Schizophrenic Thought Disorder: A Model-Theoretic Perspective

by Steven Mattheyse

Abstract

Model theory is a branch of mathematics that treats such abstruse questions as "is there another number system, different from 0,1,2,... that satisfies all the axioms of arithmetic?" (yes), and "can every mathematical hypothesis be proved true or false?" (no). It would not seem likely that any contribution could be made to our understanding of thought disorder in schizophrenia from such a remote quarter. The detailed results of model theory obviously will not apply; but I believe certain ways of thinking common in that field, but not in psychology, may help lead us toward explanations.¹

The Problem

A theory of thought disorder presupposes, first of all, an understanding of normal thinking. One could approach this subject from many perspectives, but I would like to start with the curious fact that thinking is so easy for normal people. Our thoughts effortlessly conform to an extraordinarily complicated set of syntactic, semantic, and logical constraints, as well as to whatever additional requirements must be satisfied so that they are judged "sane."² There is no sign that we struggle to make them conform. We do not seem to calculate before we think, nor do we filter out ill-formed thoughts just before speaking. Whereas some mental tasks, like mental rotation, demand concentration and require time—Shepard and Metzler (1971) showed that mental rotation requires an interval of time proportional to the amount of rotation—normal thinking demands no special concentration and takes hardly any time.

There is not even any sign that as children we once struggled to form our thoughts correctly, but now have practiced the skill to the point where it has become automatic. The normal child thinks as effortlessly as adults do. His language conforms to rules not quite like the ones he will eventually acquire, and his thinking has a different set of constraints from the rules of adult thought; but he moves as easily in that realm as we do in our own. Anyone who has tried to learn a second language through self-study has had the opposite experience. Language by rule application is not at all like natural language performance.

The major question I would like to ask is: how can it be that normal thought and language are effortless, when they must conform to a complex set of rules? Chomsky (1965, p. 9) has steadfastly maintained the distinction between competence, which the rules specify, and performance, which depends on yet-unknown processes:

¹The term "model-theoretic" is also applied to a school of thought in semantics, especially to the work of Montague. While there may ultimately be some convergence, I am using the term in a different sense, and I apologize for any confusion.

²For example, our thoughts must avoid the 20 categories of thought disorder (such as vagueness, perseveration, incongruous combination, idiosyncratic symbolism, confusion, looseness, fluidity, and confabulation) described by Johnston and Holzman (1979, pp. 69-70).
generative grammar that expresses the speaker-hearer's knowledge of the language; but this generative grammar does not, in itself, prescribe the character or functioning of a perceptual model of speech production.

And Fodor (1977, p.112):

What makes generative grammar generative is that it recursively enumerates the infinite set of well-formed sentences of a language.

This is the sense in which generation is to be understood, not physical production. Nevertheless, the fact that competence is stated in terms of phrase-structure rules and transformations easily leads the unwary to assume that the brain produces language by applying these rules. I will propose a radically different mechanism. First, a few mathematical preliminaries:

A Toy Language

The homely mathematical example I am about to give is, I think, the best way to make the ideas clear; I hope readers unfamiliar with the mathematics will not be put off by it. A finitely presented group is a set of "letters" that may be combined to form "words." The group contains all words that can be formed from the letters in any order, with the understanding that, according to certain rules (which I will specify), two words that look different may be judged to be synonyms. The system must obey certain universal syntactic and semantic rules, corresponding to the axioms of group theory.

(1) Any letter, taken by itself, is a word.

(2) Any two words may be concatenated, and the result is also a word, although it may turn out to be a synonym of another word. If two synonyms x and y are each concatenated with a third word z to form xz and yz (or zx and zy), the combined words are also synonyms.

(3) One of the letters, e (called the "identity"), has the property that any word x, concatenated with e to form xe, is synonymous with x alone. The same is true if e precedes x to form ex.

(4) Two words x and y are said to be inverse to each other if xy and yx are both synonymous with e. Every word has at least one inverse. (All inverses to a given word are necessarily synonyms.)

I have deliberately cast the rules of group theory in "linguistic" form, to make the analogy with language apparent, but this way of describing group theory does not do any violence to its axioms.

In addition, there will be some special rules for synonyms in the actual group that is being studied. For example, we might make up the rules:

(1) aaa ~ abab. [a rule of the grammar]

(2) aa ~ bab. [Concatenate any inverse of a with each side of (1).]

