Professor Raithby is quite correct in his statement that the similarity analysis is not equivalent to a solution of the boundary layer equations. Perhaps the words "essentially equivalent" are more appropriate since the reduced axial momentum equation solved by Berman [7] and others [5, 6] contains the partial derivative of pressure instead of the ordinary derivative shown in equation (2). Agreement between exact solutions of the reduced Navier–Stokes equations[5] and the BL solution has been previously established [8] for channel flow and for porous tubes [12].

Reference [13] appears to be the only experimental laminar flow work performed to date on parallel porous plate channels and it is certainly a valuable contribution. The agreement between the analytical solution and experiment is encouraging especially when one considers some of the idealizations necessary in establishing tractable boundary conditions.

Dr. Harris requested more comparative information regarding velocity profiles and development lengths. Fig. 3 shows that for $\chi^* > 0.0005$, the pressure is essentially constant across the channel. This suggests that the $\partial P/\partial y$ term could be eliminated in the normal momentum equation leaving the inertial terms to balance the shear. Although computational effort was terminated two years ago, I fortunately retained the computer output. Upon reexamination, I found good agreement between RICE and BL axial velocity profiles at $\chi^* = 0.001$. The maximum variation between the profiles is 13 percent at the channel center line and lesser amounts elsewhere across the profile. It remains that the BL solution is still reasonable for approximating the flow for $\chi^* > 0.001$ and $\mathrm{Re}_w > -60$.

In reference [8] it was found that the development length of a uniform entry velocity profile becomes nearly equal to the channel length for complete mass extraction for $\mathrm{Re}_w < -20$. For instance, the flow is 99.9 percent fully developed at $\chi^* = 0.0114$ and the point of complete mass extraction is $\chi^* = 0.0125$. As suction is increased, these two reference points become almost coincident. Consequently, the development length for $\mathrm{Re}_w = -60$ closely approaches $\chi^* = 1/240$. Thus I would expect the differences due to a particular computational procedure to be minor.

[5]Eckert, E. R. G., Donoughe, P. L., and Moore, B. J., "Velocity and Friction Characteristics of Laminar Viscous Boundary Layer and Channel Flow over Surfaces with Ejection or Suction," NACA TN 4102, 1957.

Inviscid Approximations for Wall Jet-Receiver Interaction[1]

J. B. MILES.[2] The authors have applied some very interesting fluid mechanics analyses to the study of the subject problem. As pointed out quite explicitly by the authors, a number of simplifying assumptions and approximations had to be made in order to achieve solutions in the desired form. Accordingly, potential users of the presented material must be wary when applying the solution techniques, especially when doing so for ranges of the problem parameters outside those explored by the authors.

Authors' Closure

In all fluid mechanics analyses, it is certainly necessary to insure that the assumptions are appropriate for the problem under consideration. As pointed out in this paper and a forthcoming paper,[3] possible presence of continuous wall stall and splitter wall stall must be accounted for. Suitable inviscid models can approximate the onset of continuous wall stall, and can describe overall features of the wall jet-receiver system such as influence length, spill flow angle, and receiver inlet velocity distribution. Further studies should focus on viscous and three-dimensional details of the flow field.

[1]By H. S. Shen and D. O. Rockwell, published in the March, 1975, issue of the Journal of Fluids Engineering, Trans. ASME, Series I, Vol. 97, No. 1, p. 60.

[2]Professor, Dept. of Mechanical and Aerospace Engineering, University of Missouri, Columbia, Columbia, Mo.

[3]Shen, H. S., and Rockwell, D. O., "An Experimental Investigation of Wall Jet-Receiver Interaction," to be presented at a forthcoming ASME Conference.

Flow Regimes in Two-Dimensional Ribbed Diffusers[1]

K. E. HICKMAN.[2] The reviewers of this paper brought up several issues which they wished to have discussed upon presentation of this paper. Here are these issues:

The presence or absence of "transitory stall" is a criterion used in the paper to judge the effects of changes in a number of rib design parameters. Transitory stall is said to be present if reverse flow occurs over 20 percent of the axial height of the diffuser at any axial location. Thus, the authors were required to distinguish a 3/8 in. deep stall in the 1-3/4 in. deep flow through the diffuser, where observation from the top surface only seems to have been possible. Can the authors estimate the uncertainty in this determination, and its effect on selection of optimum rib design?

