An improved method for the determination of the tectonic stress field from focal mechanism data

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SUMMARY
All conventional stress inversion methods, when applied to earthquake focal mechanism data, suffer from uncertainty as to which plane is the true fault plane. This paper deals with several problems in stress inversion brought about by this uncertainty. Our analysis shows that the direction of shear stress on the auxiliary plane does not coincide with the hypothetical slip direction unless the B-axis is parallel to one of the three principal stress directions. Based on this simple fact, we propose a new algorithm dealing with the ambiguity in fault/auxiliary plane identification. We also propose a method to handle the inhomogeneity problem of data quality, which is common and unique for focal mechanism data. Different inversion methods and algorithms are applied to two sets of 'focal mechanism' data simulated from field fault-slip measurement data. The inversion results show that, among the four stress parameters inverted, the stress ratio suffers the most from the ambiguity in fault/auxiliary plane identity, whereas the solutions for the principal stress directions are surprisingly good. The errors in inversion solutions resulting from the fault/auxiliary plane ambiguity can be significantly reduced by controlling subjectively the sample variance of the measurement errors. Our results also suggest that the fault plane cannot be distinguished correctly from the auxiliary plane with a high probability on the basis of the stress inversion alone.

Key words: earthquake-source mechanism, fault-plane solution, inversion, stress distribution.

INTRODUCTION
Over the last two decades, a number of methods have been devised to determine the tectonic stress field from the inversion of a population of fault-slip data. These methods can be classified into two different groups according to their inversion criteria. One is based on minimizing, in some way, the sum of the misfit angle between predicted shear stress and observed slip direction for each fault (Carey & Brunier 1974; Carey 1976; Angelier 1979, 1984, 1990). The other is based on minimizing the sum of the minimum rotation angle between each of the observed fault planes and any plane that can make the predicted shear stress coincident with the observed slip direction (Gephart & Forsyth 1984; Gephart 1990). More recently, Yin & Ranalli (1993), on the basis of probability and spherical statistics, proposed a unified approach that incorporates both groups of inversion techniques.

All the above-mentioned methods are suitable for fault-slip data only, and, when applied to earthquake focal mechanism data, suffer from uncertainty as to which nodal plane is the true fault plane. To overcome this ambiguity, Gephart & Forsyth (1984) proposed an algorithm in which both nodal planes are entered in inversion. At each step of the iteration (i.e., given a stress model), the misfit angle or the minimum rotation angle is calculated for both nodal planes for each focal mechanism solution, and the plane that gives rise to the smaller misfit angle or smaller minimum rotation angle is selected as the fault plane. Angelier (1984) proposed a different algorithm in which both nodal planes are entered in the inversion and are treated as if they were fault planes without identifying them. Both algorithms are empirical, however, and their physical and mathematical bases have not been fully explored. Besides, another problem that has not been considered before is the inhomogeneity in data quality; some focal mechanism solutions are known to be poorer than others due to the clarity of P-wave polarity, the number of stations available, and the azimuths of seismic stations with respect to earthquake epicentres.

The purpose of this paper is to extend Yin & Ranalli’s (1993) method to the inversion of earthquake focal mechanism data. First, we analyse the difference in mechanical properties between the fault plane and auxiliary plane. Second, having discussed the physical and mathematical basis of the two
existing algorithms from a probabilistic point of view, we propose a new algorithm that deals with the ambiguity in fault/auxiliary plane identification more appropriately. Third, we propose a method that takes into account the problem of inhomogeneity in data quality. Finally, we perform experiments on two simulated ‘focal mechanism’ data sets to study how large the errors in inversion solutions resulting from the uncertainty in fault/auxiliary plane identification are, and to what extent the fault plane can be correctly distinguished from the auxiliary plane on the basis of stress inversion alone.

**SHEAR STRESS DIRECTION ON THE FAULT PLANE AND ON THE AUXILIARY PLANE**

Unlike the fault-slip data, there are two nodal planes for each focal mechanism solution. Among them, only one is the true fault plane. If the fault plane can be distinguished from the auxiliary plane by independent geological and seismological methods, the inversion of focal mechanism data reduces to that of fault-slip data. In most cases, however, they cannot be known *a priori*. As suggested by Michael (1987), this ambiguity in fault/auxiliary plane identity can be dealt with by devising an algorithm that either can distinguish correctly the fault planes from the auxiliary planes or does not need to know them. Generally speaking, at the nucleation point of an earthquake the stress state on the fault plane is different from that on the auxiliary plane. Therefore, understanding the geometrical and mechanical difference between the fault and the auxiliary plane is essential for devising an algorithm that deals with the uncertainty in fault/auxiliary plane identification (Gephart 1985; Michael 1987; Gephart 1990).

