Adaptive water distribution networks with dynamically reconfigurable topology

Robert Wright, Ivan Stoianov, Panos Parpas, Kevin Henderson and John King

ABSTRACT

This paper presents a novel concept of adaptive water distribution networks with dynamically reconfigurable topology for optimal pressure control, leakage management and improved system resilience. The implementation of District Meter Areas (DMAs) has greatly assisted water utilities in reducing leakage. DMAs segregate water networks into small areas, the flow in and out of each area is monitored and thresholds are derived from the minimum night flow to trigger the leak localization.

A major drawback of the DMA approach is the reduced redundancy in network connectivity which has a severe impact on network resilience, incident management and water quality deterioration. The presented approach for adaptively reconfigurable networks integrates the benefits of DMAs for managing leakage with the advantages of large-scale looped networks for increased redundancy in connectivity, reliability and resilience. Self-powered multi-function network controllers are designed and integrated with novel telemetry tools for high-speed time-synchronized monitoring of the dynamic hydraulic conditions. A computationally efficient and robust optimization method based on sequential convex programming is developed and applied for the dynamic topology reconfiguration and pressure control of water distribution networks. An investigation is carried out using an operational network to evaluate the implementation and benefits of the proposed method.

Key words | district metered areas, dynamic configurability, leakage, network topology, resilience, water distribution networks

PRINCIPAL NOTATION

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<td>$A_{11}$</td>
<td>Diagonal matrix containing headloss formula parameters</td>
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<td>$A_{12}$</td>
<td>Unknown head node incidence matrix</td>
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<td>$A_{13}$</td>
<td>Control valve incidence matrix</td>
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<td>AZP</td>
<td>Average zone pressure [mH$_2$O]</td>
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<td>$\eta$</td>
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<td>$H$</td>
<td>Variable nodal heads [mH$_2$O]</td>
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<td>$i$</td>
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<td>$I$</td>
<td>Identity matrix</td>
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<td>$j$</td>
<td>Unknown head node index</td>
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<tr>
<td>$\lambda$</td>
<td>Lagrangian multiplier</td>
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<tr>
<td>$\mu$</td>
<td>KKT multiplier</td>
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<td>MFNC</td>
<td>Self-powered multifunction network controller</td>
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**INTRODUCTION**

The operational practice of sub-dividing water supply networks into small discrete areas, District Metered Areas (DMAs), has been successfully implemented by the UK water industry to reduce leakage in excess of 30% in the last 25 years (Ofwat 2013). A DMA has a fixed network topology with permanent boundaries and it includes between 1,000 and 3,000 customer connections.

The DMA network topology provides two main benefits:

1. A transparent approach to derive mass balance and non-revenue estimation as the flow into each DMA is continuously monitored. Consequently, the variability in the minimum night flow is used to: (i) substantially reduce the time for leakage detection; (ii) identify areas with a rising level of water losses so that surveys for active leakage control are carried out and repair activities prioritized; (iii) detect unreported bursts; and (iv) minimize non-revenue water. The stochastic nature of customers’ demand significantly hinders the successful execution of these tasks outside hours of minimum (night) water demand.

2. A simplistic implementation of pressure management. Pressure is a critical variable which impacts leakage and burst frequency (Germanopoulos & Jowitt 1989). Maintaining the operational pressure in a water distribution system close to the minimum pressure threshold of 1.5 bar (pressure head of 15 mH2O) defined by Ofwat – the economic regulator of the water and sewerage sectors in England and Wales – provides significant benefits both in terms of reducing water losses and bursts frequency. The control process needs to be implemented with a high degree of confidence to guarantee the required quality of service (DG3: Supply Interruptions; Ofwat DG3 2014) and minimize the risk of customers’ complaints.

Although DMAs have been successfully applied for reducing non-revenue water and managing leakage in the UK, their implementation has introduced major operational constraints which affect the quality of service. By closing boundary valves to form small metered areas, the natural redundancy of supply within these large looped networks is severely reduced; thus causing unintended consequences which are summarized as follows:

- Reduced redundancy in network connectivity and supply substantially affects the resilience of a network and thus the ability of a water utility to provide and maintain an acceptable level of service in the occurrence of bursts, failures and other deviations from normal operation. Consequently, manual valve operations are required during failures in order to connect alternative supply routes. This reactive operational approach not only fails to provide effective incident management but it frequently exacerbates a problem as the fast operation of valves causes discoloration events and bursts. Furthermore, the manual operation of valves increases the risk of breaching DMA boundaries through left open or leaking valves.

- An additional concern following the reduced redundancy in supply caused by the implementation of DMAs is the failure to provide the required pressure and fire flows.

- DMA networks often induce deterioration in water quality by introducing substantial spatial and temporal...
variations in flow velocity and residence time with the high number of dead-end branch pipes. An increase in discoloration complaints and operational challenges in dosing and maintaining residual chlorine may ensue.

- DMA networks can frequently cause higher average zone pressure (AZP) and pressure variability in comparison with a larger pressure zone with multiple feeds. This is because frictional energy losses are smaller in a network that makes use of its inherent redundancy. For example, supplying a bulk demand of 8 L/s via two identical pipe routes each carrying 4 L/s produces smaller frictional losses than one of these supply routes carrying the full 8 L/s; thus affecting both the energy use of a system and the resilience of supply. While various pressure and flow modulation techniques provide an optimal pressure control for a critical point (CP), a large proportion of a DMA might experience significant pressure variability. This can lead to pipe fatigue and more long-term network failures, a phenomenon that has been observed in the oil and gas industry (Hrabovský 2009).

