Empirical analysis of digital seismograms from a cluster of explosions at regional distances for characterizing multiple explosions

So Gu Kim¹ and Zhongliang Wu²

¹ The Seismological Institute, Hanyang University, Ahnsan, 425-791, Korea
² Institute of Geophysics, State Seismological Bureau, Beijing, 100081, China

Accepted 1995 December 18. Received 1995 December 18; in original form 1995 August 24

SUMMARY
Digital seismograms from a cluster of explosions at regional distances are analysed to investigate the similarity of seismograms with the aim of identifying possible multiple explosions. It is observed that the P wave trains from nearby explosions have considerable similarities, and the digital record of single explosions can thus be used in an empirical sense to characterize multiple explosions. By inversion and forward modelling of waveforms using the seismograms of nearby ‘single’ explosions as the pseudo-empirical Green's function, multiple explosions where the subevents occurred at nearly the same time can be clearly recognized. The resultant ‘event time functions’ correspond to a low-resolution picture of the multiplicity, which relies on both the true ground information about the nature of the pseudo-empirical Green's function and the resolution provided by the digital seismograms.

Key words: explosion seismology, Green's functions.

INTRODUCTION
It is well known that the characterization of multiple explosions plays an important role in the seismic monitoring of a comprehensive test-ban treaty (e.g. Flinn, Cohen & McCowan 1973; Husebye & Mykkeltveit 1981; Clark & Pearce 1988). In the study of this problem a practical approach is to investigate quarry and/or mining blasts, which often have considerable multiplicities due to the need to increase fracturing efficiency and reduce ground shaking. At regional distances (Δ ≤ 3000 km), such a problem is quite complex because the propagation of seismic waves is quite complex and has a strong dependence on the heterogeneities within the crust between the seismic source and the recording station. To overcome this difficulty, several approaches have been proposed and undertaken (e.g. Baumgardt & Ziegler 1988; Smith 1989; Harris 1991). Amongst these one of the most effective methods is the empirical analysis of seismograms. In the case where the crustal structure of the region under consideration is not fully understood, the empirical approach seems to be the only way by which to analyse seismograms. In this approach, empirical analysis is performed on digital seismograms from a cluster of explosions at regional distances. With a few working assumptions, the method proposed in this approach seems effective in characterizing the multiplicity of blasts, the methodology of which might be further applied to the study of multiple underground explosions.

EMPIRICAL ANALYSIS OF DIGITAL SEISMOGRAMS FROM EXPLOSIONS AT REGIONAL DISTANCES

The seismogram at regional distances may be represented by

\[ u_i(t) = M_\delta \cdot G_{ij,k}(r_0, r, t) \cdot S(t) \cdot A(t) \cdot I(t), \]

in which \(G_{ij,k}(r_0, r, t)\) is the Green's function, where \(r_0\) and \(r\) are the position vectors of the source and the receiver, respectively, \(M_\delta\) is the seismic moment tensor, \(S(t)\) is the source time function, \(A(t)\) is the attenuation operator, \(I(t)\) is the impulse response of the recording instrument, and \(*\) denotes convolution. For explosion sources, as the first-order approximation, the moment tensor may be taken as \(M_\delta \delta\). For the source time function, consider a simple case when the pressure on the elastic radius can be represented by (Mueller & Murphy 1971; Murphy & Mueller 1971):

\[ P(t) = [P_0 \exp(-\gamma t) + P_z]H(t), \]
in which

\[ \gamma = \frac{R_0}{R}, \]

\[ P_2 = P_{20} \left( \frac{h_0}{h} \right)^{1/3} \left( \frac{R_0}{R} \right)^{0.87}, \]

\[ P_0 = P_{10} \frac{h}{h_0} - P_2, \]

where \( R, h \) and \( W \) are the elastic radius, source depth and yield, respectively. In the above equations, \( y_0, P_{10}, P_{20}, h_0, R_0, \) and \( n \) in the following context, are empirical constants, and

\[ H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \]

is the unit step function. The depth and elastic radius depend empirically on the yield such that

\[ h = h_0 W^{1/3}, \]

\[ R = R_0 \left( \frac{h_0}{h} \right)^{1/n} W^{1/3}. \]