Let $x_i$ be a Cartesian coordinate system coincident with the geographical coordinate system (i.e., $x_1$, $x_2$ and $x_3$ coincide with the vertical, the east and the north directions, respectively) and $y_i$ be another Cartesian coordinate system coinciding with the direction of the principal stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$. The transformation between $y_i$ and $x_i$ can be achieved by three successive rotations of the Euler angles $\alpha$, $\beta$ and $\gamma$ (Fig. 1), and the transformation matrix is (Yin & Ranalli 1993)

$$
\begin{pmatrix}
    u_1 & u_2 & u_3 \\
    v_1 & v_2 & v_3 \\
    w_1 & w_2 & w_3
\end{pmatrix} =
\begin{pmatrix}
    \cos \alpha & \sin \alpha \sin \gamma & \sin \alpha \cos \gamma \\
    \sin \alpha \sin \beta & \cos \beta \cos \gamma & -\cos \beta \sin \gamma \\
    -\sin \alpha \cos \beta & \sin \alpha \cos \gamma & \cos \beta \sin \gamma
\end{pmatrix},
$$

(1)

where $u_i$, $v_i$, $w_i$ ($i = 1, 2, 3$) are the cosines of the angles between $y_i$ and $x_1$, $y_2$, $y_3$ and $x_1$, respectively. Let $n$, $h$ and $l$ be unit vectors denoting the direction of the normal to the fault plane, the observed slip, and the predicted shear stress on the fault, respectively. The misfit angle ($\delta$) between the predicted shear stress direction ($l$) and the observed slip direction ($h$) can be expressed as (see Yin & Ranalli 1993)

$$\cos(\delta) = 1 \cdot \mathbf{h} = \frac{-(n_1 h_1 + h_2 n_2 + h_3 n_3)}{\sqrt{(n_1^2 + \delta_1^2 n_2^2) - (n_1^2 + \delta_1^2 n_2^2)^2}},$$

(2)

where $\delta = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ is the stress ratio and $n_1$, $n_2$ and $n_3$ are the components of $n$ and $h$ in the $y_i$-coordinate system (the principal stress coordinate system); they are related to the corresponding components in the $x_i$-coordinate system (the geographical coordinate system) by

$$
\begin{pmatrix}
    n_{1y} \\
    n_{2y} \\
    n_{3y}
\end{pmatrix} =
\begin{pmatrix}
    u_1 & u_2 & u_3 \\
    v_1 & v_2 & v_3 \\
    w_1 & w_2 & w_3
\end{pmatrix} \begin{pmatrix}
    n_{1x} \\
    n_{2x} \\
    n_{3x}
\end{pmatrix},
$$

(3)

and

$$
\begin{pmatrix}
    h_{1y} \\
    h_{2y} \\
    h_{3y}
\end{pmatrix} =
\begin{pmatrix}
    u_1 & u_2 & u_3 \\
    v_1 & v_2 & v_3 \\
    w_1 & w_2 & w_3
\end{pmatrix} \begin{pmatrix}
    h_{1x} \\
    h_{2x} \\
    h_{3x}
\end{pmatrix}.
$$

(4)

According to the double-couple point-source model in which the auxiliary plane is defined as the plane perpendicular to the slip vector, there exists a simple geometrical relationship between the fault and the auxiliary plane; the slip on the fault plane is the normal to the auxiliary plane, and the normal to the fault plane is identical to the hypothetical slip on the auxiliary plane. Consequently, using this simple geometrical relation the angle ($\delta^*$) between the predicted shear stress and the hypothetical slip direction on the auxiliary plane can be readily derived by simply interchanging the places of the components of $n$ and $h$ in eq. (2), that is

$$\cos(\delta^*) = \frac{-(h_1 n_1 + h_2 n_2 + h_3 n_3)}{\sqrt{(h_1^2 + \delta^2 h_2^2) - (h_1^2 + \delta^2 h_2^2)^2}}.$$

(5)