The focus upon reducing water loss by the UK water industry has concealed the detailed consideration of these operational constraints imposed by the topology of DMAs. However, regulatory requirements and the recently introduced service incentive mechanism by Ofwat (Ofwat SIM 2013) are forcing utilities to improve and carefully manage the level of service they provide. Consequently, the water companies are required to make significant changes with regard to multiple performance indicators such as interruptions of supply, low pressure and discoloration complaints, and energy use. These indicators are reaching a point whereby the drawbacks of the current DMA approach considerably affect their performance and even become a barrier to achieve improvements. It is therefore critical that the management of leakage is not dealt with in isolation.

The motivation of this paper is to demonstrate an approach for a holistic management of water supply networks that maintains the successful application of DMAs for managing and reducing leakage while improving network resilience and quality of service by minimizing the unintended consequences and drawbacks of current DMAs. The novel approach (DMAv2O.net) includes the adaptive sectorization of water distribution systems with dynamically reconfigurable network topology for optimal pressure control, leakage management and improved system resilience.

ADAPTIVE WATER DISTRIBUTION SYSTEMS WITH DYNAMICALLY RECONFIGURABLE NETWORK TOPOLOGY

The proposed DMAv2O.net approach for the adaptive sectorization of water distribution systems with dynamically reconfigurable network topology includes the replacement of closed boundary valves with self-powered multifunction network controllers (MFNCs) that modify the network topology and continuously monitor the dynamic hydraulic conditions (Figure 1). The original DMA topology is preserved during periods of low demand (i.e. 2 am–4 am) to capture the minimum night flow within small discrete areas and maximize the management of leakage. The DMAs are then aggregated (i.e. 4 am–2 am) into larger pressure controlled zones to maximize redundancy, reliability and resilience of supply while at the same time achieving improvements in pressure management due to smaller energy losses occurring in networks with additional connectivity.

The operation of a network includes the normal diurnal cycle of segregating and aggregating a set of DMAs. A variety of objectives can be considered during the design and operation of such a water supply system. These objectives initially include pressure control and resilience and can be extended to incorporate water quality and incident management. Furthermore, the DMAv2O.net approach facilitates the formation of even smaller DMAs/zones/sectors which will drive extra reductions in leakage without affecting the quality of service during peak demand.

Control options for dynamically reconfigurable DMAs

Implementing a robust form of control for the operation of water distribution networks with adaptive topologies is a challenging problem due to the large scale of the system, poor understanding of the dynamic hydraulic conditions, and uncertainties in the hydraulic models.
The current and emerging forms of pressure control in water supply networks are summarized as follows (Bermad; Cla-Val Ltd 2013a, b, c; Dorot; i2O Water Ltd; Palmer Environmental; Technolog):

- **Open-loop controllers:**
  - fixed outlet pressure valve;
  - time varied fixed outlet pressuresettings: the outlet pressure is adjusted according to a pre-set daily or weekly time profile.

- **Feedback controllers:**
  - hydraulically derived flow modulation curves: the outlet pressure of a valve alters according to a locally measured flow and a lookup table that has been derived from a hydraulic model;
  - statistically derived flow modulation curves: the outlet pressure of a valve alters according to a locally measured flow and a lookup table that is statistically derived based on hydraulic data acquired from the CP of the network and calculated energy losses;
  - near real-time feedback control: the outlet pressure is adjusted according to feedback (∼30–60 min) from a CP;
  - model predictive control (MPC): the hydraulic status of a system and valve settings are estimated periodically for a finite time-horizon (e.g. 24 hours) and validated for pre-defined time intervals (e.g. 30 min).

These control options are most commonly applied to single-feed pressure managed zones. The implementation of multi-feed pressure managed zones is generally avoided or undertaken on a trial and error basis without a rigorous modelling and control framework that takes into account the steady and unsteady hydraulic behaviour of the system and control elements.

In the proposed DMAv2O.net approach (see below under ‘Enabling technology’), the MFNC facilitates the use and adaptive implementation of more advanced forms of feedback control as well as MPC. It also addresses the challenges of dynamically changing the topology of a network and operating multi-feed pressure managed zones by utilizing the following (Figure 2):

- A higher number of telemetry and remote control points.
- Continuous high-resolution pressure data to identify steady, pseudo-steady and unsteady-state hydraulic conditions (instabilities) which can either lead to sub-optimal control or could result from the control implementation. Optimal valve control is facilitated by the existence of steady and pseudo-steady hydraulic conditions.
The operation of MFNC as either position or pressure control valves with the capacity to follow either an open-loop profile or use feedback control from local measurements (flow and/or pressure). In this way, different control configurations and models can be implemented based on the specific hydraulic and topological conditions. For example, certain control valves could operate as master controllers and use system feedback to account for unpredicted changes in customer demand while the remainder of the control valves operate as slave controllers with a fixed position (Figure 1(b)). Other control configurations can also be accommodated with the functionalities of the MFNC. High-resolution pressure data ensure the continuous monitoring for instabilities and valve interactions (see below under ‘Enabling technology’) with the appropriate fail safe mechanisms (Figure 2).

- Fail-safe control settings that are established for each network controller, and are defined as a network configuration previously used by the water company that has been proven to be robust and safe. This may not include open boundaries, but is dependent on the type of failure that occurred. For example, sectorization would be an appropriate mitigation action in response to a water quality incident, whereas open pressure zones might be a more suitable response to a hydraulic failure resulting in low pressure. Detected hydraulic instabilities are analysed to identify the source.

- Retrospective snippets of high-resolution time-synchronized data retrieved upon request to facilitate a root-cause analysis and the prompt diagnosis of valves operation and control instabilities (Stoianov & Hoskins 2012).
Incident optimization initiated by the detection of network failures and deviations from expected hydraulic conditions.