In this case, the spectra of the source time function may be represented by:

\[ s(\omega) = \pi p a^2 \left( \frac{P_0}{\gamma + i\omega} + \frac{P_2}{i\omega} \right) \frac{\exp(i\omega R/\nu)}{\omega_0 + i\omega - \left[(\lambda + 2\rho)/4\mu\right]\omega^2}, \]

(3)

where \( \rho \) and \( \alpha \) are the density and P-wave velocity of the source region, respectively, and

\[ \omega_0 = \frac{\alpha}{R}. \]

It can be seen that the empirical constants in (2) and (3) depend mainly on the yield of the explosion. Accordingly, for explosions with yields of approximately the same order, the source time functions \( S(t) \) may be taken as approximately the same. For more general cases, the forms of eqs (2) and (3) may vary (e.g. Denny & Johnson 1991), but the qualitative conclusion that the source time function depends mainly on the yield is still valid and has been used in many studies (e.g. Baumgardt & Ziegler 1988; Smith 1989).

Several studies have concluded that seismograms can be correlated at regional distances and that this correlation can be applied in the detection of earthquakes and explosions (e.g. Israelsson 1990; Der, Hirano & Shumway 1990). Such a correlation comes from similarities in the source time functions and in the Green’s functions. For a pair of events that are

---

Figure 1. Locations of explosions (stars) and the recording station (triangle), together with the frequency response of the instrument. Two earthquakes (circles) are also plotted in the figure.
located near to one another, their Green's functions may be taken as approximately the same, i.e.
\[ G_{ij,k}(r_{0,1}, r, t) \approx G_{ij,k}(r_{0,2}, r, t) \]
where \( r_{0,1} \approx r_{0,2} \).

Therefore, the seismogram of a multiple explosion consisting of a number of single explosions, referred to as 'subevents' in this paper, can be represented as
\[ u^n(t) \approx \sum B_n u^n(t - t_n), \]
where \( u^n(t) \) is the seismogram of the multiple explosion, \( u^j(t) \) is the seismogram of the nearby single explosion recorded by the same instrument at the same station, and \( B_n \) and \( t_n \) are, respectively, the amplitude and origin time of the \( n \)th subevent relative to the first one.

Borrowed from the concept of empirical Green's functions (e.g. Harzell 1978; Mueller 1985), the term \( u^j(t) \) in eq. (4) may be referred to as a pseudo-empirical Green's function. The difference between the pseudo-empirical Green's function and the empirical Green's function is that, because the near-field source time function of the explosion cannot 'degenerate' into a step function as the event becomes smaller and smaller, \( u^n(t) \) cannot 'degenerate' into the Green's function, even for very small explosions. Another difference is that, in the analysis of earthquakes, the empirical Green's function is usually the seismogram of a smaller earthquake \( (m \leq 3.0) \), while in the analysis of explosions, the size of the single explosion and that of the multiple explosion should be of approximately the same order, so that the source time function of the single explosion and those of the subevents in the multiple explosion can be regarded as approximately the same.

More generally, eq. (4) can be written as
\[ u^n(t) \approx B(t) * u^n(t), \]
in which \( B(t) \), not necessarily a continuous function, describes the temporal distribution of the subevents in the multiple explosion, and in this paper is referred to as the 'event time function'. In principle, the inversion of \( B(t) \) using the deconvolution of \( u^n(t) \) by \( u^n(t) \) may be exactly the same as the inversions that have been carried out in the empirical Green's function approaches. In fact, neither the concept of empirical Green's functions (e.g. Harzell 1978; Mueller 1985) nor the assumption that explosions of similar yields have similar source

\[ \text{Figure 2. Comparison of seismograms. Numbers denote the different explosions listed in Table 1. Each set of three-component seismograms of a certain event is scaled according to its own amplitude. (a) Comparison of } P \text{ wave trains from nearby explosions: Event 1 and Event 3. (b) Comparison of } P \text{ wave trains from nearby explosions: Event 2, Event 6 and Event 5. (c) Seismograms of explosions referred to in Figs 2(a) and (b). Seismograms of Event 4 are shown separately in Fig. 3(c).} \]
Figure 2. (Continued.)

time functions (e.g. Baumgardt & Ziegler 1988; Smith 1989) is new to seismology. The aim of our discussion in this section is to state the relation between the empirical approaches used in earthquake seismology and the multiplicity analysis used in the study of explosions. Also in this paper the hypothesis is tested from the perspective of the similarities of seismograms, and is then applied in the characterization of a possible multiple explosion.