(3) a ~ babab. [The right side ~ baab, because bb ~ e is a rule of the grammar; substitute bab for aa, using (2); again apply bb ~ e.]

(4) a ~ aaaa. [Substitute (2) into (3).]

(5) e ~ aaa. [Concatenate both sides of (4) with any inverse of a.]

A mechanical version of this proof, suitable for computer implementation, would require many more steps than we have used. We have made guesses at each step about the best strategy for the proof, but a computer must adhere to a decision procedure fixed in advance. Let us now consider the medium-easy way. Suppose I tell you that the letter a corresponds to the matrix

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The group in question is the symmetric group on three elements, and the representations that have been given are two standard forms (see Johnson [1976] for a clear exposition).

The representation is in a world of familiar objects (familiar, at least, to mathematicians), so instead of deducing that $aaa \sim e$, one simply combines the matrices in standard ways, and—voila!—the theorem jumps out.

Now I admit that this may not seem like much of a simplification to those for whom matrix multiplication is not a way of life—but supposing it were—representation in the world of matrices would make conforming to the rules much easier. Let me give away, right here, the whole point of the argument (although it will become clearer as we go along). The way to make performance according to rules easy is to carry out the performance in a domain whose natural operations automatically satisfy the rules.

Now I will tell you the easy way. Take three balls—one red, one white, and one black—and put them in three holes. Whenever you see the symbol $a$, move the red ball where the white ball was, the white ball where the black ball was, and the black ball where the red ball was. Whenever you see the symbol $b$, swap the black and white balls. Whenever you see an $e$, don’t do anything. It is obvious that $aaa \sim e$ in this system. You have to check that the rules of the grammar are valid (a good exercise), but they are. Moving balls around is a lot easier than proving theorems. There is a catch here, which you have probably noticed: some oracle has to tell us exactly which matrices or ball-shufflings correspond to which letters. I will get to that later; it is important.

The model concept

A model of an axiomatic system is:

A system of objects which satisfy the relationships of the abstract system and have some further status as well. [Kleene 1950, p. 25]

Notice that, in this definition, the relationship between abstract and concrete levels of discourse goes in the opposite direction to the way the term “model” is usually used in psychology and biology. In these fields, the model is more abstract than the system being modeled; in mathematics, the model is more concrete.

For our purposes, everything hinges on the qualifier “objects which . . . have some further status as well.” The objects in the model come from a particular universe, unknown to the theory that is being modeled; they inherit the structure of that universe, in addition to the structure in virtue of which they satisfy the axioms of the theory. Because of the universe from which they come, they may have “natural” ways of combination that force the rules of the theory they represent to be satisfied automatically as the objects operate on each other.

A model of the axiom-system which represents the rules of thought must be directly embodied in the machinery of the brain, if we are to account for the abundance of well-formed ideas that are available at all times and in great profusion.

Man has a species specific capacity . . . which manifests itself in what we may refer to as the “cre-
atie aspect" of ordinary language use—it's property of being both unbounded in scope and stimulus-free. [Chomsky 1966, p. 19]

The language mechanism can be both "rich" and "narrowly constrained" because its natural operations always lead to productions that are well-formed, and therefore it can create valid expressions easily and freely.

**The Principle of Compositionality**

That ideas should be formed by combination, rather than calculation, is an old strain in philosophical thought. Perhaps its most famous advocate in psychology was John Stuart Mill, who coined the term "mental chemistry" in his *Logic of 1843* (Mill 1881, p. 592):

When many impressions or ideas are operating in the mind together, there sometimes takes place a process of a similar kind to chemical combination. . . . The Complex Idea, formed by the blending together of several simpler ones, should . . . be said to result from, or be generated by, the simple ideas, not to consist of them.

The logician Frege (translated in Frege 1963, pp. 1-2) struggled with this problem as well:

The question now arises how the thought comes to be constructed, and how its parts are so combined together that the whole amounts to something more than the parts separately.

Frege proposed that logical connectives like "not" and "and" serve to join thoughts together; these terms are "unfulfilled," and thus have the power of binding thoughts together into larger wholes.

There can be no negation without something negated, and this is a thought. The whole owes its unity to the fact that the thought fulfills the unfulfilled part or, as we can also say, completes the part needing completion. . . . Thoughts do not cleave to one another unless they are connected together by something that is not a thought. . . .