In this paper, a novel optimization method based on sequential convex programming (SCP) has been developed that facilitates the calculation of optimized outlet pressure profiles for a specified time horizon for a set of network controllers. Such an approach can be used as an open loop control strategy or within a MPC framework (Figure 2). As indicated, the operation of multi-feed pressure managed zones is neither well understood nor commonly implemented. There is also uncertainty with regard to the optimal configuration setup for automatic control valves (e.g. master/slave valves or independently pressure controlled valves) and the likelihood of interactions and instabilities. This is difficult to address in a purely analytical way and extensive experimental programmes are being carried out by the authors in order to investigate and validate control strategies. The following section describes one of these experimental programmes which is used to explore the concept of dynamic topologies, assess the performance of novel instrumentation and facilitate the development of the optimization algorithm presented in the paper.

**EXPERIMENTAL PROGRAMME**

The operational performance and scalability of the proposed method for dynamically reconfigurable topology is being experimentally evaluated on an operational water distribution network. The purpose of this experimental programme is to:

- test the developed sensing and control technologies;
- establish the benefits and evaluate challenges with the implementation and operation of dynamically reconfigurable DMAs;
- validate analytical optimization and control approaches with experimental data and the operational constraints of real life networks;
- assess the scalability of the proposed network management method.

The initial system consisted of two single-feed DMAs that were separated by three closed boundary valves as shown in Figure 3. Several critical customers are located in the study area, including two hospitals and large industrial customers. There is also a large variation in elevation of approximately 80 m across the DMAs.

The selected network was identified as an area that required both improvements in pressure management and the security of supply for the critical customers. Extensive pressure and flow data were collected to assess the hydraulic conditions (both steady and unsteady state) of the network and calibrate a hydraulic model, which was then used to investigate a suitable design solution that provided significant improvements in pressure management and resilience to failures. Following the initial investigation, four MFNCs were installed (Figure 4).
The experimental programme has gone through multiple stages of operation in order to rigorously assess the outlined objectives. These programme stages include the following:

1. Fixed outlet PRVs in conjunction with a closed DMA boundary.
2. Flow modulation PRVs in conjunction with a closed DMA boundary.
3. Dynamic network topologies with a combination of fixed position and flow modulation control valves (i.e. a master/slave arrangement).

The experimental programme has now entered stage 3, where the MFNCs at the boundary open daily and close during the night for leakage monitoring. Its operation is currently being closely analysed both in terms of performance (i.e. pressure management and resilience) and stability of control using hydraulic data captured by the developed instrumentation. These data will be made available once the trial has been running for a conclusive amount of time.

The valve settings used in stages 2 and 3 of the implementation schedule have been calculated using the optimization method outlined in the next section. The development of the proposed optimization method included extensive testing of various optimization approaches in order to find a method that shows robustness, scalability and computational efficiency.

**Enabling technology: self-powered multi-function network controllers**

A key component in the proposed method for dynamically reconfiguring the topology of a network is the monitoring and control system. The developed multi-function network controllers integrate a globe valve with a variety of retrofitted components, as shown in Figure 5(a), including the following:

- A pilot operated diaphragm valve (Cla-Val Series 99-01) and a pressure controller or a position controller with inlet/outlet pressure sensors, remote control/telemetry and a low-power motorized needle valve.
- A vortex flow meter inserted into the globe valve (Cla-Val e-FlowMeter).
- An energy harvester (Cla-Val e-Power MP) for powering the continuous operation of the described control and sensing components. Energy is harvested using a micro turbine installed on a bypass. A pressure differential across the valve of 6 mH2O drives the bypass flow generating 390 mW of power at 6 Vdc.
Continuous measurements of the dynamic pressure (Infrasense TS).

The increased level of control associated with dynamically reconfigurable DMAs could increase the risk of hydraulic instabilities and consequently pipe failures. As a result, novel instrumentation technologies for continuous high-resolution time-synchronized monitoring of the dynamic hydraulic conditions have been used to detect the presence of and determine the sources of pressure instabilities (Figures 5(b) and 6). The InfraSense TS continuously measures pressure (128 S/s, time synchronization of 5 ms) and performs signal processing to determine the occurrence, magnitude and sources of instabilities. Acquired raw and processed data and classified events are stored on the multifunction network controller over a period of 6 months in a rotating memory buffer thus allowing the execution of retrospective queries for snippets of high-frequency data (Stoianov & Hoskins 2012).

The continuous monitoring of pressure guarantees that unsteady-state flow conditions caused by operational changes, sub-optimal modulation design or large consumers are promptly identified before they cause a detrimental effect on the control process (Figures 6(a) and 6(b)). Furthermore, the acquired signals enable the condition assessment of network controllers based on their steady-state and dynamic responses (Figures 6(c) and 6(d)). The high-resolution data are correlated with the control signals and the response of a network controller (i.e. the position and movement of a diaphragm). These functionalities ultimately facilitate the design and implementation of optimally tuned control algorithms which operate under well-defined hydraulic conditions.

**OPTIMIZATION FOR THE OPERATION OF DYNAMIC TOPOLOGY**

**Background review**

Two areas of optimization for the operation of water distribution systems have emerged in the literature. The first area is pump scheduling for reducing energy costs, and proposed optimization methods include linear programming (Jowitt & Germanopoulos 1992), heuristics (Ormsbee & Reddy 1995), dynamic programming (Ulanicki et al. 2007), a hybrid genetic algorithm (Van Zyl et al. 2004), a generalized reduced gradient method (Skworcow et al. 2010), a combined greedy algorithm and linear programming technique (Giacomello et al. 2013), and an iterative linear programming method (Price & Ostfeld 2013). The second optimization area is valve control for leakage management and, due to the similar nature to the pump scheduling problem, a similar assortment of methods have been proposed including sequential linear programming (Sterling & Bargiela 1984; Jowitt & Xu 1990), sequential quadratic programming (Vairavamoorthy & Lumbers 1998), a generalized reduced gradient method (Ulanicki et al. 2000; AbdelMeguid 2011), and genetic algorithms (Nicolini & Zovatto 2009). Significantly less research has been conducted on valve control, which might be attributed to the fact that its benefit in previous work has been limited to leakage reduction, whereas pump scheduling may provide
water companies with greater financial incentives over advanced leakage management. The benefits gained by the valve control proposed in this project however, are not limited to leakage reduction due to the notion of a dynamic network topology.