SIMILARITIES OF SEISMOGRAMS FROM NEARBY EXPLOSIONS AT REGIONAL DISTANCES

From observations, the working assumptions described in the above discussion are strongly supported by the similarity of $P$ wave trains from a cluster of explosions. Fig. 1 shows the cluster of explosions and the recording station used in this study (Kim et al. 1995). The parameters of the explosions are listed in Table 1. The recording station Wonju is located at 37.48°N, 127.90°E and uses a KSRS digital seismometer, recording the velocity of ground motion with a sampling rate 20 SPS. The frequency response of the instrument is also shown in Fig. 1.

Figs 2(a) and (b) show the three-component $P$ wave trains from two groups of explosions that are near to each other. As a reference, Fig. 2(c) shows the entire seismogram of these events. The seismogram of Event 5 is shown in Fig. 3(b). In

![Table 1. Explosions in this study.](image)

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Date</th>
<th>O. Time</th>
<th>Latitude</th>
<th>Longitude</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88–01–17</td>
<td>12:58:59.9</td>
<td>38.3</td>
<td>126.4</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>88–01–19</td>
<td>16:13:28.0</td>
<td>38.4</td>
<td>126.2</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>88–01–24</td>
<td>13:14:28.8</td>
<td>38.3</td>
<td>126.5</td>
<td>2.7</td>
</tr>
<tr>
<td>4</td>
<td>88–01–25</td>
<td>12:13:07.6</td>
<td>38.7</td>
<td>125.3</td>
<td>3.0</td>
</tr>
<tr>
<td>5</td>
<td>88–01–28</td>
<td>16:54:31.7</td>
<td>38.5</td>
<td>126.2</td>
<td>2.7</td>
</tr>
<tr>
<td>6</td>
<td>88–01–29</td>
<td>17:00:35.3</td>
<td>38.5</td>
<td>125.9</td>
<td>2.6</td>
</tr>
<tr>
<td>7</td>
<td>88–01–30</td>
<td>11:58:07.7</td>
<td>38.3</td>
<td>126.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

\[\text{Table 1. Explosions in this study.}\]

![Figure 3. Comparison of waveforms.](image)

Figure 3. Comparison of waveforms. (a) Seismogram of Event 7. Wave trains A and B are picked out and compared with each other. (b) Seismogram of Event 5. Two wave trains are picked out and compared with each other. Note that one of the wave trains has been cut off because of problems with the instruments. (c) Seismogram of Event 4. Wave trains A and C are picked out and compared with each other.
Empirical analysis of digital seismograms

The figures an impressive similarity of P wave trains from explosions that are near to each other can be observed. Fig. 3 compares the P wave trains (A, B and C) from different groups of subevents of multiple explosions. A clear similarity can be seen between the P wave trains of different groups of subevents. The comparison shows that because the explosions that are near to each other have similar source time functions and similar Green’s functions, their seismograms are similar to each other. Accordingly, the seismogram of a single explosion may be used, in an empirical sense, to characterize multiple explosions. In contrast, for S wave trains and Lg wave trains, such similarities are not observed. The reason for such a difference needs to be studied further.

A hypothesis which may be plausible is that the P wave trains from the explosions can be described in a deterministic way by eq. (1), while the S wave trains from explosions, mainly resulting from scattering and other secondary effects, and the Lg wave train, which comes from the interaction of P and S waves at the ground surface, cannot be described by eq. (1) in a deterministic way.

