A Quantitative Interpretation of Seismicity and Seismic Activity*

W. Ullmann and R. Maaz

Summary

A new method for computation of seismicity has been worked out and the concept seismic activity has been mathematically formulated as an enlargement of seismicity. For computation of seismic activity a mathematical representation of the energy density function of an earthquake is needed. For this under the simplest suppositions a heuristic function has been constructed. The modification of the definition of seismicity by using this function causes an increase in the degree of physical information of seismicity.

1. Introduction

Investigators of seismically active territories in the Soviet Union, the U.S.A., and some other countries have made a contribution to disclosing the upper Earth structure, e.g. Allen et al. (1965), Galanopoulos (1963), Héderváry (1963, 1964 and 1965), Ritsema (1954), and Riznichenko (1958). Different procedures for cartographic representation of the investigation region Γ on the Earth’s surface Ω have been applied. A common supposition for these is to know the seismic energy $E_i$ of the $i$th earthquake considered ($i=1, \ldots, n$). Characteristic quantities of the seismic field entered on the map are the $n$ distribution functions $p_i(Q)$, defined for each point $Q \in \Omega$ for the energy values $E_i$ or the values $F(E_i)$ of a significant function $F$. Hitherto existing procedures have been based on agreements concerning the distribution of values $E_i$ or $F(E_i)$ in $\Gamma$ by subdivision of $\Gamma$ and evaluation of the subregions, e.g. those of St. Amand (1956), Benioff (1951), Bāth (1953, 1956), Riznichenko (1959, 1961). By this, the functions of $p_i(Q)$ do not appear explicitly. This has no favourable effect on the determination of seismic isolines. Besides, the expense of such work is considerable.

To enable an internationally uniform representation of ‘seismic behaviour’ of different regions and relevant comparisons for the purpose mentioned above, practicable methods must be developed. Here only a brief report of our theoretical considerations in that field can be given. Detailed articles are now published (Ullmann & Maaz 1966a, b, c). With the method proposed by us for computation of seismicity $S(\Gamma)$ and construction of characteristic isolines $S(Q) = \text{const.}$ to which we refer as isoseisms, the territory of the D.D.R. is being dealt with for the time-interval 1900–1960. Partial results will be reported on the occasion of the forthcoming meeting of the European Seismological Commission in Copenhagen.

* Publication No. 31 of the Institut für Geodynamik.
2. Seismicity and seismic activity

We define local seismicity, that is seismicity at point \( Q \) in a given time-interval \( \mathcal{I} \), by

\[
S(Q) = \sum_i E_i p_i(Q),
\]

where \( p_i(Q) \) is taken as the distribution density for the epicentre of the \( i \)th quake having occurred in time-interval \( \mathcal{I} \). This function has its maximum at point \( Q_i \) which represents the relevant epicentre with maximum probability. It monotonically decreases to zero with increasing epicentral distance \( Q_i Q \). Moreover, \( \int_{\Omega} p_i(Q) d\omega = 1 \) where \( d\omega \) means the positive surface element of \( \Omega \) at point \( Q \). The function \( p_i(Q) \) makes only quakes whose epicentres are near \( Q \) contribute significantly to seismicity \( S(Q) \).

The probability that the epicentre of the \( i \)th quake lies in a region \( \Gamma \subset \Omega \) is

\[
P_i(\Gamma) = \int_{\Gamma} p_i(Q) d\omega.
\]

With this, the definition of \( S(Q) \) enables us to write the seismicity of \( \Gamma \) in time-interval \( \mathcal{I} \) as

\[
S(\Gamma) = \frac{1}{|\Gamma|} \sum_i E_i p_i(\Gamma),
\]

where \( |\Gamma| \) means the area of \( \Gamma \). On the other hand the formula for \( S(\Gamma) \) follows from this if for the fixed point \( Q \in \Gamma \) the region \( \Gamma \) contracts to \( Q \). Between the seismicity of separate regions \( \Gamma' \subset \Omega \), \( \Gamma'' \subset \Omega \) and the seismicity of \( \Gamma' + \Gamma'' \), there exists the simple relation

\[
S(\Gamma' + \Gamma'') = \frac{|\Gamma'| S(\Gamma') + |\Gamma''| S(\Gamma'')}{|\Gamma'| + |\Gamma''|}.
\]

This is convenient for the computation of \( S(\Gamma) \) if \( \Gamma \) is a large region of investigation which is divided into several subregions.

In order to be able actually to compute the seismicity \( S(Q) \) or \( S(\Gamma) \) and especially isoseisms \( S(Q) = \text{const} \), the distribution density \( p_i(Q) \) must be suitably determined; if possible with the inaccuracy of the coordinates of \( Q_i \), the position and dimensions of the focal region should be taken into account first of all. A measurement of the dispersion of epicentres also depends upon the number \( n \) of useful quakes in time-interval \( \mathcal{I} \).

