Discussion

Eli Reshotko.[9] The paper is one of the first to consider compressible turbulent boundary-layer development with pressure gradient and heat transfer. As such it is an extremely useful contribution. Although the calculation procedure is specifically developed for flow in an axially symmetric nozzle, the same concepts could be employed in handling more general problems of turbulent boundary-layer development in favorable pressure gradient.

The energy equation is solved by the author in order to determine the variation of thermal boundary-layer thickness for use in obtaining the heat-transfer coefficient through the Reynolds analogy relation, Equation [26]. Because the ratio of thermal to dynamic thicknesses appears as $(\Delta/\delta)^{1/7}$ in Equation [31], the heat-transfer coefficient h_g is not particularly sensitive to the variation of (Δ/δ). Thus the prescribed calculation of h_g can be approximated closely, without solving the energy equation, through the assumption of a single reasonable value of $(\Delta/\delta)^{1/7}$.

Although not presented, it is interesting to note that the displacement thickness obtained for a portion of the nozzle including the throat would be negative. This result has also been obtained assuming laminar flow in a similar nozzle.[10] The negative displacement thickness is associated with the high density (relative to the free stream) in the cooled boundary layer.

Although the local heat-transfer rates in the divergent portion of the calculated nozzle ($\alpha = 15$ deg) compare favorably with the experimental determination of Saunders and Calder ($\alpha = 0.6$ deg), it is not reasonable to expect the results to fully indicate the effects of pressure gradient, especially in the convergent portion of the nozzle and in the vicinity of the throat because of the use of flat-plate relationsips for skin friction and Reynolds analogy.

The proposed method of extending the results of the boundary-layer calculation for a single nozzle to nozzles of similar or related geometric shapes is potentially very useful. It should reduce the work required for calculating large numbers of problems and provide a means for quickly estimating the effects of simple geometric changes on the nozzle boundary-layer development and heat transfer.

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[10] "The Compressible Laminar Boundary Layer with Heat Transfer and Arbitrary Pressure Gradient," by C. B. Cohen and Eli Reshotko, NACA TN 3326, 1955.

Author's Closure

The author wishes to thank Mr. Reshotko for his interesting comments. As was pointed out by the discusser, the value of h_g depends on the solution of the energy equation only through the factor $(\Delta/\delta)^{1/7}$. Specifically, the maximum values of $(\Delta/\delta)^{1/7}$ resulting from the solution of the energy equation for Cases 1, 2(a), and 2(b) of the paper (cf. Fig. 3) were 1.19, 1.27, and 1.17, respectively, near the throat. If, instead of solving the energy equation, a constant value of $(\Delta/\delta)^{1/7}$ of one was assumed for each case, the computed values of h_g would be from 17 to 27 per cent too high near the throat. It should be noted that in Equation [32] h_g is actually more sensitive to the development of the temperature boundary layer than to the development of the velocity boundary layer since the exponent of Δ is 1/7, whereas that of δ is only 3/28. Solution of the energy equation may become particularly important for cases with certain wall-temperature distributions which could exert a strong effect on the development of the temperature boundary layer.

The discusser is quite correct about negative displacement thicknesses resulting over part of the nozzle. Using the values of Δ/δ and T_w/T_0 resulting from the first approximation results to compute second approximation values of δ^*/θ from Equation [51], negative values were calculated over the range of x/x_n from 0.24 to 1 for Case 1, from 0.12 to 1 for Case 2(a), and from 0.54 to 1 for Case 2(b). As a result of the calculated boundary-layer development for the two cases in which δ^*/θ was negative at the throat, the effective throat areas were found to be 0.0027 per cent and 0.055 per cent greater than the geometrical areas, and for the other case 0.0094 per cent less than the geometrical area. Negative-displacement thicknesses are to be expected where the temperature boundary layer is much thicker than the velocity boundary layer because much higher densities in the boundary layer relative to those in the free stream result in higher boundary-layer values of ρu. The fact that δ^*/θ does go negative does not particularly retard the convergence of the momentum-equation solutions since δ^*/θ enters into the momentum equation only through the variable coefficient $\left[\frac{M^2 - 2(\delta^*/\theta) - 3}{1 - M^2}\right]$. In making a second approximation calculation, revised values of

$$\theta/\delta,\ \lambda,\ \sigma,\ \text{and}\ \frac{d}{d_x}\ln\,(T_0 - T_w)$$

must also be used.