**Problem formulation**

Optimal valve settings for the dynamic topology are specified over a time horizon from the solution of a series of nonlinear programs (NLPs) that represents steady-state hydraulic simulations in an extended-period simulation. The optimization solver determines optimal outlet pressures of control valves by varying a linear headloss parameter \( \eta \) for each control valve. Typical customer demand and reservoir profiles are used in the model as the boundary conditions. Further work will incorporate a demand forecaster in conjunction with the optimization solver.

The NLP for calculating \( \eta \) that minimizes network pressure for a single steady state hydraulic simulation is as follows:

\[
F(H) = \min \sum_{j=1}^{m} H_j
\]

Subject to equality constraints \( h(Q,H,\eta) \) defined as

\[
f(Q_i) + A12_i H + A13_i \eta + A10_i H0 = 0; \quad \forall i \in NP
\]

\[
A21_i Q - q_i = 0; \quad \forall j \in NN
\]

and inequality constraints \( g(Q,H,\eta) \) defined as

\[
- Q_j \leq 0; \quad - \eta_i \leq 0 \quad \forall i \in NV
\]

\[
H_{\text{min},j} - H_j \leq 0 \quad \forall j \in NN
\]

Figure 6 | Continuously monitored pressure signals acquired by the InfraSense TS. (a) Pressure data from a CP showing the dynamic hydraulic conditions (5 min average data (dark lines) with an envelope of the dynamic hydraulic conditions within the 5 min interval (grey lines)). (b) Pressure transients generated by a large industrial user, detail from (a), sampling rate 128 S/s. (c) Dynamic valve instabilities measured at the inlet and outlet of a PRV (dark line) showing 1 min average values, (grey line) showing the min/max envelope of the dynamic pressure for the 1 min plotting intervals. (d) Detail from (c), SR 128 S/s.
where \( H \) is a vector of piezometric heads, \( Q \) is a vector of water flow rates, \( \eta \) is a vector of valve settings, \( H_0 \) is the vector of fixed piezometric head nodes (for example, reservoirs), \( q_j \) is the customer demand at node \( j \), and \( H_{\min,j} \) is the minimum allowable head at node \( j \) and \( f(Q_i) \) is a flow dependent head loss in link \( i \). The network structure is represented using incidence matrices, where \( A_{12} \), is the \( j \)th row of a branch-node incidence matrix, \( A_{21} \), is the \( j \)th row of a node-branch incidence matrix, \( A_{13} \), is the \( i \)th row of a branch-valve incidence matrix, and \( A_{10} \), is the \( i \)th throw of a branch-fixed head node incidence matrix. The set of all variable head nodes is denoted \( NN \), where \(|NN|=nn\), and the set of all links is denoted \( NP \), where \(|NP|=np\). Control valves (MFNCs) are positioned within links, and the set of all control valves is denoted \( NV \), where \(|NV|=nv\) and \( NV \subseteq NP \). The notation used here has been adopted from Todini & Pilati (1988).

The objective function in Equation (1) is selected to minimize the summation of hydraulic head at all nodes in the network, which is equivalent to minimizing pressure.

The equality constraints in Equations (2) and (3) represent the hydraulic model consisting of \( np \) nonlinear energy conservation equations for each link and \( mn \) linear mass conservation equations at each node. The only nonlinear term in the model is the flow dependent headloss \( f(Q_i) \) which appears in Equation (2). In order to aid convergence of the optimization method, the Hazen–Williams equation is used to calculate the frictional head loss in pipes since it contains fewer discontinuities than other head loss equations. The Hazen–Williams equation is defined as follows:

\[
f(Q_i) = K_i Q_i^{|\eta_i|^{-1}}, \quad \forall i \in NP
\]

where \( \eta_i = 1.85 \) and \( K_i \) is a vector of constants calculated as follows:

\[
K_i = \frac{10.67}{C_i^{1.85} D_i^{1.87}}, \quad \forall i \in NP
\]

where \( C_i \) is a vector of Hazen–Williams coefficients and \( D_i \) is a vector of diameters for each link. However, for \( i \in NV \), \( f(Q_i) \) represents the headloss across a fully open control valve \( i \), therefore \( n_i = 2 \in NV \) and \( K_i \) is calculated as

\[
K_i = \frac{8 \zeta_0 Q_i^2}{g \pi^2 D_i^4} \quad \forall i \in NV
\]

where \( \zeta_0 \) is the valve minor loss coefficient when fully open and \( g \) is acceleration due to gravity. One of the key differences in this problem formulation in comparison to previous methods is the modeling of the control valves. Inspection of Equation (2) shows that the total headloss \( \Delta H_i \) across control valve \( i \) is defined as

\[
\Delta H_i = \frac{\zeta_0 Q_i^2}{g \pi^2 D_i^4} + \eta_i \quad \forall i \in NV
\]

where \( \eta_i \) is an additional headloss across control valve \( i \) that forms a decision variable in the optimization problem. Any valve position is modelled as the summation of headloss of a fully open valve and \( \eta \). This approach simplifies the NLP and therefore aids convergence, as opposed to defining the valve pressure differential term as a product of the valve opening and valve flow as previously defined (Jowitt & Xu 1990; Vairavamoorthy & Lumbers 1998). Because \( \eta_i \) is not a function of \( Q_i \), it is necessary to invoke an inequality constraint at all control valves so that energy losses are in the direction of the flow, as shown in Equation (4). This restricts the solution of the NLP to a particular solution point, although the problem can be solved multiple times with different configurations of valve flow direction in order to explore the solution space. In normal operation however, a valve will generally have a well-defined flow direction.