CHARACTERIZING A POSSIBLE MULTIPLE EXPLOSION USING EMPIRICAL SYNTHETIC SEISMOGRAMS

From the above discussions it seems that the seismograms of a single explosion may be used as the pseudo-empirical Green’s function to study the properties of multiple explosions that are near to each other. When dealing with a limited frequency band, such as the short-period records in this approach, time-domain inversion and forward modelling have the advantages of avoiding the instabilities caused by deconvolution in the frequency domain.

In this approach we use an inversion technique similar to that proposed by Kikuchi & Kanamori (1982). Starting from

\[ u^n_s(t) \approx \sum \alpha B_s u^n(t - \tau_s), \]

we consider \( B_s \) at every time step \( \tau_s = n\Delta t \), with \( \Delta t \) being the time step, and \( B_s = 0 \) indicating no subevent at that moment.
For instance, a multiple explosion series,
\[ B_1 = 1, t_1 = \Delta t; \quad B_2 = 2, t_2 = 4\Delta t; \quad B_3 = 1, t_3 = 6\Delta t, \]
may be represented by
\[ \{B_n\} = \{1, 0, 0, 2, 0, 1\}. \]

In the inversion, first we take a subevent with amplitude \( B_1 \) at time \( \Delta t \) by minimizing the error defined by

\[ \Delta_1 = \sum_{i=1}^{3} \left\{ \sum_n \left[ u_i^n(n\Delta t) - B_1 u_i^n(n\Delta t - \Delta t) \right]^2 \right\}. \]

Next we apply the same procedure to the residual waveform,
\[ R^1[u_i^n(n\Delta t)] = u_i^n(n\Delta t) - B_1 u_i^n(n\Delta t - \Delta t), \]
to determine \( B_2 \) for the moment \( 2\Delta t \). In general, we have to minimize

\[ \Delta_n = \sum_{i=1}^{3} \left\{ \sum_n \left[ u_i^n(n\Delta t) - \sum_{k=1}^{N} B_k u_i^n(n\Delta t - k\Delta t) \right]^2 \right\} \quad (6) \]
to determine \( B_n \) at \( n\Delta t \). This idea is almost identical to that proposed by Kikuchi & Kanamori (1982). A difference is that in their approach both the time and the amplitude of the subevent are to be determined, leading to a linear inversion, while in our approach we invert the amplitude for each time step, assuming that \( B_n = 0 \) corresponds to no subevent appearing at that time. Other differences are that in our approach all three channels have been considered, and an interactive procedure is used to put constraints (such as \( B_n \geq 0 \)) on the inversion.

Fig. 4 shows a numerical experiment to demonstrate the resolution of the technique. In the experiment a wave train is used as the pseudo-empirical Green’s function, and convolved with a given event time function. The resultant seismograms are purposely contaminated by different levels of pseudo-random noise to test the robustness of the inversion technique. Fig. 4(a) shows the seismograms produced for the experiment. In the example we take an extreme case where the two events are spaced with a delay of 0.1 s, which almost reaches the limit of the resolution provided by the seismograms with a sampling rate of 20 SPS (its time step \( \Delta t = 0.05 \) s). Fig. 4(b) shows the result of the unconstrained inversion. Fig. 4(c) shows the result of the constrained inversion with the constraint \( B_n \geq 0 \). It can be seen that the algorithm has successfully reconstructed the input event time function, while the noise contamination has caused the distortions of the resultant event time function and a decrease in the level of fitting between the synthetic seismograms and the input seismograms. Another important point is

![Figure 4](https://example.com/fig4.png)

**Figure 4.** Test of the inversion algorithm. (a) Seismograms for the experiment. (b) Results of unconstrained inversion. (c) Results of constrained inversion. (d) Seismograms of two earthquakes. Each set of three-component seismograms of a certain event is scaled according to its own amplitude. (e) Inversion results using wave train A (P wave train). (f) Inversion results using wave train B (Lg wave train).
Figure 4. (Continued.)
Figure 4. (Continued.)
that because of the disturbances in the data, as well as the errors associated with the digital waveform inversion procedure, some false signals have been introduced into the resultant event time function. As a result, the signals in the inverted event time function with an amplitude less than 30 per cent of the maximum amplitude cannot be regarded as reliable.

