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Elastoplastic Collapse of Tubes Under External Pressure[1]

R. J. FRITZ.[2] The authors' paper proposes a modification in the ASME Code criteria for the strength of tubes subjected to external pressure. The discusser agrees that these criteria should be modified and makes these comments on the paper.

The authors could have emphasized that the plastic tangent and reduced moduli used by them apply to uniaxially loaded rings, and that using these moduli in their collapse equation is inconsistent for two reasons: (*a*) plastic moduli should not be used with the elastic Poisson's ratio, and (*b*) plastic moduli for uniaxially loaded rings should not be used in the biaxial stress relationship implied by the use of the Poisson's ratio in the collapse equation. Also, the authors could better have called the reduced modulus an effective modulus rather than an average *modulus. The reduced modulus is based on a consideration of* equilibrium and compatibility and is associated with a shift in the neutral axis as the section is bent in the plastic range. In such a case the local modulus is considered to be the plastic modulus on the side with increasing compressive stress and the elastic modulus on the side being unloaded by bending. The reduced modulus is the effective bending modulus.

Using the Prager-Prandtl-Reuss type of elastic-plastic stress-strain relations, the discusser has attempted to determine the tangent modulus for a tube loaded by external pressure. His result is given below.

$$E_{tp} = \frac{E_t}{(1-\mu^2) - \left(1 - \frac{E_t}{E_s}\right)\left(\frac{1}{4} - \mu^2\right) + \frac{h}{4R}\left(1 - \frac{E_t}{E_s}\right)(1-2\mu)}$$

$$S = \frac{\sqrt{3}}{2}\frac{PR}{h}\sqrt{1 - \frac{h}{R}}$$

where E_{tp} = the tangent modulus for a pressurized tube
E_t = the uniaxial tangent modulus at stress S
$\mu = 1/2 - \left(\frac{1}{2} - \nu\right) E_s/E$, the plastic Poisson's ratio
E_s = the uniaxial secant modulus at stress S
E = the elastic modulus
h = the tube wall thickness
R = the tube outside radius
ν = the elastic Poisson's ratio
S = equivalent uniaxial stress
P = applied pressure

The above equation is based on the assumption of an axial stress due to pressure equal to one-half the hoop stress due to pressure. The tube reduced modulus is given by

$$E_r' = \frac{4E'E_{tp}}{(\sqrt{E_{tp}} + \sqrt{E'})^2}$$

where $E' = E/1 - \nu^2$

If the discusser's analysis is correct, then the critical collapse pressure should be between the values obtained by using the plastic modulus E_{tp} and the tube reduced modulus E_r' in place of $E/1 - \nu^2$ in the elastic buckling equation. The discusser intends to compare these results with experimental data that is expected to be available in the discusser's company in the near future. It is intended to propose the results of this comparison for publication.

The authors have as one of their conclusions that the main reasons for scatter are geometric imperfections for thin-walled tubes and change of material properties for thick-walled tubes. However, the authors did not logically relate the populations of expected scatter with the collapse pressure. Therefore, the discusser believes that this conclusion was not demonstrated in the paper. When the authors discussed geometric scatter, they could have discussed the geometric scatter that is presumably allowed by the Code. ASME Code tolerances are specified in Section II, if invoked by the purchasing specification, or in Sections III or VIII, if not. The discusser believes that in any review of Code requirements for external pressure strength that attention should be given to the specification of geometric tolerances which the discusser believes could be made more clear and consistent.

The discusser wishes to ask the authors to give their opinions as to why the factor of safety is lower in Section III than Section VIII for tube collapse. The consequences of failure for Section III components are usually more serious than for Section VIII components. Moreover, the inspection, analysis, and fabrication are usually identical for the case of tubing applications.

The discusser had the experience of testing tubes for external pressure strength, and the tubes to be tested were received with cold work. This is believed to be usual for tubing. Now it is known that cold work tends to create anisotropy in mechanical properties. The Code specifies the control of some tensile properties, but not of compressive properties. In order to shed some light on the degree of anisotropy that might occur, two bars of AISI type 304 stainless steel were pulled in tension to 0.4 percent total strain and then unloaded. One bar was then tested in tension and exhibited an initial modulus of 28 × 10[6] psi and a 0.2 percent yield strength of 44 ksi. The second bar was tested in compression and exhibited an initial modulus of about 16 × 10[6] psi and a 0.2 percent yield strength of about 28 ksi. The low initial modulus of 16 × 10[6] psi in compression could be particularly significant to tube collapse. This anisotrophy in flow properties is associated with the Bauschinger effect. Since the compressive properties are not specified in Code material specifications, the Bauschinger effect must be

[1] By O. Heise and E. P. Estergar, published in the November, 1970, issue of THE JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 92, No. 4, pp. 735–742.

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considered in order to rationally examine the Code factor of safety. The effect of low and high temperature creep on tube collapse should also be considered.

This discusser agrees with the authors that the factor of safety could be different in the elastic and in the elastoplastic collapse regions and that the ASME Code criteria for the strength of tubes loaded by external pressure should be reexamined. The discusser feels the authors have contributed some valuable factors to be considered in this reexamination.

ticity initiation or reduced collapse pressure sensitivity of thick tubes to ovality. Perhaps revision of the design procedure in which materials testing is separate from theoretical collapse pressure determination is in order and a design procedure based on materials behavior from actual collapse tests of tubular specimens is more realistic. In any case, including out-of-roundness effects in the theoretical determination of collapse pressure would contribute to providing uniformity in safety factor throughout the entire R/T range.

