X(3872) and Its Iso-Triplet Partners

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Decays of X(3872) and its partners as hidden-charm axial-vector tetra-quark mesons are studied. As the result, it is seen that the iso-triplet partners of X(3872) can be broad, and therefore, higher statistics will be needed to find them.

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Tetra-quark mesons are classified into the following four groups:

\[ \{qq\bar{q}q\} = [qq]_1[\bar{q}q]_1 \oplus (qq)(\bar{q}q) \oplus \{[qq](\bar{q}q) \oplus (qq)[\bar{q}q]\}, \quad (q = u, d, s, c) \quad (1) \]

in a quark model\(^1\) as in the MIT bag model\(^2\), where parentheses and square brackets denote symmetry and anti-symmetry, respectively, of flavor wave functions under exchange of flavors between them. Each term on the right-hand side of Eq. (1) is again classified into two groups\(^2\) with \(\bar{3}_c \times 3_c\) and \(6_c \times \bar{6}_c\) of the color \(SU_c(3)\). However, the former is expected to be lower in the case of heavy mesons, because forces\(^3\) between two quarks are attractive when they are of \(\bar{3}_c\), while repulsive when \(6_c\), and because a possible mixing between these two states is expected to be small at the scale of heavy meson mass.\(^4\) Regarding with the spin \((J)\), the \([qq]\) and \((qq)\) have \(J = 0 \) and \(1\), respectively, and hence the spin and parity \(J^P\) of \([qq][\bar{q}q]\) and \(\{[qq](\bar{q}q) \oplus (qq)[\bar{q}q]\}\) are \(J^P = 0^+\) and \(1^+\), respectively, in the flavor symmetry limit. In the real world, however, the flavor symmetry is broken and hence the above tetra-quark states can have \(J = 0, 1, 2\), in general. Nevertheless, no indication of tetra-quark meson with \(J^P = 2^+\) has been observed, so that the above assignment seems to be favored in the real world. Thus, we treat the \([qq][\bar{q}q]\) mesons as scalar ones\(^1\) and the \(\{[qq](\bar{q}q) \oplus (qq)[\bar{q}q]\}\) as axial-vector ones,\(^5\) in contrast with Ref. 6) in which \(X(3872)\) has been assigned to \([cn][\bar{c}n]\) with a large violation of isospin symmetry. The charm strange scalar \(D_{s0}^+(2317)\) observed in \(e^+e^−\) annihilation\(^7\) as well as in \(B\) decays\(^8\) and the \(\eta\pi^0\) peak at 3.2 GeV in two photon collision,\(^9\) respectively, are good candidates\(^{1,4,10}\) of \([cn][\bar{c}n]_{I=1} \sim \hat{F}_I^1\) and \([cn][\bar{c}n]_{I=1} \sim \hat{\delta}_c^0\), \((n = u, d)\). In addition, an indication of another charm strange scalar which has been observed in the \(D_{s0}^+\chi\) channel of \(B\) decays\(^8\) is a suitable candidate of the iso-singlet partner\(^1,4\) \(\mit{\bar{c}n}\mit{n}\) for \(D_{s0}^+(2317) = \hat{F}_0^+\). However, the hidden charm\(^11\) \(X(3872)\) has a mass much higher than that of the above candidate of the hidden-charm scalar \(\hat{\delta}_c^0\), and its spin-parity is favored to be \(J^P = 1^+\) or \(2^−\) by experiments.\(^12,13\) Therefore, we assign\(^5\) \(X(3872)\) to \(\{[cn][\bar{c}n] + (cn)[\bar{c}n]\}\) (but not to \([cn][\bar{c}n]\) with \(J^P = 1^+\).
However, we ignore\cite{14} the \((qq)(q\bar{q})\) mesons in this short note.