For analytical determination of \( p_i(Q) \) we have mapped the sphere \( \Omega \) on its tangential plane \( \Pi \) at point \( Q_i \) by stereographic projection. The probability that the image of the epicentre of the \( i \)th earthquake lies in the image region \( \Gamma^* \subset \Pi \) is denoted by \( P_i^*(\Gamma^*) \). It is necessary for this mapping to postulate

\[
P_i(\Gamma) = P_i^*(\Gamma^*) \quad (\Gamma \leftrightarrow \Gamma^*),
\]

with which a suitable relation is found between \( p_i(Q) \) and distribution density \( p_i^*(Q^*) (Q \leftrightarrow Q^*) \) defined in \( \Pi \). This is

\[
p_i(Q) = p_i^*(Q^*) \cos^{-4} \frac{Q_i Q}{2R},
\]

where \( R \) is the radius of \( \Omega \). As the simplest law for \( p_i^*(Q^*) \), the Gaussian normal distribution offers itself. If the focal region of the earthquake shows a preferred direction, a Cartesian \((x, y)\) co-ordinate system in \( \Pi \) is fixed in \( Q_i \), so that \( p_i^*(Q^*) \) appears as

\[
p_i^*(Q^*) = \frac{1}{\pi a_i b_i} \exp \left[ -\left( \frac{x}{a_i} \right)^2 - \left( \frac{y}{b_i} \right)^2 \right],
\]

where \( a_i \) and \( b_i \) are the semi-major and semi-minor axes of the Gaussian distribution respectively.
with $a_i > b_i > 0$. The focal parameters $a_i$, $b_i$ depend upon the (mean) focal depth $h_i$ and $n$. For the time being we have tentatively put $a_i = c_i' h_i \sqrt{n}$ and $b_i = c_i'' h_i \sqrt{n}$ with $c_i'$, $c_i''$ positive constants.

Now the computation of $S(T)$ causes no difficulties in principle. If the given investigation region $\Gamma$ is so large that it cannot be treated as a plane region then it may be approximated by a set of sufficiently small plane regions. The simplest case is $a_i = b_i$ for $i = 1, \ldots, n$.

Of course, these considerations do not depend upon whether or not the energy values $E_i$ or the values $F(E_i)$ of a significant function $F$ enter into the definitions. For example, several geophysicists are of the opinion that it is geologically more significant to use the values $F(E_i) = |\sqrt{E_i}|$ instead of $E_i$. Then $S(Q)$ or $S(\Gamma)$, respectively, describes the so-called tectonic flux.

Seismicity is a practical seismic field quantity because only relatively few data of physical processes during an earthquake are used in its definition. An extension, with regard also to the amount of information of the seismic field, may be constructed in the following way.

Earthquakes having occurred up to a given time $\tau$ are weighted so that the energy density $F_i(Q, \tau)$ at point $Q$ up to this time in consequence of a quake with focal time $t_i < \tau$ ($i = 1, 2, \ldots$) is multiplied by the value $v(\tau - t_i)$ of a suitable weighting function $v(t) > 0$. Corresponding to the purpose of this function, we have

$$v(t) = 0 \text{ for } t < 0, \quad \frac{dv}{dt} \leq 0 \text{ for } t \geq 0,$$

which means earthquakes are weighted less the further they lie back in time $\tau$. Then we define the function

$$A(Q, \tau) = \sum_i F_i(Q, \tau) v(\tau - t_i)$$

for seismic activity at point $Q$ up to time $\tau$.

$A(Q, \tau)$ can be normally understood as an infinite sum which is bounded if

$$\int_0^\infty v(t) \, dt$$

exists and is bounded. A very simple function $v(t)$ satisfying these conditions is

$$v(t) = v(0) e^{-\alpha t t}, \text{ for } t \geq 0,$$

where $\frac{1}{v(0)}$ is the unit of the chosen time measure and $\alpha (> 0)$ is the damping coefficient of the weighting whose determination depends on seismological considerations.

Taking suitable integral mean values of $A(Q, \tau)$ one gets the seismic activity $A(Q, \sigma)$ at point $Q$ in a time-interval $\sigma$ of seismic activity $A(\Gamma, \tau)$ of region $\Gamma$ up to $\tau$ or, finally, of seismic activity $A(\Gamma, \sigma)$ of $\Gamma$ in $\sigma$. For these functions simple addition theorems again hold.

As a rule $F_i(Q, \tau)$ can be replaced by the complete energy density

$$F_i(Q, \infty) = E_i q_i(Q).$$

Contrasting with $p_i(Q)$, the distribution density $q_i(Q)$ is a physical function. It is obvious how to modify the definition of $S(Q)$, namely by substituting the statistically motivated function $p_i(Q)$ by $q_i(Q)$. In that way also seismicity would get a higher degree of physical significance. However, to be able to compute both seismicity as defined and seismic activity, we must first solve the problem of formulating $q_i(Q)$ mathematically.

This exercise is very important for seismology in many ways. The mathematical and numerical work connected with it, however, proves to be considerable already, even for the simplest focal models. Moreover, the hypothetical character of recently devel-
oped procedures for determining $E_i$ has to be borne in mind. With regard to this we consider it useful to try to construct $q_i(Q)$ on a heuristic principle. In this way we obtain under the simplest assumptions

$$q_i(Q) = \frac{1}{4\pi h_i^2} \exp \left[ - \left( \frac{R}{h_i} \sin \frac{Q_i Q}{2R} \right)^2 \right].$$

Of course, this formula is still very defective even if it is already quite useful for seismic cartography. We are therefore still trying to improve the expression for $q_i(Q)$.

Institut für Geodynamik, Jena,
Forschungsgemeinschaft der Deutschen Akademie der Wissenschaften zu Berlin,
69 Jena, Burgweg 11, D.D.R.

References