The notion of "compositionality," as it has subsequently been called, is very close, I think, to the view being developed here. In the mind, ideas combine freely and mechanically, by "chemistry" or by "cleaving to one another." Yet in such a way that they never fail to satisfy the laws of thought. The only kinds of combinations Frege's theory encompassed, however, were those of formal logic; it is much more difficult to imagine how such a process could work for nonmathematical thought. Let us turn to the feasibility question.

**Requirements on the Model**

The strongest objection to a theory of this kind, it seems to me, is that it is hard to believe such powerful systems can exist. In our "toy language" example, we managed to find two domains that are natural models of the axioms of a specific (and deliberately chosen) finitely presented group. It is natural to wonder if any generalization is possible. The question has two halves: (a) are there domains in which very complex axiom systems can be represented? and (b) can such models be embodied in a machine, even in the human brain?

A definitive "yes" can be given to question (a). It can be proved that there exists a group in which any finitely presented group* can be embedded (Hodges 1985, pp. 65–66). In fact, there exist infinitely many such groups. It follows that a group of this kind is capable of serving as a model of any language whose grammar could be stated in the form of our "toy language." This is only an illustration of the representational power of some formal systems, since we really do not have any reason to believe that the formalism of finitely presented groups is appropriate for stating the laws of thought and language.) Question (b), on the other hand, is open. We have no systematic theory of what machines can do. I will return to this problem in the next section.

Continuing for the moment with question (a), it is important to note that although some representational domains have very powerful modeling properties, mathematical objects always have limitations. A mathematical structure can be capable of serving as a model for one axiom system, but be unable to embody

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*Strictly speaking, any finitely presented group whose "word problem" is decidable. The word problem for a group is the problem of enumerating all "words" (combinations of symbols) that can be proved synonymous with the identity. There exist groups for which the word problem cannot, in principle, be solved. In other words, in those groups there are pairs of symbol strings that cannot be proved either synonymous or not synonymous. That such groups exist is of great mathematical interest, but has little importance for this discussion, because if language were represented by the axioms of such a group, deduction of syntactic consequences from the rules of grammar would also fail.

7 Any "existentially closed" group has this property. (See Hodges 1985, Chapter 3.3.)
another. This, it turns out, will lead us to a natural way of interpreting thought disorder in schizophrenia, so I will explain it in a little more detail.

Earlier, in discussing our "toy language," we discovered that the group of two-by-two matrices of complex numbers was able to represent the axioms of a certain finitely presented group, which turned out to be the group of rearrangements of three objects (the red, white, and black balls). Suppose that we had a machine available to us that was "smart enough" to multiply two-by-two complex matrices, but could do nothing else. It could handle the group of rearrangements of three objects with ease. What about other groups? Let us consider another example—the group of rotations of a tetrahedron. A tetrahedron is a figure with four corners and four triangular faces, and our group consists of those rotations that bring the corners into register with the places where other corners were, in such a way that the rotated figure looks just the same as the original figure. If you draw a tetrahedron and label the corners a, b, c, d, and then rearrange the a, b, c, d symbols on other corners, there are 24 arrangements possible (four ways of locating "a," then three for "b," then two for "c"), but only 12 of them are realizable by rotations. For example, there is no way to swap "a" and "b" while keeping "c" and "d" fixed (as they say in Maine, "you can't get there from here.") 9 It can be proved that the group of rotations of the tetrahedron, although not a very complicated group, cannot be embedded in the group of two-by-two complex matrices (Rose 1978, p. 85). (It can be embedded in the group of three-by-three matrices.) If all we had available was a machine that could do two-by-two matrix calculations, our program of replacing computation by combination would fail.

Another way to look at this problem may be instructive. Recall that the special rules of our "toy language" were: (1) aaa ~ abab, (2) bb ~ e, and (3) aaa ~ e (which we derived from the others). It turns out that the tetrahedron rotation group can be formed by replacing (1) by the very similar rule (1') aaa ~ abab. The other two rules remain the same. In other words, here we have two grammars, nearly identical, but our two-by-two matrix machine can handle one and not the other. Evidently it is most important to have the right machine.