Now we review very briefly \(X(3872)\) for later discussions. A recent analysis\cite{15} in \(X(3872) \to \pi^+\pi^-J/\psi\) provides its mass and width as

\[
m_X = 3871.56 \pm 0.22 \text{ MeV and } \Gamma_X < 1.2 \text{ MeV (90\%CL).} \tag{2}\]

Because it decays into \(\gamma J/\psi\) state, its charge conjugation parity \((C)\) is even,\cite{16} and hence, we here take \(J^{PC} = 1^{++}\). Although the \(X(3872) \to \pi^+\pi^-J/\psi\) decay proceeds through the \(\rho^0J/\psi\) intermediate state,\cite{17,18} its isospin quantum number is favored to be \(I = 0\), because no indication of charged partner of \(X(3872)\) has been observed.\cite{19} In addition, the above isospin assignment is consistent with the observations of the \(X(3872) \to \omega J/\psi \to \pi^+\pi^-\pi^0J/\psi\) decay.\cite{13,17} Thus the \(X(3872) \to \pi^+\pi^-J/\psi\) decay is isospin non-conserving. Here, it should be noted that the rate for the \(X(3872) \to \pi^+\pi^-J/\psi\) decay is nearly equal\cite{20} to that for the \(X(3872) \to \pi^+\pi^-\pi^0J/\psi\),

\[
\frac{\text{Br}(X(3872) \to \pi^+\pi^-\pi^0J/\psi)}{\text{Br}(X(3872) \to \pi^+\pi^-J/\psi)} = 0.8 \pm 0.3. \tag{3}\]

Comparing the above ratio with the measured ratio \(\Gamma(\omega \to 3\pi)/\Gamma(\omega \to 2\pi) \approx 60\), one might feel that Eq. (3) is strange. However, these two ratios are not necessarily parallel to each other, as seen below. The \(X(3872) \to \omega J/\psi \to \rho^0J/\psi \to \pi^+\pi^-J/\psi\) decay in the denominator of Eq. (3) is extraordinarily enhanced, because of double pole contribution of \(\omega\) and \(\rho^0\) with \(m_\omega^2 - m_\rho^2 \ll m_\omega^2\), where the broad width\cite{16} \(\Gamma_\rho \simeq 150\) MeV of \(\rho\) has to be taken into account, and because the kinematical condition of the former in which \(\omega\) decays into \(\pi^+\pi^-\pi^0\) in the energy region lower than \(m_X(3872) - m_{J/\psi} \simeq m_\omega - 16\omega/2\) is different from that of the latter (on the mass-shell of \(\omega\)), so that the rate for the \(X(3872) \to \pi^+\pi^-\pi^0J/\psi\) decay might be sensitive to a mechanism of \(\omega \to 3\pi\) and hence that of \(X(3872) \to \pi^+\pi^-\pi^0J/\psi\). Nevertheless, the mechanism of \(\omega \to 3\pi\) is still uncertain. To see this, we consider the \(\omega \to 3\pi\) decay and the radiative \(\omega \to \gamma\pi^0\) in addition to \(\rho^\pm \to \gamma\pi^\pm\) under the vector meson dominance\cite{21} (VMD). By taking the measured rate\cite{16} \(\Gamma(\omega \to \gamma\pi^0)_{\text{exp}} = 701 \pm 25\) keV as the input data, our calculated rate, \(\Gamma(\rho \to \gamma\pi^\pm)_{\text{th}} \simeq 72 - 73\) keV reproduces considerably well the measured rates,\cite{16} \(\Gamma(\rho^\pm \to \gamma\pi^\pm)_{\text{exp}} \simeq 67 \pm 8\) keV and \(\Gamma(\rho^0 \to \gamma\pi^0)_{\text{exp}} \simeq 90 \pm 12\) keV, although the measured rates still have large ambiguities. From the above, it is seen that the VMD works in these decays, at least in the \(\omega \to \gamma\pi^0\) and \(\rho^\pm \to \gamma\pi^\pm\) decays. Next, we determine the \(\omega\rho\pi\) coupling strength from the above \(\Gamma(\omega \to \gamma\pi^0)_{\text{exp}}\) and apply it to the \(\omega \to \rho\pi \to 3\pi\) decay. However the resulting rate \(\Gamma(\omega \to \rho\pi \to 3\pi)_{\text{th}} \simeq 5\) MeV fails to reproduce the measured one,\cite{16} \(\Gamma(\omega \to 3\pi)_{\text{exp}} = 7.57 \pm 0.09\) MeV. It suggests that some extra contribution(s) are needed, because the contribution of \(\rho\) meson pole is sizable but insufficient, i.e., the mechanism of the \(\omega \to 3\pi\) decay and hence that of the \(X(3872) \to \omega J/\psi \to \pi^+\pi^-\pi^0J/\psi\) are still uncertain and not simple. For this reason, we have considered the \(X(3872) \to \gamma J/\psi\) decay in place of the \(X(3872) \to \pi^+\pi^-\pi^0J/\psi\) in Ref. 22). Although the measured ratio\cite{17,23} of the rates \(\Gamma(X(3872) \to \gamma J/\psi)/\Gamma(X(3872) \to \pi^+\pi^-J/\psi)\) is less than unity against the well-known hierarchy of hadron interactions,\cite{4} \(|\text{isospin conserving int. (} \sim O(1)\)| \(\gg\)
given by $X$ scheme, hidden-charm iso-singlet axial-vector tetra-quark mesons with
Here the subscripts 1, 2, and 3 have been given by $X$. If the numerical results in Eqs. (2) and (4) were literally accepted, $X(3872)$ would be different states. However, it is unnatural to assign $X(3872)$ were a charmonium, the ratio of decay rates under consideration could not be reproduced, because such an enhancement cannot work in this case.