In stress inversion, it is assumed that when faulting occurs the slip follows the direction of the shear stress on the fault plane. If there are no measurement errors, the angle ($\delta$) in eq. (2) must equal zero. Comparing eq. (2) with eq. (5), one may find that, when the above condition is satisfied, the angle ($\delta^*$) in eq. (5) is generally not equal to zero because the right-
hand sides of both equations have the same numerator, but different denominators. This implies that at the focus the shear stress on the auxiliary plane does not coincide, in most cases, with the hypothetical slip direction indicated by the normal to the fault plane. Here we are interested in conditions under which the shear stress on the auxiliary plane is coincident with the hypothetical slip, so that the two nodal planes become geometrically indistinguishable with respect to the principal stresses. The solution to this problem can be derived by equating eqs (2) and (5) to unity and solving the equations. However, it is easier to derive the solution using a graphical method. Fig. 2 shows a conventional representation of the stress tensor acting on an infinitesimal cube. A local Cartesian coordinate system $z_i$ is so devised that the $z_2$-axis is perpendicular to the auxiliary plane and the $z_3$-axis is perpendicular to the fault plane on which the shear stress is coincident with the slip (i.e. the shear stress component $\sigma_{12}$ on the fault plane vanishes). From the symmetry of the stress tensor, it can be seen clearly that the resultant shear stress on the auxiliary plane can coincide with the hypothetical slip direction only when the $B$-axis is parallel to one of the three principal stress axes, because in this case the shear-stress component $\sigma_{12} = \sigma_{23}$ vanishes.

In the next section, on the basis of the fact that the shear stress and the hypothetical slip on the auxiliary plane do not coincide unless the $B$-axis is parallel to one of the three principal stress axes, we discuss the two existing algorithms and then propose a new algorithm dealing with the fault/auxiliary plane ambiguity.

### A NEW ALGORITHM DEALING WITH THE AMBIGUITY IN FAULT/AUXILIARY PLANE IDENTIFICATION

The conventional stress inversion methods can be classified into two groups according to their inversion criteria. The first group, comprising a number of slightly different criteria, is based on minimizing in some way the sum of misfit angles between the predicted shear stress and observed slip direction for each fault. Among them, the most commonly used criterion is one that maximizes the sum of cosines of misfit angles [named $C_2$; here and hereafter, we use the same notation as Yin & Ranalli (1993) to denote different inversion criteria] (Angelier 1979, 1984, 1990):

$$C_2 = \sum_{i=1}^{n} \cos(s_i),$$  \hspace{1cm} (6)

where the subscript $i$ denotes the $i$th fault and $n$ the total number of faults. The second group, comprising only one method, is based on minimizing the sum of the minimum rotation angle ($\xi$) between each of the observed fault planes and any plane that can make the predicted shear stress coincident with the observed slip direction (named $C_4$) (Gephart & Forsyth 1984; Gephart 1990):

$$C_4 = \sum_{i=1}^{n} \xi_i.$$  \hspace{1cm} (7)

The statistical basis and physical implications of criterion $C_2$ (eq. 6) have been discussed by Yin & Ranalli (1993) in terms of probability and statistics theory. The spherical statistics theory shows that the observed slip direction and fault-plane orientation can be modelled by the von Mises and the Fisher distributions, respectively (Fisher, Lewis & Embleton 1987, pp. 81–88). Using the maximum-likelihood function, it is straightforward to prove that, if there is no measurement error in fault-plane orientation, $C_2$ (eq. 6) is the correct criterion to measure the misfit. Therefore, criterion $C_2$ implicitly assumes that the orientation of the fault plane is perfectly known and the measurement errors in slip direction follow the von Mises distribution. The statistical basis of criterion $C_4$ (eq. 7) proposed by Gephart & Forsyth (1984) has also been discussed by Yin & Ranalli (1993), but it should be pointed out that Yin & Ranalli's account is inaccurate. In Gephart & Forsyth's (1984) method, the fault plane is rotated about an arbitrary axis to make the predicted shear-stress direction coincident with the observed slip direction, and the minimum rotation angle is used as a measure of misfit. There are two ways to rotate the fault plane that can bring the predicted shear stress to the observed slip direction. One is to keep the normal to the fault plane fixed and to rotate the predicted shear-stress vector around the fault normal (we term this the in-plane rotation). The other is to rotate the fault normal around an arbitrary axis (we term this the pole rotation). The in-plane rotation is equivalent to the misfit angle ($\xi$) between predicted shear stress and observed slip direction, which can be modelled by the von Mises distribution from the probabilistic point of view. The pole rotation can be modelled by the Fisher distribution (Yin & Ranalli 1993). The angle ($\xi$) in eq. (7) represents the rotation angle of either in-plane rotation or pole rotation, whichever is the smaller. Consequently, the statistical basis of criterion $C_4$ is quite clear. It takes measurement errors both in slip direction and in fault-plane orientation into account, but not simultaneously for each fault. For some faults, criterion $C_4$ measures the measurement errors in slip direction. For the other faults, it measures the measurements errors in fault-plane orientation. Obviously, an appropriate criterion should take both kinds of measurement error simultaneously into account for each fault. Such a criterion has been proposed by Yin &