As a further test of the algorithm, Fig. 4(d) shows two earthquakes recorded at the Wonju station. The parameters of the two earthquakes have been determined by the Korean Meteorological Administration (KMA) National Seismograph Network as:

Earthquake 1: 08/12/1995 12:17:49.0 (Local Time) 38.0°N, 124.6°E, M ≈ 3.6;

Earthquake 2: 08/13/1995 05:25:37.7 (Local Time) 37.9°N, 124.8°E.

as shown in Fig. 1. The magnitude of the second earthquake has not been determined because of its weak signal. From the comparison of the seismograms, it seems likely that these two earthquakes occurred near to each other, and that their focal mechanisms are similar to each other. What is of special interest is that these two earthquakes are located near to the explosion sites under discussion. As a test of our algorithm, we use the seismogram of the smaller earthquake as the empirical Green's function to invert the source time function of the larger earthquake. Figs 4(e) and (f) shows the inversion results. The results obtained from P wave trains and Lg wave trains, which are indicated in Fig. 4(d) as wave train A and wave train B, respectively, show two inverted source time functions, which are consistent with each other and agree with previous results obtained for small earthquakes (e.g. Chen, Zhou & Ni 1991), implying that the algorithm works well in retrieving the source time function of earthquakes, and may
Figure 5. Results using wave train A of Event 4 as the pseudo-empirical Green's function to model wave train B. The positions of wave train A and wave train B are tagged in Fig. 3(a). See text for details. (a) Inversion results. (b) Forward modelling results. Explanation of the resultant event time function. See text for details.
be used to determine the event time function of multiple explosions. In conclusion, the proposed technique can resolve the time delays of two subevents down to 0.1 s, at least for the situation under discussion. On the other hand, the resultant signals with amplitudes of less than 30 per cent of the maximum cannot be regarded as reliable.

As an application of the algorithm, in Fig. 5(a) we consider two wave trains, wave train A and wave train B, of event 4. The positions of the wave trains are tagged in Fig. 3(c). From the seismograms it can be seen that wave train A and wave train B may come from nearby events. It may be further assumed that they come from the same place of blasting. For all cases, from the discussions above, the seismogram of wave train A may be used as the pseudo-empirical Green’s function to model wave train B. A possible problem is that wave train A might also be a multiple event, which will be discussed in detail in the following paragraphs.

In Fig. 5(a) unconstrained and constrained inversions have been performed. The resultant event time functions reveal four subevents which can be regarded as reliable. In Fig. 5(b), according to the inversion results, two event time functions, a low-resolution one and a high-resolution one, are used to do the forward modelling. The forward modelling is conducted by interactively changing the values of the amplitudes and the delay times of the given subevents, based on the inversion results, to get the best least-squares fit between the observed seismograms and the synthetic ones. The fitting of some important phases is also accounted for by visual inspection.
This means that solutions for the second-best least-squares fitting, rather than for the best one, may be chosen because of the better fit for certain parts of seismograms, in order to make the seismograms resemble each other more. The reason for doing this is that the least-squares fitting only considers the overall matching of the seismograms, while the matching of some particular parts of the seismograms is also important; this corresponds to a situation in which different weights are assigned to different parts of the seismogram.

From Fig. 5(b), it can be seen that the resultant event time function can be regarded as reliable. Because of the limitations of the resolution provided by the seismic records, the explanation of the event time function leads to four different situations, as shown in Fig. 5(c), which depend on the nature of wave train A, which has been chosen in the analysis as the pseudo-empirical Green's function. The assumption behind this is that wave train A represents a single explosion. As it is difficult at present to obtain the true ground information about the nature of wave train A, four situations seem possible. The simplest one is where wave train A is a single explosion, and wave train B consists of four subevents. However, because of the limitation of the resolution provided by the seismogram, it is also possible that wave train A, as well as the subevents in wave train B, are all multiple explosions; in this case the event time function reflects a 'low-resolution' picture of the multiplicity. We believe that this situation is more probable, and the resultant event time function reflects an integrated effect of the multiplicity. Another possibility is that wave train A and the subevents in wave train B are only the 'main' subevents, and there are also some smaller events, which have merged into the uncertainties in the inversion. From an engineering perspective, however, this is not as likely as the preceding suggestions.

