Skyrme-Hartree-Fock
plus Tensor Correction for Nuclear Matter

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We study the equation of states of symmetric and pure neutron matter in the Skyrme-Hartree-Fock (SHF) model with tensor corrections. We are aware now that the pion exchange interaction has a significant contribution to nuclear structure in light nuclei. The pion generates a strong tensor interaction between two nucleons, which cannot be treated within the Hartree-Fock framework for the spin-saturated system such as homogeneous nuclear matter. Therefore, we study the role of the tensor interaction based on the SHF model, in which we extend it by explicitly introducing two-particle-two-hole (2p-2h) excitations for the treatment of the tensor interaction in symmetric nuclear matter and pure neutron matter. We are able to describe infinite matter very well using the SHF model with tensor corrections. We also discuss the connection between the symmetry energy and the tensor interaction in this framework.

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§1. Introduction

The Skyrme effective interaction has been studied for a long time, which was first proposed in late 1950. By the work of Vautherin and Brink, the Skyrme interaction was introduced in the Hartree-Fock calculations of finite nuclei. Until now, the Skyrme-Hartree-Fock (SHF) framework has been successfully and widely used for the description of nuclear matter and finite nuclei. There are also some investigations for hypernuclei in the SHF model. During the development of the SHF theory, a lot of parameter sets were proposed for reproducing the properties of nuclear matter and finite nuclei. One of the very early parameterizations is SIII. The SkM parameter sets were then introduced for better description of finite nuclei. To apply the SHF model for neutron stars, further parameter sets are needed to describe not only symmetric nuclear matter (SNM), but also asymmetric nuclear matter with arbitrary proton fraction, just like pure neutron matter (PNM). The Sly230a parameter set was constructed to reproduce the results of the variational method obtained from a microscopic viewpoint by Akmal et al. The Sly230a parameter set could describe both SNM and PNM very well. Also, the SHF model with Sly230a can provide a reasonable description of neutron stars.

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Recently, an attractive model was proposed in Ref. 16). When discussing the properties of SNM, they used the $\delta$ function form for the interaction, and parameterized the coupling constants in the Skyrme form. At the Hartree-Fock level, the energy was expressed in the same form as the one in the SHF model. Furthermore, the second-order correction by two-particle-two-hole (2p-2h) diagrams was included in their model, where the high momentum contribution was cut off by introducing an integration upper limit $\Lambda$. Using this method with new Skyrme parameters, they derived reasonable saturation properties of SNM, and reproduced the equation of state (EOS) obtained with the SkP parameter set in low density region. In the present work, we compare the EOS of Ref. 16) with the result of the variational many-body theory done by Akmal et al.\textsuperscript{13)} which provides a standard EOS for SNM and PNM. However, when the model proposed in Ref. 16) is extended to the study of PNM, the results are not consistent with Akmal’s EOS. Therefore, it is very important to improve this method being able to provide reasonable descriptions of both SNM and PNM.

For this purpose, the tensor interaction of pion should be more effective than the $\delta$ function interaction in the second-order terms. The tensor contribution of pion has been studied by many authors since long time ago. It cannot be treated within the Hartree-Fock approximation in a spin-saturated system such as homogeneous nuclear matter,\textsuperscript{17,18)} but it is known that the tensor interaction plays an important role in SNM.\textsuperscript{19–21)} The tensor interaction has a strong isospin dependence, which leads to obvious difference in the EOS between SNM and PNM. Furthermore, some recent works indicate that the symmetry energy mainly comes from the contribution of the tensor force.\textsuperscript{22,23)} Hence, it is very interesting to study both SNM and PNM in the SHF model with the tensor interaction.

This article is arranged as follows. In §2, we briefly introduce our method for the calculation of EOS. We then present the numerical results in §3. Section 4 is devoted to a summary.

§2. Skyrme-Hartree-Fock model with tensor interaction

The Hamiltonian of our model can be expressed as

$$H = T + V_S + V_T.$$  \hspace{1cm} (2.1)

$T$ is the non-relativistic kinetic energy. $V_S$ is the Skyrme interaction at the Hartree-Fock level. $V_T$ is the tensor interaction of the 2p-2h correction.