The inequality constraints in Equation (5) represents operational limits on the minimum service pressure. More specifically, \( H_{\min,j} \) in Equation (5) is defined as

\[
H_{\min,j} = P_{\min} - \psi _j + \zeta _j \quad \forall j \in NN
\]

where \( P_{\min} \) is the minimum allowable pressure, \( \psi _j \) is a vector of node elevations, and \( \psi \) is a vector of pressure violations. As suggested by Vairavamoorthy & Lumbers (1998), minor violations in the pressure constraints at certain non-critical nodes are permitted in order to ensure critical nodes in the network reach the target pressure. \( P_{\min} \) may vary as it is set according to specific local requirements. For this

\[
H_{\min,j} = P_{\min} - \psi _j + \zeta _j \quad \forall j \in NN
\]
study, $P_{\text{min}}$ has been set to 20 mH$_2$O based on the operational practice adopted by the UK water industry.

**Motivation**

The major difficulty in solving optimization problems arising in water distribution networks stems from the fact that there are a high number of nonlinear constraints due to energy losses and leakage modeling (if present). In previous work on the control of water distribution systems, the optimization problem has been smaller in size as hypothetical networks have often been used or model skeletonization has been undertaken to reduce the computation burden.

For this project, an optimization method that does not rely on model skeletonization was sought for the following reasons:

- To facilitate scalability of the scheme.
- To accommodate future modes of operation and multi-objective functions that may be sensitive to the placement of customer demand.
- To retain as much information about the network as possible which helps ensure that the implementation of optimal control does not result in a suboptimal response of the distribution network.

A gradient based optimization method was initially tested to solve the NLP defined in Equations (1)–(5), where the decision variables consisted of only $\eta$, and gradients were calculated with respect to the hydraulic model variables. It was found however that the method was highly susceptible to the starting point, and convergence was often slow and unreliable. Two direct methods were also tested, an interior-point and active set method, however convergence problems were also experienced. Furthermore, because these algorithms solve the hydraulic model and optimization problem simultaneously, they cannot be stopped early and still produce a hydraulically feasible solution, which is advantageous for a near-real time control system where time might be limited, particularly for incident management. The problem was therefore reformulated with three objectives in mind that to the authors’ knowledge have not all been satisfied by any single approach in previous research on valve control:

1. Reliable convergence of a solution with minimal susceptibility to starting point.

2. Rapid convergence of a solution for very large networks without the use of network skeletonization (i.e. the scalability of DMAv2O.net).

3. A search space that remains close to the hydraulically feasible area.

By fixing and linearizing certain optimization variables, the problem can be reduced to make use of linear programming and a Newton-Raphson based hydraulic solver, both of which lead to fast and reliable iterations that remain close to the hydraulically feasible search space. This led to the construction of an optimization method that comes under the framework of SCP. Although SCP has been acknowledged as a suitable and efficient method for very large scale optimization and control problems (Zillober et al. 2004; Quoc et al. 2012), it has not yet been applied to water distribution networks.

**Linearly coherent SCP**

A traditional SCP method decomposes an optimization problem into subproblems by fixing variables, each subproblem being convex and solved iteratively, until convergence is achieved. In the case of the NLP defined in Equations (1)–(5), two convex subproblems can be formed, however the method deviates from a traditional SCP method because of the judicious use of both zero and first order approximations. The method outlined here is therefore termed Linearly Coherent Sequential Convex Programming (LCSCP).

The method solves the NLP defined in Equations (1)–(5) by first solving subproblem A, which is formed by the following:

- Fixing $\eta$ in Equation (2) according to the value of $\eta$ found in the second subproblem. For the first iteration however, this is set to zero, i.e. control valves fully open:

$$
\eta_i = 0, \quad \forall i \in NV
$$

(11)

- Eliminating the inequality constraints in Equations (4) and (5).

The resulting subproblem is a system of equations consisting of Equations (2) and (3), where $\eta$ is fixed. Its convexity and uniqueness of the solution was established by Todini & Pilati (1988), but for completeness the derivation is reviewed here.
The convexity of the subproblem is established by showing that its solution satisfies the Karush–Kuhn–Tucker (KKT) conditions of the following NLP:

$$\min_Q \sum_{i=1}^{NP} K_i Q_i^{n_i+1} / (n_i + 1) + Q_i (A10iH0 + A13i\eta)$$  \hspace{1cm} (12)$$

subject to $A21iQ - q_j = 0; \forall j \in NN$  \hspace{1cm} (13)

Since the objective function in Equation (12) is strictly convex (since $K_i$ and $n_i$ are always positive) and subjected only to linear equality constraints as shown in Equation (13), this NLP is convex. A solution $Q^*$ that satisfies the KKT conditions is therefore a global optimum and unique. The KKT conditions for this NLP consist of Equation (13) and the Lagrangian

$$\lambda(Q^*, \lambda^*) = K_i \partial \left[ \left( Q_i^{n_i+1} / (n_i + 1) \right) \right] + A10iH0 + A13i\eta + A21\lambda^* 0 \forall i \in NP$$  \hspace{1cm} (14)$$

where $\partial Q_i^{n_i}$ denotes a subgradient of $[Q_i^n]$. Since the subgradient of $[Q_i^n]$ is $-1$ if $Q_i < 0$, $+1$ if $Q_i > 0$ and $[-1,1]$ if $Q_i = 0$, it can be seen that Equation (14) is equivalent to Equation (2) where $f(Q_i)$ is defined by Equation (6), where the Lagrange multipliers of Equation (14) represent the unknown nodal heads at the solution $Q^*$. The optimality conditions of Equations (12) and (13) are rigorously justified in Rockafellar (1997). The solution to this NLP is therefore equivalent to subproblem A, which is convex and has a unique solution. Since the solution to subproblem A is unique, the objective function given in Equation (1) is not relevant. If the solution also satisfies Equation (4), then the solution is hydraulically feasible.