In the above discussion the multiplicity of wave train A and that of the subevents of wave train B are mainly confined within the resolution limits provided by the seismograms, namely within 0.1 s for a seismic record with a sampling rate of 20 SPS ($\Delta t = 0.05$ s). There is another possibility, namely that wave train A has a more complex multiplicity and wave train B is a convolution of this complex multiple event time function with its own event time function. On the bottom of Fig. 5(c) a simple example of this situation is shown. Such a situation seems unlikely for the example under discussion because the unconstrained inversion result, which does not include many negative values, indicates that either the multiplicity of wave train A can be confined within 0.1 s, or the event time functions of wave train A and wave train B have common multiples, with the probability of the latter possibility being quite small. In conclusion, without reliable observational data about the multiplicity of wave train A, the event time function obtained by the inversion and forward modelling may accommodate three possibilities, as shown in the middle part of Fig. 5(c). The one that seems the most probable is that wave train A and the subevents in wave train B are all multiple explosions, and the resultant event time function reflects an integrated picture of the multiplicity. In spite of the uncertainties in the explanation of the results, such a result can provide a 'low-resolution' picture of the multiplicities of the wave trains under consideration, which almost reaches the limit of the resolution provided by the digital seismograms.

In seismic analysis, as the origin times of the subevents are close to each other, within about 0.5 s, it is not easy to recognize them by an ordinary analysis of seismograms. On the other hand, as the structure between the source and the station is not clear, it is also difficult to make the characterization by using theoretical seismograms. Limitations of the frequency band cause difficulties in using frequency-domain approaches. In the above case the empirical analysis in the time domain has the advantage of recognizing multiple events, even when the origin times of the subevents are close to each other, as long as the working assumptions hold true.

## Conclusion and Discussion

In this approach, digital seismograms from a cluster of explosions at regional distances have been analysed in order to study the similarity of seismograms with the aim of characterizing multiple explosions. It is observed that the $P$ wave trains from explosions that are near to each other have considerable similarities, and the digital record of 'single' explosions can thus be used in an empirical sense to characterize multiple explosions. By inversion and forward modelling of waveforms using the seismograms of 'single' explosions that are near to each other as the pseudo-empirical Green's functions, multiple explosions for which the origin times of different subevents are near to each other can be clearly recognized. Usually the test sites or quarry worksites are not large compared to the distance between the source and the station. From this point of view, many explosions may be grouped into clusters, and the approach proposed in this paper can be used.

At regional distances, the characterization of multiple explosions is a complex problem because the propagation of seismic waves is quite complex and has a strong dependence on the crustal structure between the source and the receiver. In the case where the crustal structure of the region under consideration is not well understood, the empirical approach seems to be the only way in which to perform such an analysis. In this approach, we have demonstrated the potential of empirical analysis using data from a single station. It is clear that if digital waveform data from more stations are used, such an approach can be used to obtain more definite information about the properties of the artificial seismic source. It is also necessary to combine the empirical approach with other approaches, as more detailed information about the crustal structure and the sources can be obtained.

## Acknowledgments

This work was sponsored by the STEPI of Korea and the National Natural Science Foundation of China. The research was also financed in part by the Korean Ministry of Education under project number BSRI-95-5420. One of the authors (ZLW) sincerely thanks Prof. Y. T. Chen, at the Institute of Geophysics, State Seismological Bureau, China, for his directions and help in earthquake source studies, and the Seismological Institute, Hanyang University, Korea, for its support during his stay as a visiting professor. Acknowledgments are also due to the anonymous reviewers, as well as Prof. BLN Kennett, for their encouragement and helpful suggestions.

## References


© 1996 RAS, Geophys. J. Int. 125, 491–503