We first write all the energy expressions for homogeneous nuclear matter in the SHF framework. The kinetic energy per particle is written as

$$\frac{\langle T \rangle}{A} = \frac{3}{10m_N} \left( \frac{3\pi^2 \rho}{\tau} \right)^{2/3},$$ \hspace{1cm} (2.2)

where $\tau$ is the isospin factor, which is 2 for SNM and 1 for PNM due to the isospin degeneracy. Here, $A$ denotes the number of nucleons and $m_N$ the nucleon mass. We take the $V_S$ composed of central and density-dependent terms of the Skyrme effective
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interactions\textsuperscript{14) as}

\[ V_S(r_1, r_2) = t_0(1 + x_0 P_\sigma)\delta(r) + \frac{1}{6}t_3(1 + x_3 P_\sigma)[\rho(R)]^\alpha\delta(r) , \]

with

\[ r = r_1 - r_2 , \quad R = \frac{1}{2}(r_1 + r_2) , \quad P_\sigma = (1 + \sigma_1 \cdot \sigma_2)/2. \]

Here, we have dropped the terms containing \( t_1, t_2 \) and \( W_0 \) to simplify our model following the work of Ref. 16). \( t_0, x_0, t_3, x_3 \) and \( \alpha \) are the parameters of SHF model, which are determined by reproducing the properties of finite nuclei usually. Some of the parameter sets like SkP can be applied to SNM, while another one like Sly230a works very well for both SNM and PNM.

At the Hartree-Fock level, we consider the above \( \delta \) function interaction with the central and density-dependent terms as adopted in Ref. 16), and add the terms of the spin-dependent operator. This choice is due to the fact that the tensor interaction is handled in terms of the 2p-2h excitation. For the Hartree and Fock parts of the Skyrme interaction, the energy per particle can be expressed as

\[
\frac{\langle V_S \rangle_H}{A} = \frac{1}{2\rho} \sum_{k_1,k_2} \langle k_1 k_2 | V_S | k_1 k_2 \rangle , \quad (|k_1|, |k_2| < k_F) \quad (2.4)
\]

\[
\frac{\langle V_S \rangle_F}{A} = -\frac{1}{2\rho} \sum_{k_1,k_2} \langle k_1 k_2 | V_S | k_2 k_1 \rangle , \quad (|k_1|, |k_2| < k_F) \quad (2.5)
\]

where

\[
\langle k_1 k_2 | V_S | k_1 k_2 \rangle = \int d^3r_1 d^3r_2 e^{-i k_1 \cdot r_1} e^{-i k_2 \cdot r_2} V_S(r_1, r_2) e^{i k_1 \cdot r_1} e^{i k_2 \cdot r_2} . \quad (2.6)
\]

In the sum of spin matrix elements,\textsuperscript{24) different states have to be considered in Hartree and Fock terms, which are

\[
\sum_{s_1, s_2} \langle s_1 s_2 | (1 + x_0 P_\sigma) | s_1 s_2 \rangle = 2x_0 + 4 ,
\]

\[
\sum_{s_1, s_2} \langle s_1 s_2 | (1 + x_0 P_\sigma) | s_2 s_1 \rangle = 4x_0 + 2 , \quad (2.7)
\]

where \( |s_1\rangle \) and \( |s_2\rangle \) are spin wave functions, which are the eigenstates of the spin operator \( \sigma_1 \) and \( \sigma_2 \). There is a similar analysis in the density-dependent term. Thus, the energy per particle of central force at the Hartree-Fock level is written as

\[
\frac{\langle V_S \rangle_{HF}}{A} = \frac{1}{4\rho} t_0 [(2 + x_0)\rho^2 - (2x_0 + 1)(\rho_p^2 + \rho_n^2)]
\]