![Figure 2. Tectonic stresses acting on the fault plane (F) and the auxiliary plane (A). Vector h denotes the slip and B denotes the B-axis.](https://academic.oup.com/gji/article-abstract/125/3/841/679401/843)
Ranalli (1993). Using numerical analysis, Yin & Ranalli (1993) found that the misfit angle between the predicted shear stress and the observed slip direction can be modelled approximately by the von Mises (or the normal) distribution with a varying variance, and proposed two inversion criteria. These two criteria give rise to nearly the same result, but one criterion is much faster than the other in terms of computing time. Here we use the faster one (named $C_6$), which is based on modelling the misfit angle as the normal distribution. The criterion is (Yin & Ranalli 1993)

$$C_6 = \frac{1}{\prod_{i=1}^{n} \sqrt{\nu(1+b_i)}} \exp \left[ -\frac{1}{2} \sum_{i=1}^{n} \frac{s_i^2}{\nu(1+b_i)} \right],$$

where $\nu$ is the variance of measurement errors both in slip direction and in fault-plane orientation, and $b$ is a factor which is a function of the principal stress directions, the stress ratio $(\delta)$, and the orientation of the fault plane. The factor $b$ can be calculated using Yin & Ranalli's (1993) eq. (31). The variance $\nu$ can be estimated independently by (Yin & Ranalli 1993)

$$\bar{\nu} = \frac{1}{n} \sum_{i=1}^{n} \frac{s_i^2}{1+b_i}.$$

These three inversion criteria $C_2, C_4, C_6$ are suitable for fault-slip data only, and, when applied to focal mechanism data, must be modified because of the ambiguity in fault/auxiliary plane identification. To date, there have been two different algorithms available to deal with the ambiguity. One selects, at each step of the inversion (or iteration), the nodal plane as the fault plane that gives rise to the smaller misfit angle (for criterion $C_2$) or smaller minimum rotation angle (for criterion $C_4$) (Gephart & Forsyth 1984; Gephart 1990). The other enters both nodal planes into the inversion and treats both as if they were fault planes without discrimination (Angelier 1984). The former is called algorithm 1 and the latter is called algorithm 2 for convenience. Here, we discuss these two algorithms. As mentioned above, the fault plane differs from the auxiliary plane in the relation between the shear stresses and the slip directions. On the fault plane, the slip coincides with the shear-stress direction. On the auxiliary plane, the hypothetical slip does not coincide with the shear-stress direction, unless the $B$-axis is parallel to one of the three principal stress directions. If the misfit angle is taken as a random variable (Yin & Ranalli 1993), this geometrical difference between the fault plane and the auxiliary plane can also be interpreted in terms of the probability distribution of the misfit angle. The misfit angle (s) and/or the minimum rotation angle ($\zeta$) for the fault plane have zero means, whereas the misfit angle ($s^*$) and/or the minimum rotation angle ($\zeta^*$) for the auxiliary plane have non-zero means unless the $B$-axis is parallel to one of the three principal stress directions.