In order to solve the subproblem defined strictly by Equations (2) and (3), the nodal Newton–Raphson method, as proposed by Todini & Pilati (1988) and Salgado et al. (1988), is used due to its speed and efficiency. This iterative method for calculating nodal hydraulic heads and pipe flows is modified in order to take into account the valve settings $\eta$

$$H_i^{k+1} = -(A21iN^{-1}A11^{-1}A12)^{-1} \left( A21iN^{-1} \left( Q_i^k + A11^{-1}(A10iH0 + A13i\eta) \right) + (q - A21iQ^k_i) \right)$$  \hspace{1cm} (15)$$

$$Q_i^{k+1} = (I - N^{-1})Q_i^k - N^{-1}A11^{-1}(A12iH_k^i + A10iH0 + A13i\eta)$$  \hspace{1cm} (16)$$

where $H_k$ is a vector of simulation piezometric heads, $Q_k$ is a vector of simulation flow rates, $N$ is a diagonal matrix containing the head loss formula exponents, $A11$ is a diagonal matrix defined as $K_iQ_i^{n_i-1}$ for $i \in NP$, $I$ is the identity matrix, and $k$ indicates the hydraulic simulation iteration number.

Having solved subproblem A, a linear program is formed for subproblem B. The only nonlinear term in the original NLP defined in Equations (1)–(5) is $f(Q_i)$ in Equation (2), and two approaches were originally considered for its linearization:

- Fixing $Q$ in the NLP defined in Equations (1)–(5). This eliminates the nonlinearity of $f(Q_i)$ which results in the NLP being converted to a linear program. This is the approach typically undertaken in a traditional SCP method.
- Linearizing $f(Q_i)$ using a first order Taylor expansion (as used by Sterling & Bargiela (1984) on a small illustrative network with a sequential linear programming method).

The first of these approaches occasionally resulted in an infeasible linear program being formed and was therefore disregarded. When using the second approach, convergence between subproblem A and B was poor when tested on large and complex networks. One possible reason for this is that the linear approximation of $f(Q_i)$ did not pass through the origin, i.e. energy losses are not always in the direction of flow. A different type of linear approximation was therefore made for $f(Q_i)$ that follows the linear form used by Jowitt & Xu (1990). The linear approximation of $f(Q_i)$ in Equation (2) is constructed at the solution $Q_i$ calculated in subproblem A and takes the general form

$$F(Q_i) = A_i Q_i + b_i, \forall i \in NP$$  \hspace{1cm} (17)$$

with the following boundary conditions for pipes:

$$F(0) = 0$$
$$F(Q_{Q.i}) = K_i Q_i^{0.85}, \forall i \in NP$$  \hspace{1cm} (18)$$
and the following boundary conditions for valves:

\[ F(0) = 0 \]
\[ F(Q_{s,i}) = \frac{8Q_i^2}{g^2 \pi D_i^4}, \quad \forall i \in NV \]  \hspace{1cm} (19)

These boundary conditions ensure that the linear head loss equation is representative of energy conservation, i.e. that energy losses always occur in the direction of flow. This form of linear approximation of \( f(Q) \) was found to produce a flexible second subproblem that is always feasible.

Solving Equation (17) using Equations (18) and (19) results in the following coefficients for pipes:

\[ A_i = K_i Q_{bs,i} \]  \hspace{1cm} (20)
\[ b_i = 0, \quad \forall i \in NP \]

and the following coefficients for valves:

\[ A_i = \frac{8Q_{s,i}}{g^2 \pi D_i^4} \]
\[ b_i = 0 \quad \forall i \in NV \]  \hspace{1cm} (21)

The optimization problem defined in Equations (1)–(5) is now fully linear in its objective function and constraints, therefore subproblem B is convex (Luenberger & Ye 2003). Solving this linear program produces new valve settings, \( \eta \), and this is once again used as a fixed variable in the first subproblem.

The method finds a solution to the NLP by iterating between the two subproblems described above. At each iteration, convergence of the method is checked using one of two methods. The first convergence check examines the KKT conditions which, if satisfied, are first order necessary conditions for a solution to a NLP to be optimal (Luenberger & Ye 2005). With reference to the NLP defined in Equations (1)–(5), the KKT conditions are defined as follows:

\[ h(Q, H, \eta) = 0 \]  \hspace{1cm} (22)
\[ g(Q, H, \eta) \leq 0 \]  \hspace{1cm} (23)

\[ \nabla_H f(H) + \nabla_{Q,H,\eta} h(Q, H, \eta) \lambda + \nabla_{Q,H,\eta} g(Q, H, \eta) \mu = 0 \]  \hspace{1cm} (24)
\[ g(Q, H, \eta) \mu = 0 \]  \hspace{1cm} (25)
\[ \mu \geq 0 \]  \hspace{1cm} (26)

where \( \lambda \) and \( \mu \) are the KKT multipliers. The purpose of using the KKT conditions as a convergence check is to show that the proposed optimization method is capable of finding local minima. For large and complex networks, however, this may take many iterations which for near real-time control purposes is unsuitable. In practice, it is logical to stop the optimization process earlier once a control solution has been found that sufficiently decreases the objective function and meets all constraints. Therefore an alternative convergence check is used for large or real networks where near real-time control could be implemented. The second convergence check calculates the difference in the flow solutions provided by each subproblem

\[ \epsilon_i = |Q_i - Q_{s,i}|, \quad \forall i \in NP \]  \hspace{1cm} (27)