At the low water velocities used, the bottom surface boundary layer grows rapidly; for a flat plate, $d\delta/dx \sim 0.4$ in./ft. Thus, the boundary layer depth is significant in comparison to the flow depth. Knowledge and control of the inlet velocity profile should be important in these studies. Can the authors comment on the extent to which the boundary layer and velocity profile effects present in their flow test arrangement, and the low aspect ratio of the test diffuser, may affect the applicability of the results to conventional diffuser design?

How does the pressure recovery effectiveness of the ribbed diffuser compare to the effectiveness of an equivalent wide-angle diffuser with splitter vanes?

C. R. SMITH.[3] This flow visualization study of flow regime characteristics for ribbed wall diffusers indicates the presence of some interesting rib generated flow behavior which is apparently responsible for substantial improvement in diffuser flow regime characteristics. However, there are several points regarding apparatus employed for this study which in the view of this discusser need clarification.

It is of concern that the flow regime positions of lines *a-a* and lines *b-b* for the non-ribbed diffusers are not in better agreement

[1]By F. D. Stull and H. R. Velkoff, published in the March, 1975, issue of the Journal of Fluids Engineering, Trans. ASME, Series I, Vol. 97, No. 1, p. 87.

[2]York Division, Borg-Warner Corp., York, Pa.

[3]Assistant Professor, School of Mechanical Engineering, Purdue University, West Lafayette, Ind. Mem. ASME.

with the results of Fox and Kline [1].[4] Examination of the authors' Fig. 3 indicates that their results fall from 20 percent ($N/W_1 = 2.5$) to 60 percent ($N/W_1 = 25$) of 2θ below the results obtained in [1]. Although there is better correlation with the results of Moore and Kline [2], the location of line a-a and b-b in [2] was done in a much less systematic manner and with a much cruder apparatus than [1]. It should be pointed out that other investigators (e.g. [3]) have been able to reproduce the results of [1] to within ± 10 percent of 2θ at worst. One possible explanation for the lack of agreement in flow regime characteristics is the very low throat aspect ratio employed in this study ($b/W_1 \approx 1/2$). With such a low aspect ratio the presence of end-wall boundary layers connot be neglected, as can be done in high throat aspect ratio studies [1], [3]. The effect of this end wall boundary layer would be to increase the inlet blockage effects, and to make it difficult to maintain a turbulent throat boundary layer at the low throat Reynolds numbers employed. It is conceivable that the boundary layer trip employed in this study was insufficient to trigger the transition to a turbulent throat boundary layer, which would explain the premature transition locations of lines a-a and b-b. This possibility of a laminar throat boundary layer is further supported by the very smooth, laminar-like appearance of the dyed fluid used to illustrate the behavior of the flow in and adjacent to the ribbed walls of the diffusers just downstream of the throat (e.g. authors' Figs. 6, 8, and 13).

From the description of the experimental apparatus it is not clear whether the bottom surface of the diffuser was sealed to prevent leakage under the edges of the diffuser walls. Personal experience has shown that failure to provide an adequate bottom surface seal for a water channel diffuser can result in the gross deterioration of the flow behavior in a fixed geometry diffuser due to small leaks near the throat. If such leaks were present, they could have had an effect on both the variation in flow regime behavior and in the character of the ribbed wall flow behavior.

Two comments can be made regarding the hot film anemometer measurements of this study. First, the authors have included rms voltage measurements using a single element hot film in regions of obvious reversed flow. The relevance of such measurements to physical flow characteristics (i.e., turbulence) is unclear, since a single element anemometer cannot determine flow direction. In addition, the anemometer output was apparently not linearized, and no reference is made to the level of the mean voltage measured at any position. Under the circumstances, one can only hope to monitor qualitative steadiness effects at best. Secondly, the oscilloscope traces obtained with the hot film sensor located in and at the edge of the rib cavity indicate a periodic, laminar-like behavior (c.f. authors' Figs. 17 and 19). This would seem to further support the existence of a laminar boundary layer at the throat. Indeed, the vortex behavior initiated in the rib cavities is not unlike the formation of mixing vortices at the free shear interface of two initially laminar flows [4].