**Realization of Models as Machines**

Let us return to question (b), about the possibility of embodying models as machines. Although we have used the term "two-by-two matrix machine," it is not really a machine, only the abstract idea of a machine. A further step toward concreteness must be made if a model-theoretic approach to cognitive and language performance is to have value as a psychological theory. The model must be capable of realization in the brain. At this stage of knowledge about the brain, it is not possible to say very much about what kinds of models it is capable of embodying. On the other hand, given a hypothetical model system, we can ask a simpler question: whether any physically realizable machine can be designed to carry out its operations. While it is known that any effective calculating procedure can be carried out by a simple device known as a Turing machine, our requirements on the machine are much more stringent. It must carry out the operations of the model in a natural way, so that its built-in actions essentially coincide with the combining relationships of the model. In a word, the model must be its natural language.

The concept of "naturalness," like that of an "effective calculating procedure," is difficult to define precisely, but for digital machines that operate in clock cycles, let us agree that the "natural operations" of a machine include anything that can take place in just one clock cycle. 9 Machine designers frequently combine several clock cycles into a larger cycle, so it would be reasonable to broaden our definition to include as "natural" any operation that the machine can carry out in a time interval strictly independent of the size of the objects being manipulated, even if that time interval includes several clock cycles.

An example will make this idea clear. Hillis has designed a machine that consists of many small processors ("cells") operating in parallel, but able to send messages to each other and to reconfigure their pattern of communication to fit the demands of the problem. A set of objects can be represented in this machine by placing a single bit in each cell, which indicates whether that cell is a member of the set. The machine forms the union or intersection of two sets in a fixed amount of time, however large the sets may be, because each cell can decide simultaneously whether it is or is not

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9The group is called the "alternating group," because of the kinds of permutations that are allowed.

9For analog computers, "naturalness" has an obvious definition, namely, the equations of the system being represented on the computer (e.g., atmospheric dynamics) should be identical with the equations governing the computer itself (e.g., current flow).
in the union or intersection, just by using the bits in its own internal memory (Hillis 1985, pp. 92–93). Elementary set theory (or Boolean algebra, which comes to the same thing) is, so to speak, the "natural language" of this machine.

Since group theory has been the inspiration for our model-theoretic approach to language performance, it is reasonable to ask whether there is a machine for which the operations of group theory are "natural," i.e., are carried out in time independent of the size of the group represented. The answer is yes; I will briefly sketch the design of such a machine.¹⁰

The process that makes the design work is nothing more than indirect addressing, which is a standard feature of traditional computer architecture. "Indirect addressing" means the ability to read (or write to), not only a memory register whose address is specified, but also the memory register whose address is specified in the first register. Every finite group can be thought of as a group of rearrangements (or permutations) of N objects among themselves, where N is the number of elements in the group. That is because the action of group elements on each other amounts to rearranging them. Let the memory registers of the machine be labeled by (i,j), where i varies from 1 to N, and j varies from 0 to 3. I chose 3 simply because, in our example, we will multiply two group elements (with indices j = 1 and j = 2) to get a group element that we will represent with index j = 3. As we shall see, j = 0 has a special purpose. In general, as many values of j will have to be allowed as there are group elements the machine needs to keep in working memory at any one time. Let r(i,j) stand for the contents of the register with label (i,j). A group element will be represented by the sequence r(1,j), r(2,j), ..., r(N,j) for some j. The contents of these registers specify where the group element sends each member of the group when it acts as a permutation. Consequently, r(1,j), r(2,j), ..., r(N,j) are all different from each other, and range from 1 to N. The machine consists of N identical processors working in parallel, labeled i = 1 to i = N. When the machine is initialized, r(i,0) is set equal to i, and will never be changed. In other words, j = 0 represents the identity element of the group.

Let us now describe the multiplication of any two group elements. Let the first be stored in registers (1,1), (2,1), ..., (N,1); the second in registers (1,2), (2,2), ..., (N,2). The product will be stored in registers (1,3), (2,3), ..., (N,3). Each processor (from i = 1 to i = N) will work simultaneously to carry out its part of the group multiplication. Processor 1 reads register (1,1) and, through indirect addressing, finds the contents of register (r(1,1),2). In general, processor i reads register (i,1) and looks up register (r(i,1),2). In other words, if register (1,1) contained a 4, processor 1 would indirectly read register (4,2). Processor 1 then writes the contents of this register, say r(4,2) = 6, into register (1,3). In the general case, processor i writes the contents of (r(i,1),2) into register (i,3). All these events take place in a fixed number of clock cycles, however large the group may be, because they occur in parallel. It is easy to see that none of the actions of the processors will "collide" or interfere with each other (the great worry in designing algorithms for parallel processing), because r(1,j), r(2,j), ..., r(N,j) are all different from each other; no two processors will ever try to address the same register at the same time.