Analyses in the $D^0 D^{*0} + c.c.$ ($\rightarrow D^0 D^{*0}$ and $D^0 D^{0,\gamma}$) channels also have reported observations$^{25}$ of $X(3875)$. Recent results$^{26}$ on its mass and width are

$$m_{X(3875)} = 3872.6^{+0.6+0.4}_{-0.4-0.5} \text{ MeV and } \Gamma_{X(3875)} = 3.9^{+2.8+0.2}_{-1.4-1.1} \text{ MeV.} \quad (4)$$

If the numerical results in Eqs. (2) and (4) were literally accepted, $X(3875)$ and $X(3872)$ would be different states. However, it is unnatural to assign $X(3875)$ and $X(3872)$ to different states as will be discussed later, and therefore, we here presume that the narrow $X(3875)$ and $X(3872)$ are identical. In this case, the averaged ratio of rates for the $X(3872) = X(3875) \rightarrow D^0 D^{*0} + c.c.$ decay to the $X(3872) \rightarrow \pi^+ \pi^- J/\psi$ has been given by$^{20}$

$$\frac{\Gamma(X(3872) \rightarrow D^0 D^{*0} + c.c.)}{\Gamma(X(3872) \rightarrow \pi^+ \pi^- J/\psi)} = 9.5 \pm 3.1. \quad (5)$$

Now, we study possible decay modes of $X(3872)$ and its partners. In the present scheme, hidden-charm iso-singlet axial-vector tetra-quark mesons with $C = \pm$ are given by $X(\pm) = \{X_u(\pm) + X_d(\pm)\}/\sqrt{2}$, where $X_u(\pm)$ and $X_d(\pm)$ are provided by

$$X_u(\pm) = \frac{1}{2\sqrt{2}} \left\{ (cu)^{1_s}_{3_c}(\bar{c}\bar{u})^{3_s}_{3_c} \pm (cu)^{3_s}_{3_c}(\bar{c}\bar{u})^{1_s}_{3_c} \right\}_{1_c}^{3_s}, \quad (6)$$

$$X_d(\pm) = \frac{1}{2\sqrt{2}} \left\{ (cd)^{1_s}_{3_c}(\bar{c}\bar{d})^{3_s}_{3_c} \pm (cd)^{3_s}_{3_c}(\bar{c}\bar{d})^{1_s}_{3_c} \right\}_{1_c}^{3_s}. \quad (7)$$