First of all, we consider a special case, i.e. assuming that either the slip direction or the fault-plane orientation is perfectly known without uncertainty (note that in this case s, $\zeta$, $s^*$ and $\zeta^*$ have constant variances). In the case where all the $B$-axes are parallel or close to one of the three principal stress directions (i.e., the mean of $s^*$ or $\zeta^*$ is equal or close to zero for each auxiliary plane), the two nodal planes become geometrically indistinguishable, and the density functions of the angles $s$ and $s^*$ (or $\zeta$ and $\zeta^*$) are identical or nearly equal (Fig. 3a). Thus, algorithm 2 is appropriate since it treats both nodal planes as fault planes and does not need to distinguish them. In the case that the absolute value of the mean of $s^*$ (or $\zeta^*$) $> 0$ for each auxiliary plane, the overlapping portion between the two distributions of $s$ and $s^*$ (or $\zeta$ and $\zeta^*$) is small and the fault plane can be distinguished with a high probability from the auxiliary plane, based on the principle of selecting the plane that has the smaller misfit (or minimum rotation) angle as the fault plane (Fig 3b). Algorithm 1 is therefore appropriate. In practice, however, the absolute value of the mean of $s^*$ (or $\zeta^*$) for most auxiliary planes is neither equal to 0 nor $> 0$. In this case, algorithm 1 works better than algorithm 2. The probabilistic basis behind algorithm 1 can be explained as follows. Given two nodal planes, we have no a priori knowledge about which is the true fault plane, but we know that the misfit angle (s) or the minimum rotation angle ($\zeta$) should follow the von Mises or Fisher distribution with zero mean and constant variance, independent of the orientation of the nodal planes. Consequently, we select the plane that gives rise to the smaller misfit (or minimum rotation) angle as the fault plane, because the smaller angle means higher distribution density and consequently higher probability. This algorithm is consistent with the fundamental principle of the maximum-likelihood function. However, it should be pointed out that some errors that are brought about...
by this algorithm are inevitable, both in inversion solutions and in the identification of fault planes, and these will be discussed later on.

Now we consider the general case, i.e. take both types of measurement errors in slip direction and in fault-plane orientation simultaneously into account for each fault. Yin & Ranalli (1993) have shown that when both types of measurement errors are considered the misfit angle \( s \) can be modelled approximately by the von Mises distribution (or the normal distribution). The mean of \( s \) is still zero for the general case. Unlike for the special case, however, the variance of \( s \) is no longer constant, but a function of the principal stress directions, the stress ratio, and the orientation of the fault plane. Given a stress model, the principal stress directions and the stress ratio \( \delta \) are the same for both nodal planes. Nevertheless, the variances of the misfit angle are not identical for both nodal planes since their orientations are different. Therefore, when the fault plane is selected from the two nodal planes, algorithm 1 is no longer valid because a smaller misfit angle does not necessarily mean a higher distribution density (or probability). We propose a new and simple algorithm (algorithm 3). We enter both nodal planes into the inversion. At each step of the iteration, we calculate the density function according to eq. (8) for both nodal planes of each focal mechanism solution and select as the fault plane the plane whose misfit angle has the higher distribution density. One may find from eq. (8) that, in order to calculate the density function, the sample variance given in eq. (9) must be known. However, the sample variance cannot be calculated unless the fault planes have been selected \textit{a priori}. This difficulty can be readily overcome by assigning arbitrary initial labels of 'fault plane' and 'auxiliary plane' to each focal mechanism solution at the beginning of the inversion, and, at each step of the iteration, using the fault planes selected at the previous step to calculate the sample variance. The new algorithm described above is consistent with the inversion criterion \( C_\alpha \), which is constructed on the basis of the maximum-likelihood function. Also, when either the measurement errors in slip direction or in fault-plane orientation vanish or are neglected, algorithm 3 reduces to algorithm 1.

It is inevitable that algorithm 3 (or its variant algorithm 1 for the special case) brings about some errors in identification of the fault planes and consequently in inversion solutions. In the case where the stress field is uniform, to what extent can the fault planes be distinguished correctly from the auxiliary planes on the basis of the stress inversion method alone, and how large are the errors in inversion solutions that result from the misidentification of fault planes? These two questions are important. From the probabilistic point of view, the chance of correctly distinguishing the fault plane from the auxiliary plane depends on the probability distribution of the misfit angle \( s \) for the fault plane as well as on the distribution of the misfit angle \( s^* \) for the auxiliary plane. As addressed previously, for a given stress model the misfit angle \( s \) for the fault plane follows approximately the normal distribution with mean zero and variance dependent on the stress field and the orientation of the fault plane. Similarly, the angle \( s^* \) for the auxiliary plane can also be modelled approximately by the normal distribution, but with both the mean and the variance varying with the principal stress directions, the stress ratio, and the orientation of the auxiliary plane. Fig. 4 shows a common distribution of the two random variables \( s \) and \( s^* \). The shaded area is identical to the probability that the true fault plane is misidentified as the auxiliary plane, because when the value of \( s \) lies in the shaded area the auxiliary plane will be selected as the fault plane according to algorithm 3. Thus, if the two distributions are identical (i.e. exactly overlapping), the probability of misidentification of the true fault plane is 0.5. If the mean of \( s^* \gg 0 \), the probability of misidentification of the true fault plane goes to zero. Generally speaking, the probability that the fault plane can be correctly distinguished from the auxiliary plane based on the stress inversion alone lies between 0.5 and 1.0.