\[ + \frac{1}{24} t_3 \rho^{\alpha-1} [(2 + x_3)\rho^2 - (2x_3 + 1)(\rho_p^2 + \rho_n^2)] . \quad (2.8)
\]
We now add the contribution of the tensor interaction. This component cannot be handled within the Hartree-Fock approximation in a spin-saturated system, such as homogeneous nuclear matter, due to the fact that the tensor operator involves spin operators of rank two. Hence, for the treatment of the tensor interaction, we have to introduce 2p-2h excitations (Fig. 1). We take into account the tensor interaction from one-pion exchange with a dipole form factor, which takes care of the finite size effect of the nucleon. We assume that other meson contributions to the tensor interaction are included by taking different values of the form factor for simplicity,\(^25\)

\[
V_T(q) = -g_\pi^2 \frac{3(\sigma_1 \cdot q)(\sigma_2 \cdot q) - \sigma_1 \cdot \sigma_2 q^2}{3(q^2 + m_\pi^2)} \left( \frac{A^2 - m_\pi^2}{A^2 + q^2} \right)^2 (\tau_1 \cdot \tau_2) .
\] (2.9)

Then, the direct and exchange terms in the tensor contribution are given by

\[
\frac{\langle V_T \rangle_D}{A} = \frac{1}{2\rho} \sum_{k_1,k_2,q} \frac{\langle k_1 k_2 | V_T | k_1' k_2' \rangle^2}{\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_1+q} - \epsilon_{k_2-q}} ,
\]

\[
(|k_1|, |k_2| < k_F; |k_1 + q|, |k_2 - q| > k_F)
\] (2.10)

\[
\frac{\langle V_T \rangle_E}{A} = -\frac{1}{2\rho} \sum_{k_1,k_2,q} \frac{\langle k_1 k_2 | V_T | k_1' k_2' \rangle \langle k_2' k_1' | V_T^\dagger | k_1 k_2 \rangle}{\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_1+q} - \epsilon_{k_2-q}} ,
\]

\[
(|k_1|, |k_2| < k_F; |k_1 + q|, |k_2 - q| > k_F)
\] (2.11)

where

\[
k_1' = k_1 + q , \quad k_2' = k_2 - q .
\] (2.12)

\(\langle V_T \rangle_E\) has the same form as \(\langle V_T \rangle_D\), while \(q\) is replaced by \(q' = q + k_1 - k_2\), where \(q\) and \(q'\) are the transferred momentum between two nucleons. Similar to the Hartree-Fock treatment, the sum of spin and isospin must be derived first. For the spin part, we obtain

\[
\sum_{s_1,s_2,s'_1,s'_2} \langle s_1 s_2 | T | s'_1 s'_2 \rangle \langle s'_2 s'_1 | T | s_2 s_1 \rangle = \frac{8}{3} q^4 ,
\]

\[
\sum_{s_1,s_2,s'_1,s'_2} \langle s_1 s_2 | T | s'_1 s'_2 \rangle \langle s'_2 s'_1 | T^\dagger | s_1 s_2 \rangle = q^2 q'^2 \left[ 4 \cos^2(q, q') - \frac{4}{3} \right] ,
\] (2.13)
where
\[ T = \frac{1}{3} [3(\sigma_1 \cdot q)(\sigma_2 \cdot q) - \sigma_1 \cdot \sigma_2 q^2], \]

is the tensor operator of spin. For SNM, the sum of isospin is also different in direct and exchange parts for the tensor interaction
\[
\sum_{\chi_1, \chi_2, \chi_1', \chi_2'} \langle \chi_1 \chi_2 | \tau_1 \cdot \tau_2 | \chi_1' \chi_2' \rangle \langle \chi_1' \chi_2' | \tau_1 \cdot \tau_2 | \chi_1 \chi_2 \rangle = 12,
\]
\[
\sum_{\chi_1, \chi_2, \chi_1', \chi_2'} \langle \chi_1 \chi_2 | \tau_1 \cdot \tau_2 | \chi_1' \chi_2' \rangle \langle \chi_1' \chi_2' | \tau_1 \cdot \tau_2 | \chi_1 \chi_2 \rangle = -6,
\]

where \( \chi_1 \) and \( \chi_2 \) are isospin wave functions.