Here the subscripts $1_c, 3_c, 3_c$ denote the color multiplets, and the superscripts $1_s$ and $3_s$ the spin multiplets. The above $X_u(\pm)$ can be decomposed as

$$X_u(+) = \frac{1}{2} \sqrt{\frac{1}{6}} \left\{ \sqrt{2}(c\bar{c})^{3_s}_{1_c}(u\bar{u})^{3_s}_{1_c} + (c\bar{c})^{3_s}_{1_c}(u\bar{u})^{1_s}_{1_c} + (c\bar{c})^{1_s}_{1_c}(u\bar{u})^{3_s}_{1_c} \right\}_{1_c}^{3_s} + \cdots$$

$$-\frac{1}{2} \sqrt{\frac{1}{6}} \left\{ (u\bar{c})^{1_s}_{1_c}(\bar{c}\bar{u})^{3_s}_{1_c} + (u\bar{c})^{3_s}_{1_c}(\bar{c}\bar{u})^{1_s}_{1_c} + \sqrt{2}(u\bar{u})^{3_s}_{1_c}(c\bar{c})^{3_s}_{1_c} \right\}_{1_c}^{3_s} + \cdots, \quad (8)$$

$$X_u(-) = \frac{1}{2} \sqrt{\frac{1}{6}} \left\{ (c\bar{c})^{1_s}_{1_c}(u\bar{u})^{3_s}_{1_c} + (c\bar{c})^{3_s}_{1_c}(u\bar{u})^{1_s}_{1_c} + \sqrt{2}(u\bar{u})^{3_s}_{1_c}(c\bar{c})^{3_s}_{1_c} \right\}_{1_c}^{3_s} + \cdots$$

$$-\frac{1}{2} \sqrt{\frac{1}{6}} \left\{ (u\bar{c})^{1_s}_{1_c}(\bar{c}\bar{u})^{3_s}_{1_c} + (u\bar{c})^{3_s}_{1_c}(\bar{c}\bar{u})^{1_s}_{1_c} + \sqrt{2}(u\bar{u})^{3_s}_{1_c}(c\bar{c})^{3_s}_{1_c} \right\}_{1_c}^{3_s} + \cdots. \quad (9)$$

where $\cdots$ denotes a color-singlet sum of products of color-octet $\{q\bar{q}\}$ pairs. Decompositions of $X_d(\pm)$ are obtained by replacing $u$ by $d$ in the above equations. Replacement of the color singlet $\{q\bar{q}\}$ pairs, $\{u\bar{u} - d\bar{d}\}^{1_s}_{1_c}/\sqrt{2}$, $\{u\bar{u} + d\bar{d}\}^{1_s}_{1_c}/\sqrt{2}$, $(c\bar{c})^{1_s}_{1_c}$, etc. by the ordinary $\pi^0, \eta_0, \eta_c, \ldots$, respectively, leads to

$$X(+) = \frac{1}{4} \sqrt{\frac{1}{3}} \left\{ 2(J/\psi \omega - \omega J/\psi) + (D^0 D^{*0} - \bar{D}^{*0} D^0) + (D^{*0} D^0 - \bar{D}^0 D^{*0}) \right\} \quad (10)$$
Their iso-triplet neutral partners \( X_0^0(\pm) \) also can be decomposed as

\[
X_0^0(+) = \frac{1}{4} \sqrt{\frac{2}{3}} \left\{ (2 J/\psi \rho^0 - \rho^0 J/\psi) + (D^0 \bar{D}^{*0} - \bar{D}^{*0} D^0) + (\bar{D}^{0*} D^0 - D^{0*} \bar{D}^0) \right. \\
- (D^+ \bar{D}^{*-} - D^{-*} D^*) - (D^{*-} D^+ - D^{-*} D^*) \left\} + \cdots, 
\]

\[
X_0^0(-) = \frac{1}{4} \sqrt{\frac{2}{3}} \left\{ (\eta_c \omega - \omega \eta_c) + (J/\psi \eta_0 - \eta_0 J/\psi) \\
+ (D^{0*} \bar{D}^{*-} - \bar{D}^{*-0} D^{0*}) + (D^{*-0} D^{0*} - D^{*-0} \bar{D}^{0*}) \right\} + \cdots. 
\]