Misidentification of the fault planes will certainly induce errors in the inversion solution, and sometimes the errors may be serious. This is because, at each step of iteration, the algorithm will automatically select the plane whose misfit angle has the higher distribution density as the fault plane, and the selection process degenerates until the solutions for the three principal stress directions and the stress ratio have been reached that maximize criterion \( C_\alpha \). As a consequence, the sample variance calculated using eq. (9) is always, under any circumstances, smaller than that obtained when fault planes are known \textit{a priori}. Because inversion results for criterion \( C_\alpha \) are affected by the variance of measurement errors (Yin & Ranalli 1993), there are two ways to prevent the selection process from degenerating and consequently to reduce the errors in inversion solutions. The first is described as follows: (1) after completion of the stress inversion, calculate the sample (or estimated) variance or standard deviation of the measurement errors according to eq. (9); (2) add a few degrees, as a modified amount, to the sample standard deviation; (3) insert the modified sample standard deviation into eq. (8) and repeat the inversion process. The second way is to insert directly into eq. (8) a variance about the measurement errors, estimated \textit{a priori} by an independent method or empirically, and then to perform the inversion. The two examples in the last section demonstrate that controlling the sample variance can significantly reduce the inversion errors.

**INHOMOGENEITY IN DATA QUALITY**

Another common problem in stress inversion using focal mechanism data is the inhomogeneity in data quality. Unlike
the fault-slip data, the quality of which can be assumed to be the same for each fault in a given area, the measurement errors for focal mechanism solutions are often different for different earthquakes. The quality of some solutions is known a priori to be poorer than others due to the clarity of P-wave polarity, the number of stations available, and the azimuths of seismic stations with respect to the earthquake hypocentres. For instance, Bergerat, Angelier & Vilemin (1990) classify the focal mechanism solutions into the three groups A, B and C, with decreasing quality from A to C. A common treatment of this problem is to take all the data as having the same quality, or to abandon a few data whose quality is very poor and take the rest as having the same quality (e.g. Rivera & Cisternas 1990; Reches, Baer & Hattor 1992). This treatment faces a difficulty sometimes, especially when the poor data are more numerous than the good and fair data. The poor data can neither be totally abandoned nor used confidently. To deal with this difficulty, we propose two approaches: one subjective and the other objective. Suppose a data set can be classified into good, fair and poor groups, and each group has a relatively large size (if not, merge them). The subjective approach is to assign empirically a standard deviation of measurement errors to each group of data, with an increasing amount, say $3^\circ$--$5^\circ$, from the good to the fair and the poor. Accordingly, the inversion criterion $C_6$ in eq. (8) can be divided into several parts, each of which comprises one group of data. After the appropriate values of the standard deviation have been inserted, the inversion proceeds as usual until the best solutions are obtained that minimize the product of the three parts of $C_6$. The objective approach is similar to the subjective one, except that the variance ($\nu$) in eq. (8) is substituted by the sample variance calculated using eq. (9) separately for each group of data. These two approaches, like weighted functions commonly used in geophysical inversions, allow data of different quality to constrain the tectonic stress field differently.