The optimization method is terminated if \( \epsilon_i \) is below a specified tolerance, which for this study has been set to \( 10^{-3} \) L/s. This tolerance was selected because it results in a reasonably low number of iterations to converge (as shown in the results section) but still provides sufficient accuracy in comparison to more rigorous convergence checks such as the KKT conditions. The overall optimization algorithm is summarized by the flowchart shown in Figure 7 and its iterations and convergence can also be visualized, as shown in Figure 8. With reference to Figure 8, the first subproblem (A) solves a steady state simulation with all control valves open, the convergence of which will lead to a feasible solution, albeit not optimal. The next subproblem (B) then finds a solution with better optimality, but one that may not be feasible due to the linearization. Further iterations of subproblem A may push the solution back into the feasibility region if Equations (4) and (5) are satisfied, whereas further iterations of subproblem B will push the solution back into infeasibility at the cost of higher optimality. The subproblems converge when the convergence check is successful, i.e. the
solution to both subproblems is effectively the same point in the feasibility region. The convergence of both subproblems to the same solution point is defined mathematically in Equation (27).

**Case studies**

The developed optimization method is demonstrated by optimizing valve settings in two different hydraulic models that are shown in Figure 9. Case study (a) is a variant of a small illustrative network used by Todini (2003) and case study (b) is the network model used in the experimental programme.

**Case study A**

The small example network in case study (a) (Figure 9(a)) consists of one pressure control valve and nine variable head nodes with an allocated demand that varies according to a data-derived time-dependent demand factor (shown in Figure 10). The properties of this network are shown in Tables 1 and 2. The developed optimization method for minimizing pressure was applied to this network and the optimized pressure control valve settings are shown in Figure 10. The purpose of optimizing for the small illustrative network is firstly to demonstrate the solution of the KKT condition as a stopping criterion and therefore prove that the developed optimization method is capable of finding local minima and secondly to provide an example network that is replicable. The
optimization method was applied to case study (a) and converged at every time step in two iterations. The minimum allowable pressure $P_{\text{min}}$ is set to 20 mH$_2$O and the solution shows this pressure value at the CP node after convergence at each time step (Figure 10). As expected, the outlet pressure of the PRV increases as the demand increases in order achieve $P_{\text{min}}$ consistently at the CP.
Case study B

The hydraulic model of the experimental programme in case study (b) consists of approximately 2,300 nodes, 2,400 links and four control valves (Figure 9(b)). The optimization method was applied using the same objective function of minimizing pressure in order to calculate optimized outlet pressure profiles for all four control valves. The stopping criteria defined in Equation (27) was used as the convergence check due to the requirements of near real-time control.

The hydraulic response of the four MFNCs for the optimized valve control settings is shown in Figure 11. The optimization solver indicated that a minimal pressure reduction is required at the boundary (valves two and four in Figure 11), therefore these were set to a fully open position in order to avoid problems in valve stability. In addition, during the night the boundary valves close down and the network reverts back to the original DMA structure for leakage monitoring purposes. During the leakage monitoring period, optimized settings are calculated for valves one and three only for this closed network configuration under the same objective function of minimizing pressure. It is anticipated however that water companies may wish to operate their DMAs in a different way during this time, for example by increasing pressure in order to accentuate any bursts or leakage and improve their leakage detection practices.

The convergence of the developed optimization method for a single NLP (time step) is shown in Figure 12. The error defined in Equation (27) is plotted together with the objective function (i.e. average pressure) as well as the pressure at the CP of the network. A solution is found after 12 iterations, where each iteration consists of one solution from subproblem A (Newton–Raphson method) and one solution from subproblem B (linear program). It is observed that the error approaches zero (i.e. both subproblems converge to the same point in the feasible solution space), and the pressure at the CP of the network reaches the minimum allowable pressure $P_{\text{min}}$, which has been set to 20 mH2O.

In order to test the robustness of the proposed optimization method, a sensitivity analysis is undertaken for case study (b) as shown in Figure 13. The customer demand $q$ is randomly generated based on a log-normal distribution which was established using collected hydraulic data in this area, and the number of iterations required for convergence is recorded for a series of NLPs. The optimization method converges in every instance, and the number of iterations required for the stopping criterion follows a right-skewed normal distribution. The distribution produced is dependent on the size and complexity of the network as well as the termination criteria used. For case study (b),

### Table 1 | Node properties for case study network (a)

<table>
<thead>
<tr>
<th>Node</th>
<th>Elevation (m)</th>
<th>Demand (L/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>–98</td>
</tr>
<tr>
<td>2</td>
<td>85</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>30</td>
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<tr>
<td>8</td>
<td>90</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>85</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 2 | Link properties for case study network (a)

<table>
<thead>
<tr>
<th>Link</th>
<th>Length (m)</th>
<th>Diameter (mm)</th>
<th>HW coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
<td>300</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>1,000</td>
<td>500</td>
<td>65</td>
</tr>
<tr>
<td>3</td>
<td>1,000</td>
<td>150</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
<td>250</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>1,000</td>
<td>200</td>
<td>70</td>
</tr>
<tr>
<td>6</td>
<td>1,000</td>
<td>200</td>
<td>65</td>
</tr>
<tr>
<td>7</td>
<td>1,000</td>
<td>250</td>
<td>65</td>
</tr>
<tr>
<td>8</td>
<td>1,000</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>9</td>
<td>1,000</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>1,000</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>11</td>
<td>1,000</td>
<td>250</td>
<td>70</td>
</tr>
<tr>
<td>12</td>
<td>1,000</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>13</td>
<td>1,000</td>
<td>200</td>
<td>65</td>
</tr>
<tr>
<td>14</td>
<td>1,000</td>
<td>150</td>
<td>65</td>
</tr>
<tr>
<td>15 (PRV)</td>
<td>n/a</td>
<td>400</td>
<td>n/a</td>
</tr>
<tr>
<td>16</td>
<td>1,000</td>
<td>500</td>
<td>65</td>
</tr>
</tbody>
</table>
the convergence check is satisfied most commonly in 12 iterations, with some variance depending on the hydraulic parameters used. Smaller networks are likely to require a lower number of iterations (i.e. case study (a) which converges typically in two iterations). The stopping criteria can be modified to suit the level of accuracy or computational time required in a near real-time control solution.