A concrete example will help clarify what is going on. Let the machine be representing the group of permutations of three objects (the red, white, and black balls of our previous example). That group has six elements, because there are six ways of rearranging three objects. Suppose the contents of memory, before the group multiplication, are as follows:

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Regarded as permutations, the first column represents the actions indicated by solid arrows, the second the actions indicated by dashed arrows.

Consider what processor 1 does during the multiplication. It reads register (r(1,1),2) through indirect addressing (i.e., register (2,2)), which contains the number 2; and writes that value into register (1,3). Meanwhile, processor 2 is reading
register \((r(2,1),2)\), which contains 3, and writes that value into register \((2,3)\). Each processor carries out the same operation. After the cycle is over, registers \((1,3), (2,3), \ldots, (6,3)\) look like this:

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\begin{align*}
    r(1,3) &= 2 \\
    r(2,3) &= 3 \\
    r(3,3) &= 6 \\
    r(4,3) &= 1 \\
    r(5,3) &= 4 \\
    r(6,3) &= 5
\end{align*}
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Examine the diagram, and you will see that the sequence \((i,3)\) correctly represents the product of the two group elements.  

The importance of all this is that we have designed a machine that speaks group theory as its natural language. Designing a machine that speaks English as its natural language will not be so easy, but I think it is the same in principle. First you construct a model of the formal rules of the grammar in which the elements of the model come from a universe with a sufficiently rich structure (above and beyond their function in representing the grammar) that a machine could conceivably carry out the fundamental operations of the model in a natural way. Then you try to embody the model in hardware, and ultimately in that very special hardware which is the human brain.

There is one other difficulty, hinted at before, to which we must return. In the “toy language,” the medium-easy and easy ways of solving the word problem presupposed that some “oracle” had revealed what matrices or ball-shufflings should be selected to represent a

and b. The choices were by no means obvious. In the more general context, there has to be a way of fixing enough of the elements of the map from the axiom system to the model that the operations of the model can be guaranteed to represent the axioms faithfully. In modeling the specific group we discussed, only the three elements a, b, and e had to be mapped in advance. Group operations took care of everything else. In any axiom system, the terms that need anchoring to make the model work are the “constants,” the specific entities that are mentioned in the set of rules that we want to model. If the representation is a faithful one, the rest of the relationships will take care of themselves.

The brain is likely to anchor constants through learning, by trial-and-error adjustment until they are embedded in such a way that the axioms are properly obeyed. A complete theory of language or thought, based on these principles, must include proof of learnability of the fundamental embeddings. The concept of learned anchoring of fundamental constants is not very different from Chomsky’s (1986) notion that language learning should be regarded not as a problem in acquiring rules but one of fixing parameters in a largely determined system. [p. 151]

**Implications for Functionalism**

Without diverting attention too much from our main theme, I would like to discuss the implications of the model-theoretic approach for the functionalist program. Johnson-Laird (1983, pp. 9–10) may be cited as an articulate exponent of functionalism.

The neurophysiological substrate must provide a physical basis for the processes of the mind, but granted that the substrate offers the computational power of recursive functions, its physical nature places no constraints on the patterns of thought.

The functional organization of mental processes can be characterized in terms of effective procedures. . . . Computers can think because thinking is a computational process.

Any future theory of the mind will be completely expressible within computational terms. The computer is the last metaphor; it need never be supplanted.

In contrast to functionalism, our approach requires that a theory of the mind be expressible within compositional terms. Functionalism ignores the clue that the subjective mental experience associated with the activity of our brains provides. Accounting for the naturalness of thought is a challenge and an opportunity for psychological theory.

My own view is closer to Searle (1984, pp. 28–29), who satirizes functionalism with characteristic wit:

So, for example, if you made a computer out of old beer cans powered by windmills; if it had the right program, it would have to have a mind.

“Could a machine think?” he asks.

