Scattered Surface Waves from a Surface Obstacle

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Summary

This paper is concerned with the interpretation of a surface wave arrival at the Eskdalemuir seismological array which is apparently generated by the scattering of the initial $P$ wave.

The theory of scattering of a plane wave by a surface obstacle is developed in order to apply to a three-dimensional surface inclusion of any shape. Results are approximate, depending on the depth of the obstacle and the slope of all boundaries being small.

Comparison of theory with data from four events leads to an estimate of the breadth of the scatterer. A study of the relief in the area of the neighbourhood of the array strongly suggests that scattering is due to irregularities of surface topography.

1. Introduction

In a recent paper Key (1967) described how the Eskdalemuir array was used to filter signal-generated seismic noise for a variety of directions and velocities. A section of the records following the first onset of $P$ was used and a number of apparent noise sources were identified on the map of the area adjacent to the array. The most prominent of these was located in a deep river valley called Moffat Water, 13 km from the array. It appears that some kind of variation in elastic properties or change in surface topography is causing the incident $P$-wave to be scattered with a result that Rayleigh waves travel outwards from the source to the array.

Assuming that the irregularity of the crust giving rise to this scattering lies near or on the surface—which is reasonable considering that the Rayleigh waves are only about 2–3 km in wavelength—the properties of the scattered wave may be compared with those predicted by the theory on the subject.

Gilbert & Knopoff (1960) gave expressions for the scattered waves emanating from an irregularity in the surface of a half-space. However, only the two-dimensional problem was considered, and the method can in fact be used to deal with scattering from variations in elastic properties at the surface as well. We shall therefore extend the application of this method to three-dimensional obstacles of an arbitrary nature.

The results will be compared with Key's data in order to check the theory—i.e. to see if the simplifications involved are justified—and to ascertain something more of the nature of the obstacle.
2. Scattering from a surface obstacle

We consider a uniform half-space, whose properties are given by density \( \rho \) and Lamé constants \( \lambda \) and \( \mu \), indented at the surface by an intrusion of an elastic solid with properties given by \( \rho' \), \( \lambda' \) and \( \mu' \) (see Fig. 1).

\[ z = 0 \]
\[ y = f(x, y) \]
\[ y = g(x, y) \]

FIG. 1. The geometrical description of the obstacle.

If we use Cartesian axes \( Oxyz \) with \( Oz \) vertical and \( z = 0 \) on the surface outside the indentation, we may describe the upper surface of the intrusion by

\[ z = f(x, y), \]

and the interface between the two solids by

\[ z = g(x, y). \]

We denote by \( u^0 \) and \( \tau_{ij}^0 \) the displacement and stress tensor due to the incident wave being reflected from the surface of a perfectly uniform half-space; in other words, the displacement which would occur if the indentation were not present.

The displacement and stress in the half-space we write as

\[
\begin{align*}
\mathbf{u} &= \mathbf{u}^0 + \mathbf{u}' \\
\tau_{ij} &= \tau_{ij}^0 + \tau_{ij}'
\end{align*}
\]

where we assume

\[ |\mathbf{u}'| \ll |\mathbf{u}^0| \]

and

\[ |\tau_{ij}'| \ll |\tau_{ij}^0|. \]

The assumption that the scattering is small enables us to construct a first-order theory. It is justified if we take \(|f|\) and \(|g|\) to be everywhere small and the difference between \( \rho \) and \( \rho' \), \( \lambda \) and \( \lambda' \) and \( \mu \) and \( \mu' \) to be not large.

Following Gilbert & Knopoff (1960) we now show that the presence of the intrusion can be allowed for by using certain boundary conditions on the stresses at \( z = 0 \).

From ordinary elasticity theory we have the boundary conditions for a free boundary on \( z = f \),

\[ \tau_{nn}' = \tau_{n1}' = \tau_{n2}' = 0 \quad \text{at} \quad z = f(x, y), \]

and continuity of stress and displacement across \( z = g \),

\[
\begin{align*}
\mathbf{u} &= \mathbf{u}' \\
\tau_{nn} &= \tau_{nn}', \tau_{n1} = \tau_{n1}', \tau_{n2} = \tau_{n2}'
\end{align*}
\]

where \( \mathbf{u}' \) and \( \tau_{ij}' \) denote the displacement and stress within the intrusion and \( \tau_{nn}' \), \( \tau_{n1}' \) and \( \tau_{n2}' \) denote the normal and tangential stresses on the surface or interface (similarly \( \tau_{nn}, \tau_{n1}, \tau_{n2} \)).