From Eqs. (10) – (13), we can see (i) \( X(+) \), to which \( X(3872) \) is assigned, couples to \( \omega J/\psi \) and \( D^0 \bar{D}^{0*} + c.c. \) therefore, it can decay into \( \pi^+ \pi^- \pi^0 \) or \( \pi^+ \pi^- J/\psi \) through \( \omega \) pole (with the \( \omega \rho^0 \) mixing) and into \( D^0 \bar{D}^{0*} \) or \( \gamma \) through \( D^0 \bar{D}^{0*} + c.c. \), as have been observed. It is also seen that rates for these decays are small because of small overlap of flavor, color and spin wave functions as the \( \hat{F}^+ = D^+_{s0}(2317) \rightarrow D^+_s \pi^0 \) decay, isospin non-conservation in the \( \pi^+ \pi^- J/\psi \) decay and very small phase space volumes in the \( D^0 \bar{D}^{0*} \) decay through the \( D^0 \bar{D}^{0*} + c.c. \) and the \( \pi^+ \pi^- \pi^0 J/\psi \) decay through the \( \omega J/\psi \). (ii) Its opposite \( C \)-parity partner \( X(-) \) couples to \( J/\psi \eta_0, \eta_c \omega \) and \( D^* \bar{D}^* \). Nevertheless, the threshold of \( D^* \bar{D}^* \) decay is beyond \( m_{X(-)} \), and hence probably beyond \( m_{X(3872)} \). Therefore, \( X(+) \) might be observed in the \( \eta J/\psi \) channel. (iii) The iso-triplet partners \( X_1(+) \) of \( X(3872) \) couple to \( J/\psi \rho^0 \) \( D^0 \bar{D}^* \) (and \( \bar{D} \bar{D}^* \)). Therefore, one might identify \( X_0^0(+) \) with \( X(3875) \) observed in the \( D^0 \bar{D}^{0*} + c.c. \) channel. However, it should be noted that the rate for the isospin conserving \( X_0^0(+) \) \( \rightarrow \rho^0 J/\psi \rightarrow \pi^+ \pi^- J/\psi \) decay will be much larger than that for the isospin non-conserving \( X(3872) \rightarrow \omega J/\psi \rightarrow \rho^0 J/\psi \rightarrow \pi^+ \pi^- J/\psi \), as will be explicitly seen later. Therefore, \( X_1(+) \) can be broad, and hence it seems to be unnatural to assign the narrow \( X(3875) \) to the hypothetically broad \( X_1(+) \), as noted before. (iv) The iso-triplet partners \( X_1(-) \) with negative \( C \)-parity couple to \( \pi J/\psi \) and \( \eta_c \rho \). If the spatial wave function of \( X_1(-) \) is not very much different from that of \( X_1(+) \), the rate for the \( X_1(-) \) would be much larger than that for the \( X_0^0(+) \) because of much larger phase space volume. 

We here study the rate for the \( X_0^0(+) \rightarrow \rho^0 J/\psi \rightarrow \pi^+ \pi^- J/\psi \) decay to see why \( X_0^0(+) \) has not been observed. In Eq. (22) of Ref. 22, we have calculated the rate for the \( X(3872) \rightarrow \omega J/\psi \rightarrow \rho^0 J/\psi \rightarrow \pi^+ \pi^- J/\psi \) decay with the \( \omega \rho^0 \) mixing. The rate for the above decay of \( X_0^0(+) \) can be obtained by replacing \( X(3872) \) by \( X_0^0(+) \) and eliminating the contribution of \( \omega \) pole with the \( \omega \rho^0 \) mixing in the equation. Taking \( m_{X_1(+)} \approx m_{X(3872)} \) (because both of them consist of the same quarks and their flavor wave functions are of the same type, as in the case of \( \hat{F}^+_I = D^+_s(2317) \) and \( \hat{F}_0^+ \) which have been observed as signal and indication, respectively, at the same
mass in \( B \) decays\(^9\) as discussed before), we get the ratio of rates
\[
\frac{\Gamma(X^0_I(+) \to \rho^0 J/\psi \to \pi^+\pi^- J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^- J/\psi)} \sim 200, \tag{14}
\]
where it has been assumed that the size of the \( X^0_I(+)\rho^0 J/\psi \) coupling is approximately equal to that of the \( X(+)\omega J/\psi \), because the spatial wave functions of \( X_I(+) \) and \( X(+) \) are expected to be not very much different from each other, as in the case\(^4\) of \( \hat{F}_I^+ \) and \( \hat{F}_0^+ \).