My own view is that only a machine could think, and indeed only very special kinds of machines, namely brains and machines that had the same causal powers as brains. [Searle 1980, p. 424]

I interpret “having the same causal powers as brains” as possessing the brain’s facility for free but lawful combination. One more quotation from Searle (1984, p. 52):

11 Inverses are found in the following way: if element j is to be inverted and represented as element k, each processor i simultaneously reads r(i,0) and writes it into r(i,j),k.
Thought Disorder in Schizophrenia

The model-theoretic viewpoint suggests a way of understanding disorders of thought. First, let us assume that there exist "laws of thought," a set of necessary and sufficient conditions for thoughts to be well formed. Although we do not yet possess a compendium of rules of competence for thinking, no normal observer has trouble judging some schizophrenic utterances to be ill formed. Consider, for example, the following responses of a schizophrenic woman who was asked to complete the sentence "a fish can live in water because...":

Because it's learned to swim.

[What if it couldn't swim?] Not naturally, he couldn't. Why do certain gods have effects on seas like that? What does the earth have such an effect to break their backs? The fishes near home come to the surface and break. [Why?] I think it is due to bodies that people lose. A body becomes adapted to the air. Think thoughts and break the fishes. [Cameron 1938, p. 25]

The patient's expressions do not seriously violate the rules of grammar. Nor are they semantically anomalous in the same way as Chomsky's famous example, "colorless green ideas sleep furiously." That example looks like a valid expression, but it actually says nothing at all. In contrast, this patient seems to be trying terribly hard to say something. We get an inkling of her meaning if we add the information that among her psychotic preoccupations was a fear that her back might be broken. By "laws of thought" I mean a hypothetical set of rules of competence that permits any normal speaker to decide that thoughts like these are improperly formed.

The model-theoretic program connects rules of competence to brain function by postulating that some assembly of neurons is a model for the formal system specified by the rules. In other words, the states and natural operations of the assembly are a domain in which the axioms of the formal system are necessarily satisfied.

Axiom-systems powerfully constrain their models. Recall that in our toy language the model constructed out of two-by-two matrices was capable of representing one set of axioms but not another. Each system of mathematical objects has a range of expressive powers, and its power to represent formal systems has absolute bounds. Conversely, each system of axioms makes different demands on its models. For example, the axioms of group theory can be satisfied by finite sets of objects (the finite groups), whereas the axioms of arithmetic require infinite models (such as the natural numbers), and the axioms of the real number system require as models sets even larger than the set of all integers.

Taking this analysis one step further, every physical system, be it a manmade machine or the brain, has possibilities and limitations for carrying out the operations of models, and therefore can represent some formal systems and not others. If an inadequately formed, damaged, or malfunctioning brain has lost the capacity to serve as a model for the laws of thought, there is no longer any guarantee that thinking will conform to its normal laws of competence.

At this stage of development the model-theoretic perspective provides only an approach to a theory of thought disorder, not a concrete and testable theory. Nevertheless, we can draw some conclusions, and even make a few predictions.

(1) There is not likely to be any single defect in schizophrenic thinking, from which all the others follow as consequences. If the brain is unable to embody a sufficiently powerful model, the laws of thought will not all be simultaneously satisfied by its natural operations; but the specific compromise made by each patient will be hard to predict. Trying to find a fundamental deficit in schizophrenic thinking from which all others follow is a path that many have trod, but if this analysis is correct, it is not likely to be rewarding. Holzman (1986, p. 343) reached essentially the same conclusion:

Two impediments have slowed...
the investigation of cognitive disorders. The first is the tendency of many investigators to search for a single quality of thinking that would encompass all of schizophrenic thinking. Thus, for example, disorders of concept formation, of logical inference ("paralogical thinking"), and of narrow or wide inclusiveness have been introduced.... It is noteworthy that Bleuler considered that the "disorder of association" encompassed many different types of thought disorder, and he considered them all to be symptomatic of the basic defect, which in his opinion, was a brain disorder.\footnote{12}

(2) Over the years, a recurrent theme in theorizing about schizophrenia is that schizophrenic patients are unable to "filter" or "screen" thoughts as normals do, and therefore cannot prevent poorly formed thoughts from entering awareness and being communicated. For example, Chapman, Chapman, and Miller (1977, p. 166) propose:

> Appropriate screening of potential responses must take into account the context in which the word appears. Schizophrenics fail to screen their own potential responses for appropriateness.

Our point of view is not consistent with "filtering" of improperly formed thought by normal people. If the machinery of thought is a model for the axioms that thought must satisfy, normal people will not produce a mixture of well-formed and ill-formed thoughts that subsequently have to be separated by filtering or screening. Normal people do suppress thoughts that interfere with the task at hand (keeping one's mind on the topic in problem solving, avoiding embarrassing remarks in conversation), but they do not have to be on the alert against thoughts that are improperly formed.