**EXAMPLES**

Two data sets taken from field fault-slip measurements are studied in this paper to understand better the effect of ambiguity in fault/auxiliary plane identity on the stress inversion. The two data sets were first published by Angelier (1990), and were used by Yin & Ranalli (1993) to test their new methods for the determination of the tectonic stress field. The first data set is composed of 33 faults, taken from Neogene reef limestone near Agia Vavara, central Crete, Greece (site AVB). The second data set, comprising 38 faults, was collected in Neogene marly limestone near Tymbaki, southern Crete, Greece (site TYM). According to Yin & Ranalli's inversion results, both data sets are used to simulate two 'focal mechanism' data sets. We use algorithm 1 for $C_2$ and algorithm 3 for $C_6$ to distinguish the fault plane from the auxiliary plane at each step of the inversion. We also study the effect of the variance of measurement errors on stress inversion. For the method $C_6$, we use two different inversion procedures. One is to use eq. (9) to calculate the sample variance at each step of the inversion, as described previously. The other is to substitute in criterion $C_6$ (eq. 8) the constant standard deviation estimated from the fault-slip data, i.e. $7^\circ$ for site AVB and $12.1^\circ$ for site TYM (Yin & Ranalli 1993); these two values are regarded as the 'true' standard deviations of the measurement errors for the two data sets. The first procedure is called $C_6$ and the second one is called $C_6^*$. Table 1 lists the inversion results by different methods from the simulated 'focal mechanism' data. For the sake of comparison, the inversion results obtained by Yin & Ranalli (1993) from the original fault-slip data are also listed in Table 1. One can see from Table 1 that for both examples the estimated standard deviations from the 'focal mechanism' data are smaller than those from the fault-slip data by, on average, $4.7^\circ$ for $C_6$ and $2.1^\circ$ for $C_6^*$. Among the four inverted parameters of the stress tensor (the three principal stress directions and the stress ratio $\delta$), the one that suffers the most from the uncertainty in fault/auxiliary plane identity is the stress ratio. For both examples (sites AVB and TYM) and for both methods (criteria $C_2$ and $C_6$), the stress ratio obtained from the 'focal mechanism' data differs from that obtained from the fault-slip data by as much as 0.26. This value is much larger than the 90 per cent confidence limits for the stress ratio obtained from the fault-slip data (see Yin & Ranalli's (1993) Table 2). However, the errors in the three principal stress directions that result from the uncertainty in fault/auxiliary plane identity are relatively small. For the method $C_6$, the maximum difference between two principal stress axes is about $7^\circ$ (angular distance in space) for site AVB and is less than $3^\circ$ for site TYM. A similar size of errors in the principal stress directions is found for the method $C_6^*$. As addressed above, the errors in inversion solutions induced by the ambiguity in fault-slip plane identity can be reduced by controlling the variance of the measurement errors. To test this hypothesis, we list the inversion results obtained by $C_6^*$ ($C_6^*$ is nearly the same as $C_6$ except for using the 'true' variance estimated from the fault-slip data). Comparing the inversion results by $C_6$ and $C_6^*$, one finds that $C_6^*$ significantly improves the inversion results, that is, reduces the errors. For both data sets, the four parameters of the stress tensor inverted by $C_6^*$ are either within or close to the 90 per cent confidence limits for those inverted from the original fault-slip data [see Yin & Ranalli's (1993) Table 1]. Therefore, in the case where the variance of the measurement errors is difficult to estimate independently, one may use the above-proposed procedure that after the first inversion we add $3^\circ$--$5^\circ$ to the sample standard deviation and repeat the whole inversion process using the modified sample standard deviation.

We also perform experiments on the two data sets to study the problem regarding the differentiation of the fault plane from the auxiliary plane on the basis of stress inversion alone. We apply algorithm 1 (select the nodal plane which has the smaller misfit angle as the fault plane) to the stress solutions inverted by $C_2$, and algorithm 3 (select the nodal plane whose...
Table 1. Comparison of stress inversion results obtained by different methods from the original fault-slip data with those from the simulated 'focal mechanism' data.

<table>
<thead>
<tr>
<th>Sites</th>
<th>Data Type</th>
<th>Method</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
<th>$\delta$</th>
<th>$\psi^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Plunge</td>
<td>Trend</td>
<td>Plunge</td>
<td>Trend</td>
<td></td>
</tr>
<tr>
<td>Fault-Slip</td>
<td></td>
<td>$C_3$</td>
<td>73°</td>
<td>72°</td>
<td>17°</td>
<td>240°</td>
<td>4°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_6$</td>
<td>75°</td>
<td>63°</td>
<td>15°</td>
<td>244°</td>
<td>0°</td>
</tr>
<tr>
<td>AVB</td>
<td>&quot;Focal</td>
<td>$C_2$</td>
<td>67°</td>
<td>68°</td>
<td>23°</td>
<td>244°</td>
<td>2°</td>
</tr>
<tr>
<td></td>
<td>Mechanism&quot;</td>
<td>$C_6$</td>
<td>68°</td>
<td>65°</td>
<td>22°</td>
<td>243°</td>
<td>1°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_6^*$</td>
<td>74°</td>
<td>70°</td>
<td>16°</td>
<td>247°</td>
<td>1°</td>
</tr>
<tr>
<td>TYM</td>
<td>&quot;Focal</td>
<td>$C_2$</td>
<td>81°</td>
<td>281°</td>
<td>6°</td>
<td>55°</td>
<td>7°</td>
</tr>
<tr>
<td></td>
<td>Mechanism&quot;</td>
<td>$C_6$</td>
<td>83°</td>
<td>202°</td>
<td>6°</td>
<td>57°</td>
<td>4°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_6^*$</td>
<td>84°</td>
<td>218°</td>
<td>6°</td>
<td>60°</td>
<td>3°</td>
</tr>
</tbody>
</table>