To estimate the denominator of Eq. (14), we assume that the full width of \( X(3872) \) is approximately saturated as
\[
\Gamma_{X(3872)} \simeq \Gamma(X(3872) \to \pi^+\pi^- J/\psi) + \Gamma(X(3872) \to \pi^+\pi^- 0 J/\psi) + \Gamma(X(3872) \to D^0\bar{D}^{*0} + \text{c.c.}). \tag{15}
\]
By using Eqs. (3) and (5), \( \Gamma(X(3872) \to \pi^+\pi^- J/\psi) \) can be given by \( \Gamma_{X(3872)} \). We have listed two different values, \( \Gamma_{X(3872)} \) in Eq. (2) and \( \Gamma_{X(3875)} \) in Eq. (4), where \( X(3872) \) and \( X(3875) \) are now identified. The latter is consistent with the measured width\(^11\) 2.5 ± 0.5 MeV of \( X(3872) \). However, this is narrower than the experimental energy resolution, and therefore, some corrections might be needed. Such a corrected width has been given in Eq. (2).

Taking \( \Gamma_{X(3875)} \) in Eq. (4), as an example, we obtain \( 0.1 \lesssim \Gamma(X(3872) \to \pi^+\pi^- J/\psi) \lesssim 0.9 \) MeV, and therefore,
\[
\Gamma(X^0_I(+) \to \rho^0 J/\psi \to \pi^+\pi^- J/\psi) \sim (20 - 200) \text{ MeV}. \tag{16}
\]

The above rate is much larger than that for the near threshold \( X_I(+) \to D^0\bar{D}^{*0} + \text{c.c.} \) decay, because \( \Gamma(X_I(+) \to D^0\bar{D}^{*0} + \text{c.c.}) \simeq \Gamma(X(+) \to D^0\bar{D}^{*0} + \text{c.c.}) \) will be obtained, if \( m_{X_I(+) \simeq m_{X(3872)}} \) and \( |g_{X_I(+)D^0\bar{D}^{*0}}| \simeq |g_{X(+)D^0\bar{D}^{*0}}| \) as discussed before. Therefore, the full width of \( X_I(+) \) would be dominated by \( \Gamma(X_I(+) \to \rho J/\psi \to \pi\pi J/\psi) \), and hence \( X_I(+) \) would be much broader than \( X(3872) \). In this case, it is expected that the broad enhancement of the \( \pi^+\pi^- J/\psi \) mass distribution from \( X^0_I(+) \) would be behind the background of the narrow \( X(3872) \) peak, unless the production rates of \( X_I(+) \) are much larger than that of \( X(3872) \). Although the existing search for the charged partner of \( X(3872) \) mentioned before has reported\(^{19}\) no indication of a narrow \( \pi^-\pi^0 J/\psi \) resonance around the mass of \( X(3872) \), this does not necessarily exclude the existence of iso-triplet partners, because their production rate has not been known yet and, in addition, the present statistical accuracy might be insufficient to observe the broad \( X^0_I(+) \) in the \( \pi\pi J/\psi \) mass distribution. On the other hand, if \( \Gamma_{X(3872)} \) in Eq. (2) as another example is taken, a small \( \Gamma_{X_I(+) \to D^0\bar{D}^{*0} + \text{c.c.}} \) could have been observed in the \( \pi\pi J/\psi \) channels. However, the negative result on the search for the \( (\pi\pi)^- J/\psi \) might imply that the true width of \( X(3872) \) is near the upper bound of \( \Gamma_{X(3872)} \) in Eq. (2), and therefore \( X_I(+) \) would be considerably broad, if its production rate is of the same order of magnitude as that of \( X(3872) \). Therefore, more precise determination of intrinsic widths of \( X(3872) \) and its partners in addition to their production rates will provide important information to search for these partners.
In summary we have studied $X(3872)$ and its partners, assigning these axial-vector mesons to \{$(cn)(\bar{c}\bar{n}) \pm (cn)(\bar{c}\bar{n})_I = 1,0$\}. As the results, we have discussed their possible decay modes, and pointed out that the iso-triplet partners of $X(3872)$ can be considerably broad and therefore higher statistics will be needed to find them.

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