On the other hand, there is an interpretation of the "filter" theory that accords well with our point of view. If cerebral function is inadequate and cannot support a model of the laws of thought, a filter becomes necessary, and screening out improperly formed thoughts becomes a demanding and even crippling task. After they fall ill, schizophrenic patients do try to screen out inappropriate behavior, unless their contact with society is utterly lost. The effort slows their responses and impairs their performance on nearly all psychological tests (which is why psychological testing has generally been unrevealing in schizophrenia; there are just too many deficits in performance). The failure of filters to make up for brain deficiency is additional reason to expect that the laws of thought are normally built directly into cerebral architecture, not superimposed upon it.

**Predictions**

The theory, even though it is incomplete, makes two predictions that can be tested experimentally. The first prediction is that if a psychological test of thought disorder were to be carried out under conditions which encouraged the subject to relax filtering operations, normal subjects would still not produce ill-formed thoughts. If the procedure succeeded in provoking subjects to produce response content that they would not normally express, the unusual response would be a sign that the conditions of the experiment did indeed relax the screening process.

At least two experiments along these lines have already been carried out.\footnote{13} Exner, Armbuster, and Mittman (1978) gave the Rorschach test to normal subjects with instructions that, for each card, they had 60 seconds to report "as many things as they could find." The number of responses increased markedly over the number given during standard administration, but there was no indication of thought disorder; the "form quality" of the percepts did not suffer. Holzman and Rousey (1970) administered the Thematic Apperception Test (TAT) to normal subjects (the subject's task in the TAT is to make up stories to ambiguous pictures) under standard conditions and while the subject heard loud white noise through earphones. The white noise condition dramatically increased the level of impulse expression in the stories, while decreasing the degree of defensiveness (e.g., intellectualization, denial, and undoing). Despite the increase in expressive content in the presence of white noise, the stories produced by normal subjects under these conditions did not show thought disorder.

A second prediction of the theory is that deficits in schizophrenia are likely to spread across several domains of intellectual function. Let us return to our mathematical examples: Recall that a system of mathematical objects that is capable of representing one axiom system will in general be capable of representing other axiom systems as well. We saw that matrices of numbers, endowed with their standard combining properties, are capable of expressing the axioms of both the group of permutations of three objects and the group of rotations of a...
tetrahedron (although three-by-three matrices were the smallest size that could represent the tetrahedral group). Matrices are also capable of representing a great variety of other groups, including groups that are not finite, such as the group of permissible transformations in projective geometry (views of a scene in perspective drawing, or through a pinhole camera). A machine whose natural language was matrix multiplication could be adapted to any of these diverse mathematical domains. The same logic applies to models in general although, lacking any systematic theory of the powers of machines, we cannot make any general claims about how widely the competence of a machine might be extended.

Because the number of genes available to code for neural structures is relatively small (see below), it is likely that if the embedding of several cognitive processes in a single type of neuronal assembly is a possibility, the brain will capitalize on it. Conversely, a defective neuronal assembly would fail in each of the domains that it is called upon to represent.

The model-theoretic approach to intellectual deficit in schizophrenia leads us, therefore, to look for inadequate performance in other domains where normal competence can be described by formal rule systems. Syntax is one such area; perception (through the influence of the Gestalt school) is another. In syntactic performance, however, our search does not turn up anything positive. Schizophrenics conform remarkably well to the rules of grammar.

Consider the following conversation between a schizophrenic patient, a nurse, and a psychiatrist.  

Patient: I'm responsible for my own motives. I keep my mouth closed and my nose open.
Nurse: Can you say things a bit more clearly to let us know what's going on?
Patient: Just ask my autograph book who was signing it all the time. It's not my fault it's ripped up.
Psychiatrist: Did you think we'd know what you meant when you said that?
Patient: I know you all know what I meant.
Psychiatrist: I didn't.
Patient: That's not your fault.
Nurse: I suspect no one else in this room knew what you were talking about.
Patient: I said I could remember when my mother's hair was down her back and she kept cutting it off.
Psychiatrist: I don't know what that means.
Patient: That's what I mean. There's been a pass over me. I've been passed over.
Psychiatrist: I still don't know.
Patient: Well, look at the dark shadows. What do you see? Same old monkeys.

What I would like you to notice is about this conversation is not the bizarre thinking, but the perfect syntax, which persists despite severe thought disorder.