In general, clarification is needed regarding the influence of aspect ratio on the results of this study, the possible existence of bottom wall leakage and/or a laminar throat boundary layer, and the relevance and interpretation of the hot film anemometer results. Qualification of the present results as to their extensibility to high aspect ratio diffusers with higher throat Reynolds numbers would also be desirable.

References

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[4]Numbers in brackets designate References at end of discussion.

Authors' Closure

The authors wish to thank Kenneth E. Hickman and C. R. Smith for their stimulating discussion of our paper. It is true that the diffuser model tested had a low aspect ratio, which was about 40 percent of the model used by Moore and Kline [5][5] and appreciably smaller than that used by Fox and Kline [6]. With the exception of the depth of the model, all dimensions including the entrance section were patterned after the model used by Moore and Kline, and testing was conducted on a water table very similar to the one used by them. With boundary layer and inlet velocity profiles more closely resembling that of Moore and Kline's apparatus, it is not surprising that closer agreement was obtained with their data than that of Fox and Kline. An exception to this would be line a-a, which was unobtainable from Moore and Kline's results because of their reliance on the fixed injector observation method. Extrapolating the absolute flow regime results of the present paper to high aspect ratio diffusers (which are essentially independent of inlet Reynolds number) and higher Reynolds numbers must be done with caution; however, the trends of this study in regards to the relative effect of the ribs on flow regimes are valid. Tests conducted on a small scale air diffuser [7] at much higher Reynolds numbers showed the ribs to be effective over a wide Mach number range.

The ability to estimate a 3/8 in. deep stall in the 1-3/4 in. deep flow was not a problem in determining transitory stall, since changes as small as 1/2° in 2θ caused a significant visual difference in the wall traces when approaching line a-a. The uncertainity in obtaining line a-a and line a'-a' was $\pm 1°$. The location of defining line b-b was somewhat more subjective and its uncertainty was estimated as $\pm 2°$. Such uncertanties would have little effect on the selection of optimum rib design. Checks for leakage were made prior to each run by injecting a small amount of dye at the junction of the water table glass bottom and diffuser walls. This was not a problem with the smoothly varnished diffuser models, but leaks were detected on some of the early ribbed sections tested. A clay like material was thinly applied to the bottom of these for a sealant.

The original purpose of the hot film anemometer was to document the turbulence level of the flow entering the diffuser, since high inlet turbulence can delay the onset of two-dimensional stall. The maximum values of turbulence intensity ($u'/\bar{U}$) measured on the center line at the throat position of the straight wall diffuser ranged from 0.3 to 0.6 percent for all values of W_1 tested. Turbulence intensities lower than 0.25 percent were difficult to measure because of 60 cycle background noise which was present. The hot film sensor was later found useful in monitoring the periodic oscillations caused by the ribbed sections and the extent to which these velocity fluctuations persisted throughout the diffuser. The frequency of such pulsations could be related to visual observations of surface waves generated in the ribbed sections. No attempt was made to quantify "turbulence" in regions of reverse flow.

Although pressure recovery measurements could not be obtained in the water table diffuser experiments described in this paper, performance data obtained on similar small scale air diffusers [7] showed that for an unstalled diffuser, the addition of ribs decreased performance, but for a moderately stalled dif-

[5]Numbers in brackets designate References at end of closure.

fuser, the addition of ribs generally increased static pressure recovery, C_p, up to 25 percent. In comparison Feil [8] has shown that properly designed splitter vanes can increase C_p of an equivalent wide-angle diffuser more than 60 percent. The ribbed diffuser would be preferred in applications where a wide-angle diffuser was required and mechanical complexity of using splitter vanes would not be desirable, such as in high temperature flows.