**DYNAMIC NETWORK TOPOLOGY: PERFORMANCE ASSESSMENT**

The benefits of operating a water distribution network with dynamic topology are evaluated by comparing three different control configurations for the case study (Figure 9(b)):

1. Closed boundary valves and constant outlet PRVs at locations one and three.
2. Closed boundary valves and modulating PRVs at locations one and three.
3. Optimized valve settings at all locations (dynamic topology) and the boundary closing at night.

The PRV settings for case 1 are chosen so that the pressure at the CP during peak demand hours is close to but does not violate the minimum allowable pressure (20 mH2O). For case 2, the PRVs control the pressure at the CP and keep them steady at approximately the minimum allowable pressure. Case 3 uses MFNC settings calculated by the optimization solver (dynamic topology).

**Average zone pressure and pressure variability**

A comparison of the AZP for the three configurations is shown in Figure 14. By using an open, dynamic network configuration where energy losses are smaller, pressure is reduced on average by 17.1% in comparison to case 1, and by 8.0% in comparison to case 2. The following observations can also be made from Figure 14:

- During peak demand hours, the pressure management performance of flow modulation (case 2) is equivalent to the fixed outlet configuration (case 1). This is because there is always excessive pressure with a fixed outlet configuration except at peak hours. Flow modulating PRVs are configured to reduce excessive pressure at all times.
- During the night, the boundary valves of the dynamic topology close, and case 2 becomes equivalent to case 3 in terms of pressure management performance.

Another important aspect of pressure management is the diurnal pressure variability across all nodes in a network which has an impact on corrosion fatigue and fatigue related pipe failures. The analysis of this factor (Figure 15) shows that 87% of all nodes in case 3 have diurnal pressure variability of no more than 10 mH2O, whereas 72% of nodes in case 2 and 73% in case 1 see pressure variability below 10 mH2O. This is because both a fixed outlet pressure and flow modulation configurations inherently produce a higher pressure differential. In the case of fixed outlet pressure PRVs, the settings are chosen so that the CP does not fall below the minimum allowable pressure during peak hours. However for the remainder of the day and night pressure
will be much higher, therefore the nodes around the CPs will have high pressure variability. Whilst flow modulation aims to keep pressure at the CP at a target pressure, therefore alleviating this problem, the flow modulating PRV must change its settings in order to achieve this, therefore nodes further upstream from the CPs and close to the PRV experience high pressure differential. Since energy losses are smaller in case 3, valve settings do not vary as much and better pressure variability results are achieved.

Resilience to failure

An investigation into the resilience (i.e. the security of supply during network failures) of the water supply system with open and closed network topology was carried out by simulating a burst of 5 L/s as shown in Figure 16. In the open network, only the two boundary valves from the case study are opened. In order to provide an objective comparison, in both the open and closed configurations, the PRV settings of valves one and three are chosen so that pressure at the CP is 20 mH2O in normal operation. The burst is then simulated at 08:00 and the pressure is recorded. Less than 0.2% of nodes in an open configuration experience pressure below the minimum allowable pressure, and these were at non-critical nodes, whereas over 10% of nodes in a closed configuration violate the minimum allowable pressure. The flexibility in network topology provides system operators with unique capabilities to manage both leakage and mitigate the consequences of failures thus maintaining a high quality of service to customers.

CONCLUSIONS

The UK water utilities are under increasing regulatory and financial pressure to develop integrated operational strategies for managing leakage and provide high quality of supply to customers. The current practices in leakage management which segregate water supply networks into DMAs have significantly reduced the redundancy in connectivity and resilience, thus hindering the ability to provide higher levels of customer service and security of supply. As the improvements achieved by a DMA-based network topology begin to plateau and new service incentive mechanisms are forcing utilities to improve the level of service they provide, the presented study describes the development of a novel approach for the adaptive sectorization of water distribution networks with dynamically reconfigurable topology. To the authors’ knowledge, the developed control method presents for the first time the concept of a dynamically reconfigurable water supply network which integrates the benefits of DMAs for managing leakage with the resilience of supply and optimal pressure management of large area networks.

The main contributions of this study are as follows:

- Advances in energy harvesting, monitoring, control, and optimization facilitates the concept of adaptive sectorization of water supply networks with dynamically reconfigurable technology, a notion that will enable water companies to improve the resilience of their distribution systems, further reduce leakage and extend the life cycle of their ageing infrastructure.
- The DMAv2O.net method is being implemented on a water distribution network in the UK in order to test the technology and provide experimental data that are used to validate analytical work and control methodologies. The on-going study also aims to investigate the benefits and optimal scalability of this novel approach.
- Furthermore, since DMAs are increasingly being implemented in many countries around the world, an opportunity exists for incorporating dynamic topology into the design of these systems and maximizing the benefits that they provide.
- This paper has begun formulating the control algorithms for dynamically reconfigurable topologies. Within the control framework exists a strong optimization component and a novel method based on SCP that is fast, reliable and scalable has been demonstrated. Its future development will include a more detailed exploration of its convergence, robustness and scalability.
- Preliminary results based on modelling demonstrate that the concept and technology can provide significant improvements in pressure management and network resilience, and therefore has the ability to bring water distribution closer to the current driver for ‘smarter water networks’.
Figure 16 | Analysis of resilience.
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In memory of Debbie Scull, 1983–2011, who joined Bristol Water in 2007 and initiated the close links with Imperial College. She is sadly missed by all her friends and colleagues in both institutions.

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