Thus, the energies per particle of SNM are written in the direct term and exchange term as
\[
\frac{\langle V_T \rangle_D}{A} = -\frac{16 m_N g^4}{\rho(2\pi)^9} \int_{|k_1|,|k_2|<k_F,|k_1+q,|k_2-q|>k_F} \frac{d^3k_1 d^3k_2 d^3q}{q^4 + \frac{q^2}{(q^2 + m_\pi^2)^2}} \left( \frac{A^2 - m_\pi^2}{A^2 + q^2} \right)^4,
\]
\[
\frac{\langle V_T \rangle_E}{A} = -\frac{6 m_N g^4}{\rho(2\pi)^9} \int_{|k_1|,|k_2|<k_F,|k_1+q,|k_2-q|>k_F} \frac{d^3k_1 d^3k_2 d^3q d^3q'}{q^2 q'^2 [4 \cos^2(\hat{q}, \hat{q'}) - \frac{4}{3}]} \left( \frac{A^2 - m_\pi^2}{A^2 + q^2} \right)^2 \left( \frac{A^2 - m_\pi^2}{A^2 + q'^2} \right)^2.
\]

Furthermore, the direct part of the tensor interaction can be expressed in an analytic form as
\[
\frac{\langle V_T \rangle_D}{A} = -\frac{2 m_N g^4 k_F^7}{15 \rho \pi^6} \left( \int_0^1 u du F_1(u) + \int_1^\infty u du F_2(u) \right),
\]
where
\[
F_1(u) = \left[ \left( 4 + \frac{15}{2} u - 5 u^3 + \frac{3}{2} u^5 \right) \ln(1 + u) + \left( 4 - \frac{15}{2} u + 5 u^3 - \frac{3}{2} u^5 \right) \ln(1 - u) + 29 u^2 - 3 u^4 - 40 u^2 \ln 2 \right] \frac{u^4}{(u^2 + \mu^2)^2} \left( \frac{\lambda^2 - \mu^2}{\lambda^2 + u^2} \right)^4,
\]
\[
F_2(u) = \left[ (4 - 20 u^2 - 20 u^3 + 4 u^5) \ln(u + 1) + (-4 + 20 u^2 - 20 u^3 + 4 u^5) \ln(u - 1) + 22 u + 4 u^3 + (40 u^3 - 8 u^5) \ln u \right] \frac{u^4}{(u^2 + \mu^2)^2} \left( \frac{\lambda^2 - \mu^2}{\lambda^2 + u^2} \right)^4,
\]

with \( u = q/2k_F, \lambda = \Lambda/2k_F, \mu = m_\pi/2k_F. \) \( k_F \) is the Fermi momentum of nuclear matter. However, for the exchange part of the tensor interaction, the integration is very complicated. After simplifying the angle parts, we take the left five-fold integrations by a numerical method.
For PNM, the total isospin is 1, and therefore the values of these two terms become

$$\frac{\langle V_T \rangle^N_D}{A} = -\frac{4m_N g_A^4}{3\rho(2\pi)^9} \int_{|k_1|, |k_2| < k_F, |k_1+q|, |k_2-q| > k_F} \frac{d^3k_1 d^3k_2 d^3q}{q^2 + q \cdot (k_1 - k_2)}$$

$$\frac{q^4}{(q^2 + m^2_\pi)^2} \left( \frac{A^2 - m^2_\pi}{A^2 + q^2} \right)^4,$$

$$\frac{\langle V_T \rangle^N_E}{A} = -\frac{m_N g_A^4}{\rho(2\pi)^9} \int_{|k_1|, |k_2| < k_F, |k_1+q|, |k_2-q| > k_F} \frac{d^3k_1 d^3k_2 d^3q d^3q'}{q^2 + q' \cdot (k_1 - k_2)} \delta(k_1 - k_2 + q - q')$$

$$\frac{q^2 q'^2 [4 \cos^2(\hat{q}, \hat{q}') - \frac{4}{\pi}]}{(q^2 + m^2_\pi)(q'^2 + m^2_\pi)} \left( \frac{A^2 - m^2_\pi}{A^2 + q^2} \right)^2 \left( \frac{A^2 - m^2_\pi}{A^2 + q'^2} \right)^2.$$  \hspace{1cm} (2.20)

We can see that the tensor contribution of PNM is much smaller than that of SNM.