References

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Solid Particle Demixing in a Suspension Flow of Viscous Gas[1]

GEORGE RUDINGER.[2] It is perhaps worthwhile to point out that the existence of a demixed region in capillary flow of suspensions is known as the Fahraeus-Lindquist effect. Its consequence is that the effective viscosity of a suspension becomes a function of the tube diameter [1].[3] In the case of blood flow through narrow blood vessels, the uneven distribution of the red blood cells has an influence on the distribution in branching vessels [2].

When the plate is set in motion, the particles can acquire a velocity only through interaction with the fluid phase. A rotating particle that is moving away from the plate therefore might enter a region where its velocity is larger than the fluid velocity. Maude and Whitmore [3] hinted at such a velocity inversion in the flow of a suspension through a capillary tube. It would be interesting to see whether the authors of the present paper observed such flow regions.

The definition of the thickness of the demixed region is not clearly defined. In the context of computations, in which the particle diameter appears merely as one of the constant parameters involved, a particle can have zero distance from the wall. In reality, it cannot be closer than one-half diameter. A particle "at the wall" therefore moves under the influence of the gas that is one-half particle diameter away from the wall. The question therefore arises whether the lower limit of the left integral of equation (20) should not be $d/2$ instead of zero. Since the demixed region is only a few particle diameters thick, according to Fig. 4 of the paper, a correction of the thickness might be relatively large.

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[2]Principal Scientist, Textron Bell Aerospace, Buffalo, N.Y. Fellow ASME.

[3]Numbers in brackets designate References at end of discussion.

L. A. SPIELMAN.[4] This work constitutes a valuable step in the development of realistic and complete equations describing flow of particulate suspensions. The most important addition is the incorporation of Saffman's lift force calculation, which here is crucial to the occurrence of the demixed zone. Of lesser importance, at least for the parameter values selected, is the incorporation of the concentration correction factor on drag; here the authors cite the work of Tam who attempted to rationalize a formula which was originally derived by Brinkman [4].[5]

The authors are well aware of the difficulties in developing fully rigorous equations of this kind. Nevertheless, it is useful to examine some of those features which are most seriously in need of improvement in order to facilitate future progress.

The particular problem of motion near a flat plate which was selected by the authors to demonstrate the effect requires a closer examination of both wall boundary conditions and wall influences on particle-fluid interactions, especially since Fig. 4 of the paper shows for the parameters chosen, that the demixed zone extends just a few particle diameters from the wall For the largest Re = 100, the equilibrium demixed zone is only about five particle diameters. Although the authors state in the paragraph following equation (18) that the particles are free to slide, collide or stick to the surface, their analysis apparently ignores such complications since particle-wall sliding friction and wall-directed momentum exchange due to collisions are not aspects of this calculation. Here also, adhesion forces which might cause sticking have been ignored.

While the above wall complications are difficult to treat, there are simpler effects which could be handled readily. Presumably y_d refers to the coordinate of the particle centers at the edge of the demixed zone. This would seem to require $y_d = d/2$ at $t = 0$, rather than $y_d = 0$ as shown in Fig. 4. This discrepency would be unimportant were it not for the fact that y_d never exceeds more than a few diameters throughout the calculations shown.

Another important modification which should be incorporated is the influence of the wall on both translational and rotational particle Stokes interactions with the fluid. Various authors [5–9] have rigorously treated wall effects for Stokes interactions for perpendicular, tangential and rotational motions, including shear flows, showing such effects to be significant up to many particle diameters from the wall. Using qualitative reasoning, one would expect these interactions to retard formation of the demixed zone.

The concentration correction factor, α, strictly applies to Stokes drag, yet in equation (13), the factor multiplies the nonStokesian terms as well. This seems to be somewhat arbitrary, although nothing more rigorous readily suggests itself. For $\chi \cong 0.001$, the concentration correction on drag force amounts to about 7 percent, which is not very large anyway.

One also wonders how important is including the antisymmetric stress due to particle torque interaction with the fluid. Perhaps the authors can comment on how much their specific results are affected, as well as more generally.

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[5]Numbers in brackets designate References at end of discussion.