt{-\frac{1}{2}(\nabla_\mu \chi_\nu)(\nabla^\mu \chi^\nu)},$$

(3.7)
where $\chi$ is the Killing vector $\partial_t$, we can write

$$T_+ = \frac{U'(r)}{4\pi} = \frac{1}{4\pi} \left( \frac{(n-2)m}{r_{+}^{n-1}} - \frac{4Ar_+}{n(n-1)} + \Psi \right),$$  

(3.8)

$$\Psi = \begin{cases} 
0, & s = 0, 1/2 \\
\frac{2^{n/2}q^n}{(n-2)^{n}r_+} \left[ 1 - (n-2) \ln r_+ \right], & s = n/2 \\\rac{2(2s-1)(ns-3s+1)}{(n-1)(n-2s)} \left( \frac{2(n-2s)^2}{(n-2)(2s-1)^2} \right)^s r_+^{(4s-2ns-1)/(2s-1)}, & \text{otherwise}
\end{cases}.$$  

(3.9)

Using the fact that the temperature of the extreme black hole is zero, it is easy to show that the condition for having an extreme black hole is that the mass parameter is equal to $m_{\text{ext}}$, which is given as

$$m_{\text{ext}} = \frac{4Ar_+^n}{n(n-1)(n-2)} - \begin{cases} 
0, & s = 0, 1/2 \\
\frac{2^{n/2}q^n}{(n-2)^{n}r_+} \left[ 1 - (n-2) \ln r_+ \right], & s = n/2 \\\rac{2(2s-1)(ns-3s+1)}{(n-1)(n-2s)} \left( \frac{2(n-2s)^2}{(n-2)(2s-1)^2} \right)^s r_+^{(2s-n)/(2s-1)}, & \text{otherwise}
\end{cases}.$$  

(3.10)

One can show that the metric of Eqs. (3.1)–(3.6) presents a slowly rotating black hole solution with two inner and outer horizons provided that the mass parameter $m$ is greater than $m_{\text{ext}}$, an extreme black hole for $m = m_{\text{ext}}$, and a naked singularity otherwise.

In what follows we investigate the other conserved and thermodynamics quantities. The entropy of the black hole typically satisfies the so-called area law which states that the entropy of the black hole is a quarter of the event horizon area.\(^{20}\) This near universal law applies to almost all kinds of black holes in Einstein gravity.\(^{21}\) Since the area of the event horizon does not change up to the linear order of the rotating parameter $a$, we can easily show that the entropy of black hole on the outer event horizon $r_+$ can be written as

$$S = \frac{V_{n-1}^{n-1}}{4},$$  

(3.10)

where $V_{n-1}$ represents the volume of constant curvature hypersurface of an unit $(n-1)$-sphere, described by $d\Omega_{n-1}^2 = d\theta_1^2 + \sum_{i=2}^{n-1} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2$.

Next, we calculate the mass, angular momentum, electrical charge and the gyromagnetic ratio of these rotating nonlinear charged black holes which appear in the limit of slow rotation parameter. The mass and angular momentum of the black hole can be calculated through the use of the quasi-local formalism of the Brown and York.\(^{22}\) According to the quasilocal formalism, the quantities can be constructed from the information that exists on the boundary of a gravitating system alone. Such quasilocal quantities will represent information about the spacetime contained
within the system boundary, just like Gauss’s law. In our case the stress-energy tensor can be written as

\[ T^{ab} = \frac{1}{8\pi} \left[ \Theta^{ab} - \Theta \gamma^{ab} \right], \quad (3.11) \]

which is obtained by variation of the action (2.1) with respect to \( \gamma_{ab} \). To compute the angular momentum of the spacetime, one should choose a spacelike surface \( \mathcal{B} \) in \( \partial M \) with metric \( \sigma_{ij} \), and write the boundary metric in ADM form

\[ \gamma_{ab} dx^a dx^b = -N^2 dt^2 + \sigma_{ij} \left( d\varphi^i + V^i dt \right) \left( d\varphi^j + V^j dt \right), \]

where the coordinates \( \varphi^i \) are the angular variables parameterizing the hypersurface of constant \( r \) around the origin, and \( N \) and \( V^i \) are the lapse and shift functions, respectively. When there is a Killing vector field \( \xi \) on the boundary, then the quasilocal conserved quantities associated with the stress-energy tensors of Eq. (3.11) can be written as

\[ Q(\xi) = \int_{\mathcal{B}} d^{n-1}x \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (3.12) \]

where \( \sigma \) is the determinant of the metric \( \sigma_{ij} \), \( \xi \) and \( n^a \) are the Killing vector field and the unit normal vector on the boundary \( \mathcal{B} \), respectively. For boundaries with timelike (\( \xi = \partial/\partial t \)) and rotational (\( \xi = \partial/\partial \varphi \)) Killing vector fields, one obtains the quasilocal mass and angular momentum

\[ M = \int_{\mathcal{B}} d^{n-1}x \sqrt{\sigma} T_{ab} n^a \xi^b = \frac{V_{n-1}(n-1)m}{16\pi}, \quad (3.13) \]

\[ J = \int_{\mathcal{B}} d^{n-1}x \sqrt{\sigma} T_{ab} n^a \varsigma^b = \frac{V_{n-1}ma}{8\pi}, \quad (3.14) \]

provided the surface \( \mathcal{B} \) contains the orbits of \( \varsigma \). For \( a = 0 \), the angular momentum vanishes, and therefore \( a \) is the rotational parameter of the nonlinear charged black hole. The quasilocal mass that presented here, is the same as ADM mass of calculated by Abbott and Deser.\(^{23}\) Combining Eq. (3.13) with Eq. (3.14), we get

\[ J = \frac{2Ma}{(n-1)}. \quad (3.15) \]