§3. Numerical results

In Ref. 16), the Hartree-Fock term and the 2p-2h correction with δ function interactions were taken into account in their calculation. For SNM, they could reproduce the EOS obtained with the SkP parameter set in the low density region.
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Fig. 3. The EOS for the parameter sets PS1, PS2 and PS3 in the present model. Akmal’s results given in Ref. 13) are denoted by squares for comparison. The upper panel is for SNM, while the lower panel for PNM.

and also the EOS obtained by Akmal et al. using a variational method in the entire density region by adjusting the form factor $\Lambda$.\textsuperscript{13}) The EOS of SNM are compared in the upper panel of Fig. 2, while the EOS of PNM are shown in the lower panel of Fig. 2. It is seen that the simple model given in Ref. 16) provides a softer EOS of PNM than the one given by Akmal et al. in Ref. 13). The symmetry energy in the model of Ref. 16) is around 27 MeV, which is smaller than the experimental value ($30 \sim 35$ MeV).\textsuperscript{26}) Comparing with the EOS of Akmal, the model of Ref. 16) cannot provide similar EOS for both SNM and PNM at the same time. If the model of Ref. 16) is applied to a neutron-rich matter such as neutron star matter, it cannot provide reasonable results.

For the EOS of PNM, the spin-dependent terms containing $x_0$ and $x_3$ play very important roles, which do not give any contribution to the energy of SNM.\textsuperscript{14}) Furthermore, as discussed in §2, we consider the contribution of the tensor part from one-pion exchange potential instead of the $\delta$ interaction for the 2p-2h correction used in Ref. 16). In this case, only one parameter is adopted to control the energy coming from the tensor interaction, which is the cutoff momentum $\Lambda$. We first fix the value of $\Lambda$ to be 1.0 GeV, which comes from the Bonn A potential,\textsuperscript{27}) and then determine the parameter set PS1 by fitting the EOS of both SNM and PNM obtained in the variational method.\textsuperscript{13}) For comparison, the parameter sets PS2 and PS3 with different cutoff momentum $\Lambda$ are also fitted to Akmal’s EOS. All of the parameter sets mentioned above are listed in Table I, and the corresponding EOS are plotted
Table I. Three parameter sets PS1, PS2 and PS3 of the SHF model with tensor force and the corresponding properties of nuclear matter. The last column $E_T$ is the contribution of the tensor interaction at the saturation point.

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>$t_0$ (MeV$^{-1}$)</th>
<th>$t_3$ (MeV$^{-1}$fm$^{-3}$)</th>
<th>$x_0$</th>
<th>$x_3$</th>
<th>$\alpha$</th>
<th>$\rho_0$ (fm$^{-3}$)</th>
<th>$E/A(\rho_0)$ (MeV)</th>
<th>$K$ (MeV)</th>
<th>$E_{\text{sym}}$ (MeV)</th>
<th>$E_T$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS1</td>
<td>1000</td>
<td>−26</td>
<td>4812</td>
<td>−40.0</td>
<td>−1.40</td>
<td>0.67</td>
<td>0.100</td>
<td>−16.00</td>
<td>251.1</td>
<td>32.58</td>
</tr>
<tr>
<td>PS2</td>
<td>850</td>
<td>−364</td>
<td>6105</td>
<td>−1.60</td>
<td>−0.71</td>
<td>0.67</td>
<td>0.100</td>
<td>−16.00</td>
<td>251.5</td>
<td>34.23</td>
</tr>
<tr>
<td>PS3</td>
<td>650</td>
<td>−913</td>
<td>8153</td>
<td>−0.40</td>
<td>−0.55</td>
<td>0.50</td>
<td>0.100</td>
<td>−16.00</td>
<td>237.4</td>
<td>33.60</td>
</tr>
</tbody>
</table>

Fig. 4. Contributions of the kinetic energy $E_k$, the SHF energy $E_{\text{SHF}}$ and the tensor interaction $E_T$ with the PS2 parameter set. The upper panel is for SNM, while the lower panel for PNM.