Although syntactic performance may be normal in schizophrenic patients, there is evidence that something is unusual about their strategy for organizing the perceptual world. I will cite one particularly striking research finding. Since nonspecific deficits such as low motivation and impaired task comprehension can readily cause schizophrenic patients to perform more poorly than normals on almost any task, it is particularly noteworthy when a deviation from the norm shows up in the context of adequate or superior performance. Place and Gilmore (1980) were able to design experimental conditions under which schizophrenic patients actually performed a task better than normal controls.

Schizophrenic patients and normal subjects were presented with sets of lines, either horizontal or vertical, grouped into simple or complex geometrical patterns, and varying in the number of lines present. Their task was simply to count the lines. Presentation of the stimuli as increasingly complex arrays interfered with the performance of normal subjects, but the schizophrenic patients did about as well with complex as with simple groupings, actually performing better than normals in counting complex arrays of lines. The authors conclude:

These results strongly support the hypothesis that schizophrenics do not respond to organizational factors in a stimulus array the way normals do. The control subjects, it is proposed, engaged an initial, passive, global analysis of the visual array ... a global analysis of the heterogeneous arrays would be a hindrance to efficient processing for it breaks the target elements into separate groups, each of which the subject must focus his attention on. [Place and Gilmore 1980, pp. 415-416]

Whether the deviant performance of schizophrenic patients in both thought and spatial perception is a consequence of similarities in the cerebral architecture that serves the two functions is unknown, but it is certainly worth further investigation.

In our studies of the genetics of schizophrenia (Matthysse, Holzman, and Lange 1986), we have been led to postulate the existence of "latent traits" that can express themselves in observable behavior in several ways. Each possible expression is called a "manifest trait."
The reason for introducing latent traits into genetic analysis is that their transmission patterns in families may more closely approximate mendelian inheritance than any of the manifest traits related to them. In our view, it is not likely that any single family member will express all the manifest traits related to a genetic latent trait; the family has to be taken as a whole to see the spectrum of behaviors associated with the latent trait.

The model-theoretic approach to mental function fits in well with the "latent trait" concept. Inadequate functioning of a neuronal system that normally serves as a model for several different domains of competence would be regarded as a latent trait, and the effects of this deficiency in each domain of competence as the associated manifest traits. If the architecture of the neuronal assembly were replicated in several brain regions, a genetic disease process affecting that mechanism would have different cognitive effects in different family members, depending on where in the brain it happened to strike. Unusual strategies of perceptual organization, although harmless from a clinical point of view, might indicate the presence of the underlying disease process of schizophrenia just as surely as the clinical syndrome. It would be most interesting, for example, to find out whether the family members of schizophrenic probands have a greater probability than a random population sample of showing the Place and Gilmore (1980) effect.

First Steps

We are very far from a concrete model-theoretic analysis of thought disorder in schizophrenia. All that has been proposed here is an outlook. We had some success with our "toy language," but it is not clear how far this approach can go. It seems to me that the first concrete steps should be taken, not in the analysis of thinking or perceptual organization, but in some domain where the laws of normal competence have been better codified—for example, in syntax. As a beginning, one might select a subset of transformational grammar, and try to construct a model for that subset from mathematical objects with appropriate combining properties. The second step is to work out designs for machines whose natural operations embody the combining relationships of the model. The third step is to search for a structure in the brain capable of functioning as such a machine.

The biggest problem at the third stage will be that we know so little about the brain that our knowledge does not impose many constraints on what it might be imagined to do. There is, however, one exception. We know that the architecture of the brain develops under the control of a surprisingly small number of genes—perhaps five orders of magnitude fewer than there are neurons in the brain. Whatever cerebral architecture carries out any hypothetical process must be capable of being specified genetically in a succinct way. It is possible to arrive at estimates of the information needed to specify the construction of a machine. For example, Atrubin (1965) designed a machine that carries out multiplication in time proportional to the number of digits to be multiplied, using an array of identical modules with only nearest-neighbor connections. The information needed to construct the array was, therefore, independent of its digit capacity. To multiply more digits, it had only to increase in size. Constructibility constraints may be very useful in deciding whether a particular model could be physically embodied in the brain.

After these tasks have been carried out in a more well-defined domain, one might hope to return to the problem of schizophrenic thought disorder with a theory that has more scientific flesh on its philosophical bones.

References


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