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A. LOHMEIER.[2] The writer appreciates the specific objective of the authors to provide external pressure design safety factors consistent with theory, test results and service experience for application in pressure vessel codes. The relation to design reliability and economy of tubular heat exchangers should be underscored. It is the writers concern, however, that the objective might better be served through more realistic theoretical approaches to prediction of collapse pressure than to provide a "variable safety factor" based on a limited field of experimental results.

In reviewing the collapse pressures of tubes from Table 1 materials plotted in Figs. 8 and 11, it is not clear that the theoretical collapse pressures have included corrections for out-of-roundness. The formulas cited in Table 1 for computing theoretical collapse pressures (equations (3), (4), (5) and (6)) do not include correction for ovality. If the effect of out-of-roundness were included in the theory, agreement of experimental results with theory might be more consistent over the entire R/T range and the upward trend of test agreement with theory for lower characteristic ratios less apparent.

Examination of data plotted in Fig. 8 indicates that the tests for tubes of the same material were restricted over a limited range of R/T. Tubular specimens of 304 stainless steel were tested within the R/T range of approximately 12 to 25. For these specimens, the conclusion that "thick-walled tubes are characterized by a definite trend of improved correlation of tests and prediction" has not been justified by experiment. In the same sense, the "adjusted lower bound of test results" of Fig. 11 appears to have insufficient experimental justification from the limited number of data points presented.

The proposed recommendation for initiating the onset of safety factor reduction at the characteristic ratio as defined at the material proportional limit is of further concern. For example, equation (10) shows the initiation of plastic behavior in tubes to be a function of the tube out-of-roundness. Hence, if one considers the effect of ovalization in the theoretical analysis for collapse pressure, the onset of plasticity will not be determined simply from membrane analysis of the circular tube and an effective modulus determined from the stress strain curve shape.

The concept of a variable safety factor as recommended by the authors appears to continue the inclusion within the safety factor of inherent inaccuracies resulting from exclusion of geometric abnormality in the theory. While judgement would indicate a lesser factor for thick tubes, the point of onset of safety factor reduction remains in question, whether the basis be plas-

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Author's Closure

The comments of both discussers call for more rigorous theoretical treatment of title problem. Mr. Lohmeier thinks if the effect of out-of-roundness were included in the theory, agreement of experimental results with theory might be more consistent. Mr. Fritz suggests an expanded formula for the reduced modulus to improve the correlation with test data. We appreciate these comments all the more, because similar expectations motivated us to start the work resulting in the paper.

Twenty years have elapsed since the present ASME design rules were developed. We hoped the theory had advanced significantly, and new procedures could now be proposed on a more theoretical basis. Unfortunately, this is not the case, as shown in Section 4, where we reviewed the theories for out-of-roundness correction. The results are summarized in Fig. 7, showing the scatter in both the corrected and uncorrected test results.

If the deviations from the theoretical predictions were caused by initial ovality, as suggested by Mr. Lohmeier, the correction would eliminate or at least reduce the scatter of the experimental results. It is clearly demonstrated in Fig. 7 that the opposite is the case; the corrective methods increase the scatter rather than reduce it. The average value of the collapse pressure is decreased by the corrections (shown by the shift in the ratio P/P_{th}). The magnitude of the shift is quite arbitrary as it depends on the definition of out-of-roundness (e.g., Donnell uses "unevenness factors" derived from an analogy with beams; Corum suggests an "average" value since the maximum ovality overcorrected the theoretical values by almost a factor of 2).

Since the corrections do not reduce the scatter, the arbitrary shift in the average value is not an improvement on the theory, and the correction methods have the same validity as the factor of safety applied to the uncorrected theoretical values. Therefore, the present practice of using a "safety" factor, as an empirical correction factor, is justified, until the deviation from ideal shape is better defined and a more effective corrective theory is developed. This observation is applicable to both the elastic and plastic ranges of collapse. We found that the scatter in the plastic range is just as large as the scatter shown by numerous investigators for the elastic range. The average values, however, and even the lower-bound values, are well above the predicted collapse pressure for perfect shells. The change in the elastic modulus formula suggested by Mr. Fritz would further increase the discrepancy between the theory and experiment, which, at this stage of empirical adjustment of an approximate theory, would demand an even larger reduction of the safety factor than what is suggested in the paper.

We believe that the real reason for the discrepancy between the buckling formulas' predictions and the test results lies in the fact that in the plastic range buckling does not take place in the classical snap-through mode, caused by the bifurcation of energy states. The collapse occurs as a continuous deformation, terminated by the complete flattening of the tube. The plastic load-carrying capacity of the shell is increased by the strain hardening characteristics of the material. Strain hardening is not factored into the theory at the present, hence the theoretical values underestimate the plastic strength of the tubes. The adjustment of the safety factor is simply expressing the change in the failure mode from buckling to a fairly predictable plastic bending mode. The important contribution of our paper is the separation of the elastic buckling and plastic flattening mode as identified by the buckling index defined by equation (14).

Mr. Fritz's practical comments concerning the measurements of the elastic modulus values are well taken. The numerical value quoted further substantiates our thesis that the solution to the problem is not to be found in partial improvements in the theory; rather, a better understanding of the geometry factors and the role of material properties should be developed. Until this is achieved empirical correction factors are needed if improvements in the design rules are desired.