Here, we can compute the electrical charge of the solutions. To determine the electric field we should consider the projections of the electromagnetic field tensor on special hypersurfaces. The normal to such hypersurfaces for spacetimes with longitudinal magnetic field is

\[ u^0 = \frac{1}{N}, \quad u^r = 0, \quad u^i = -\frac{N^i}{N}, \quad (3.16) \]

and the electric field is \( E^\mu = g^{\mu\rho} F_{\rho\nu} u^\nu \). The electric charge, \( Q \) can be found by calculating the flux of the electric field at infinity, yielding

\[ Q = \frac{V_{n-1}}{4\pi} \times \begin{cases} 0, & s = 0, \frac{1}{2} \\ \frac{2(2s-n)q}{(2s-1)(n-2)} \frac{q^{n-1}}{(n-2)^{n-1}}, & s = \frac{n}{2} \\ \frac{2s}{2\pi} \left[ \frac{2(2s-n)q}{(2s-1)(n-2)} \right]^{2s-1}, & \text{otherwise} \end{cases}, \quad (3.17) \]
At last, we calculate the gyromagnetic ratio of this rotating nonlinear charged black holes. One of the important subjects about the 4-dimensional charged black hole in the Einstein gravity is that it can be assigned a gyromagnetic ratio $g = 2$ just like the electron in Dirac theory. Here we want to know how the value of the gyromagnetic ratio changes for slowly rotating nonlinear charged black holes in higher dimensions. The magnetic dipole moment for this slowly rotating black hole is

$$\mu = Qa.$$  

Therefore, the gyromagnetic ratio is given by

$$g = \frac{2QM}{\mu J} = n - 1,$$  \hspace{1cm} (3.18)

which is the gyromagnetic ratio of the $(n + 1)$-dimensional Kerr-Newman black holes. Since both of angular momenta and the magnetic dipole momenta of these black holes first appear at the linear order in rotation parameter $a$, we have led to the conclusion that the value of the gyromagnetic ratio remains $g = n - 1$. Also, we find that, the nonlinearity of electromagnetic field does not change the gyromagnetic ratio of the rotating black hole.

\section*{§4. Conformally invariant electromagnetic field}

As one can find, the clue of the conformal invariance of Maxwell source lies in the fact that we have considered power of the Maxwell invariant, $F = F_{\mu\nu}F^{\mu\nu}$. Here we want to justify the nonlinearity parameter $s$, such that the electromagnetic field equation be invariant under conformal transformation ($g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$ and $A_\mu \rightarrow A_\mu$). The idea is to take advantage of the conformal symmetry to construct the analogues of the four dimensional Reissner-Nordström solutions in higher dimensions. It is easy to show that for Lagrangian in the form $L(F)$ in $(n + 1)$-dimensions, $T^{\mu}_\mu \propto \left[ F^{\frac{dL}{dF}} - \frac{n+1}{4} L \right]$; so $T^{\mu}_\mu = 0$ implies $L(F) = \text{Constant} \times F^{(n+1)/4}$. For our case $L(F) \propto F^s$, and subsequently, $s = (n+1)/4$. It is worthwhile to mention that since $n \geq 3$ and therefore $s \geq 1$, one can show that the slowly rotating black holes with conformally invariant Maxwell source are asymptotically AdS in arbitrary dimensions. In this case the functions $U(r)$, $F(r)$ and $h(r)$ reduce to

$$U(r) = 1 - \frac{2Ar^2}{n(n-1)} - \frac{m}{r^{n-2}} - \frac{2(n-3)/4}{r^{n-1}} \left( \frac{q}{n-2} \right)^{(n+1)/2}, \hspace{1cm} (4.1)$$

$$F(r) = \frac{(n-2)}{r^{n-2}} \left[ m + (n-3)r^{n-2} + \frac{2(n-3)/4}{r} \left( \frac{q}{n-2} \right)^{(n+1)/2} \right], \hspace{1cm} (4.2)$$

$$h(r) = \frac{1}{r}, \hspace{1cm} (4.3)$$

and therefore $F_{tr} \propto r^{-2}$ in arbitrary dimensions.
§5. Summary and conclusion

In this paper, we presented higher dimensional slowly rotating nonlinear charged black hole solutions in Einstein gravity in the presence of negative cosmological constant. We discarded any terms involving $a^2$ or higher power in $a$. The nonlinearity did not change to $O(a)$ and $A_\phi$ is the only component of the vector potential that change to $O(a)$. These solutions may be interpreted as black hole solutions with inner (Cauchy) and outer (event) horizons provided that the mass parameter $m$ is greater than an extremal value given by Eq. (3.9), an extreme black hole if $m = m_{\text{ext}}$ and a naked singularity otherwise. We showed that the vector potential and the metric functions are logarithmic form for special case, $s = n/2$. This case is analogous with the BTZ solutions. Also, we presented the effects of nonlinearity on the solutions and discussed the asymptotic behavior of them. For the special choices of the nonlinearity parameter $s$, these solutions do not have asymptotic AdS behavior. Calculations of the electromagnetic field, electrical charge and temperature showed that they are sensitive to the nonlinearity of the electromagnetic field. The expressions of the mass, temperature, and entropy of the black hole solution show that they do not change up to the linear order of the angular momentum parameter $a$. One may think there is not any rotational effect at infinity for the linear order in rotation parameter $a$.

Then, we obtained the ADM mass $M$, the angular momentum $J$ and the gyromagnetic ratio $g$ of the black hole and found that they do not depend on the nonlinearity parameter $s$. Finally, we adjusted the nonlinearity parameter, $s$, such that, in higher dimensions, the electromagnetic field equations be invariant under conformal transformations and then $F_{tr} \propto r^{-2}$.

Acknowledgements

This work has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

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