The three principal stress axes are denoted by $\sigma_1$, $\sigma_2$ and $\sigma_3$, respectively. Stress ratio and estimated standard deviation are denoted by $\delta$ and $\psi^{1/2}$. The computing precision is 1" for $\sigma_1$, $\sigma_2$ and $\sigma_3$, 0.01 for $\delta$, and 0.1" for $\psi^{1/2}$. For the 'focal mechanism data', algorithms 1 and 3 are associated with criteria $C_2$ and $C_6$, respectively. $C_6^*$ denotes a different inversion procedure of $C_6$ in which the constant standard deviation estimated from the fault-slip data rather than from the 'focal mechanism' data is used, i.e. a standard deviation of 7.4° for site AVB and 12.1° for site TYM.

Table 2. Results of differentiation of fault planes from auxiliary planes based on the stress solutions obtained with various methods from inversion of the original fault-slip data.

<table>
<thead>
<tr>
<th>Site</th>
<th>Inversion solutions obtained by</th>
<th>Total number of faults</th>
<th>Number of faults being correctly identified</th>
<th>Ratio of the identified to the total faults</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVB</td>
<td>$C_2$</td>
<td>33</td>
<td>22</td>
<td>66.7%</td>
</tr>
<tr>
<td></td>
<td>$C_6$</td>
<td>33</td>
<td>24</td>
<td>72.7%</td>
</tr>
<tr>
<td>TYM</td>
<td>$C_2$</td>
<td>38</td>
<td>32</td>
<td>84.2%</td>
</tr>
<tr>
<td></td>
<td>$C_6$</td>
<td>38</td>
<td>26</td>
<td>68.4%</td>
</tr>
</tbody>
</table>

Fault planes are distinguished from auxiliary planes by algorithm 1 for the inversion solutions with $C_2$ and by algorithm 3 for the solutions with $C$. © 1996 RAS. GJI 125, 841–849
mechanism data collected in the western coastal areas of Canada. The work is in progress.

**CONCLUSIONS**

The stress analysis shows that the direction of shear stress on the auxiliary plane does not coincide with the hypothetical slip direction (indicated by the normal to the fault plane) unless the B-axis is parallel to one of the three principal stress directions, different from the fault plane on which the shear-stress direction is assumed to coincide with the slip direction. This geometrical difference can also be described in terms of the probability distribution of the misfit angle. The misfit angle between predicted shear stress and observed slip direction for the fault plane has zero mean, whereas it has non-zero mean for the auxiliary plane, unless the B-axis is parallel to one of the principal stress directions.

Two previous algorithms dealing with the ambiguity in fault/auxiliary plane identification, namely algorithm 1 and algorithm 2, were discussed from the probabilistic point of view. The results suggest that algorithm 1, which is better than algorithm 2, is suitable for the inversion criteria C₂ and C₆. However, it is inappropriate for the criterion C₅. We propose a new algorithm (algorithm 3) which is suitable for C₅. The previous algorithm 1 can be found to be a special case of our new algorithm. A procedure is described to reduce the errors in inversion solutions through subjectively controlling the sample variance of measurement errors.

We propose two techniques to deal with the inhomogeneity problem in data quality, i.e. a subjective and an objective technique. For both techniques, a data set is divided into several subsets. Within each subset, the data are assumed to be of the same quality. The subjective technique is to assign empirically each subset a different constant value of variance. The objective technique is to calculate the sample variance separately for each subset of data at each step of the inversion. These techniques allow data of different quality to constrain the tectonic stress field differently.

Different inversion methods and algorithms are applied to two sets of 'local mechanism' data simulated from field fault-slip measurement data. The inversion results show that, in the case where the fault plane is unknown, the algorithm proposed in this paper can allow us to obtain good inversion solutions for the three principal stress directions, which deviate from the solutions obtained in the case where the fault plane is known by only a few degrees (angular distance in space). However, the uncertainty in fault/auxiliary plane identity leads to relatively large errors for the stress ratio, sometimes as large as 0.26. Both errors in principal stress directions and the stress ratio can be reduced if the variance of measurement errors in focal mechanism solutions can be estimated independently. The inversion results also suggest that the chance of distinguishing the fault plane from the auxiliary plane is low based on the stress inversion alone, even if the stress field is uniform and known, and consequently the previous algorithms and the new algorithm proposed in this paper are not suitable for distinguishing the fault planes from the auxiliary planes.

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Determination of the tectonic stress field