in Fig. 3. We determine the parameters in the present model by reproducing the saturation properties, the energy per particle $E/A = −16$ MeV and the saturation density $\rho_0 = 0.16$ fm$^{-3}$, as shown in Table I. We are able to reproduce the EOS of Akmal for both SNM and PNM as shown in Fig. 3. The incompressibility $K$ comes out to be in a reasonable range 230 $\sim$ 250 MeV. The symmetry energy $E_{\text{sym}}$ has the value around 30 $\sim$ 35 MeV. The energy contribution of the tensor interaction $E_T$ in SNM at the saturation point is listed in the last column of Table I. The values of all the SHF parameters are changed largely by changing the cutoff parameter $\Lambda$ in PS1, PS2 and PS3.

In Fig. 4, we show various contributions from the kinetic energy $E_k$, the SHF energy $E_{\text{SHF}}$ and the tensor interaction $E_T$ with the PS2 parameter set. For SNM, the tensor contribution is very large, while for PNM it is almost zero through the whole density region. The tensor contributions obtained for the other parameter sets, PS1 and PS3, are also very small in PNM. The most important reason for
the difference between SNM and PNM is that the tensor interaction is strongly isospin dependent, which is mentioned in §2. Not only the absolute values, but also the signs of the direct and exchange energies of the tensor interaction are largely different between SNM and PNM. In SNM, these two parts are both negative, while in PNM they are opposite in sign. Therefore, for all the densities the contribution of the direct term in PNM is almost canceled by the exchange term to make the tensor contribution very small. In Fig. 5, we show the dependence of the tensor contribution on the cutoff momentum $\Lambda$ in SNM. We compare the results obtained with the parameter sets PS1, PS2 and PS3, which have different cutoff momenta $\Lambda = 1.0$, 0.85 and 0.65 GeV as shown in Table I. It is seen that a larger $\Lambda$ provides a larger tensor contribution in SNM.

Recently, some authors investigated the relation between the tensor interaction and the symmetry energy.\(^{22},^{23}\) They claimed that the main contribution to $E_{\text{sym}}$ is due to the tensor force, which has been shown in Table IV of Ref. 23). It is easy to imagine that due to the strong isospin dependence the tensor contribution is very large in the difference of the total energies between SNM and PNM. In our work, we also show the symmetry energy and the tensor contribution of the three parameter sets through the whole density region in Fig. 6. We found that the tensor contribution derived from PS2 is in good agreement with the symmetry energy.
§4. Conclusion

We have studied a Skyrme-Hartree-Fock (SHF) model with tensor force. In the present work, we have taken the Skyrme interaction at the Hartree-Fock level, which contains the central and density-dependent terms with spin-exchange operators. The tensor interaction cannot be treated within the Hartree-Fock approximation in a spin-saturated system, such as homogeneous nuclear matter. Hence, we have added the tensor interaction explicitly in the form of one-pion exchange interaction and treated the tensor interaction by the 2p-2h diagrams. In the SHF interaction, there are two parameters, $x_0$ and $x_3$, which are very important to reproduce the EOS of both SNM and PNM, because the tensor contribution is essentially zero in PNM. When $Y_p = 0.5$ for SNM, the terms with these two parameters vanish automatically. So they can only affect the EOS of PNM. The tensor term includes one important parameter in the form factor, the cutoff momentum $\Lambda$, in order to simulate the contributions of the tensor interaction by the one-pion exchange form. The form factor $\Lambda$ directly determines the contribution of the 2p-2h diagrams. Therefore, we take $\Lambda \sim 1.0, 0.85$ and $0.65$ GeV to obtain three parameter sets PS1, PS2 and PS3. We also compared the symmetry energy with different strength of tensor interaction in the three parameter sets. There is one group of parameters which is in good agreement with the symmetry energy.

With the present model, it is possible to calculate asymmetric nuclear matter with arbitrary proton fraction, and discuss the properties of neutron stars. This will
be our further work. We are able to apply this model for finite nuclei by working out the direct and exchange 2p-2h diagrams in the angular momentum basis.

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