The normal to the upper surface is given by \((f_x, f_y, -1)\) to the first order, assuming that the slope of the surface, given by \( f_x \left( = \frac{\partial f}{\partial x} \right) \) and \( f_y \left( = \frac{\partial f}{\partial y} \right) \) is everywhere small.
Therefore on $z = f$,
\[
\begin{align*}
\tau_{nn'} &= \tau_{xx'} - 2f_x \tau_{xx} - 2f_y \tau_{xy'}, \\
\tau_{nl'} &= l_i f_x \tau_{xx'} + m_i f_y \tau_{yy'} - n_i \tau_{xx} - m_i \tau_{xy'} - l_i \tau_{xx'} + (l_i f_y + m_i f_x) \tau_{xy'} \quad (i = 1, 2),
\end{align*}
\]
(2.6)
to the first order, where $(l_i, m_i, n_i)$, $i = 1, 2$, are two perpendicular directions in the tangent plane of the surface $z = f(x, y)$. They satisfy the equations
\[
\begin{align*}
l_i l_i + m_i m_i + n_i n_i &= \delta_{ij}. \quad (2.7)
\end{align*}
\]
From these we see that $n_i$ is of the first order in small quantities.

If we now assume that $|f|$ is very much less than the wavelength of the disturbance in the medium and that no shadow zones are formed by the waves, we can use Taylor's expansion to write
\[
\begin{align*}
\left[ \tau_{ij}' \right]_{z = f} &= \left[ \tau_{ij}' \right]_{z = 0} + f \left[ \frac{\partial \tau_{ij}'}{\partial z} \right]_{z = 0}.
\end{align*}
\]
(2.8)
Substituting into equation (2.6) we get
\[
\begin{align*}
\left[ \tau_{nn'} \right]_{z = f} &= \left[ \tau_{xx'} + f \frac{\partial \tau_{xx'}}{\partial z} - 2f_x \tau_{xx} - 2f_y \tau_{xy'} \right]_{z = 0}, \\
\left[ \tau_{nl'} \right]_{z = 1} &= \left[ l_i \left( -\tau_{xx'} - \frac{f \tau_{xx'}}{\partial z} + f_x \tau_{xx} + f_y \tau_{xy} \right) \right. \\
&\quad + m_i \left( -\tau_{yy'} - \frac{f \tau_{yy'}}{\partial z} + f_x \tau_{xy} + f_y \tau_{yy'} \right) \left. \right]_{z = 0}
\end{align*}
\]
(2.9)
to the first order.

We can write down similar equations for the stresses on $z = g(x, y)$ assuming that $|g|$ is very much less than the wavelength of the disturbance, that no shadow zones are formed, and the slopes $g_\alpha$ and $g_\beta$ are small.

The boundary conditions are now reduced to conditions on the plane $z = 0$. They are
\[
\begin{align*}
\tau_{xx'} + f \frac{\partial \tau_{xx'}}{\partial z} &= 0, \\
\tau_{xx'} + f \frac{\partial \tau_{xx'}}{\partial z} - f_x \tau_{xx} - f_y \tau_{xy'} &= 0, \\
\tau_{yy'} + f \frac{\partial \tau_{yy'}}{\partial z} - f_x \tau_{xy} - f_y \tau_{yy'} &= 0, \\
\tau_{zz'} + g \frac{\partial \tau_{zz'}}{\partial z} &= \tau_{zz'} + g \frac{\partial \tau_{zz'}}{\partial z}, \\
\tau_{xx'} + g \frac{\partial \tau_{xx'}}{\partial z} - g_x \tau_{xx} - g_y \tau_{xy} &= \tau_{xx'} + g \frac{\partial \tau_{xx'}}{\partial z} - g_x \tau_{xx} - g_y \tau_{xy}', \\
\tau_{yy'} + g \frac{\partial \tau_{yy'}}{\partial z} - g_x \tau_{xy} - g_y \tau_{yy} &= \tau_{yy'} + g \frac{\partial \tau_{yy'}}{\partial z} - g_x \tau_{xy} - g_y \tau_{yy}', \\
u_i + g \frac{\partial u_i}{\partial z} &= u_i' + g \frac{\partial u_i'}{\partial z} \quad (i = x, y, z),
\end{align*}
\]
(2.11)
to the first order, where all the stresses and their derivatives are evaluated at \( z = 0 \).

We have used the fact that \( \tau_{zz}^0 = \tau_{zx}^0 = \tau_{yz}^0 = 0 \) on \( z = 0 \) in the above equations.

These equations can now be reduced to boundary conditions on \( \tau_{zz}^s, \tau_{zx}^s \) and \( \tau_{yz}^s \) at \( z = 0 \).

To the zeroth order, at \( z = 0 \),

\[
\frac{\partial \tau_{zz}'}{\partial z} = \rho' \frac{\partial^2 u_z'}{\partial t^2} = \frac{\rho'}{\rho} \frac{\partial \tau_{zz}^0}{\partial z} \tag{2.13}
\]

and therefore equations (2.10) and (2.11) give

\[
[t_{zz}']_{z=0} = \left[ (g-f) - g \right] \left[ \frac{\partial \tau_{zz}^0}{\partial z} \right]_{z=0} . \tag{2.14}
\]

Furthermore, at \( z = 0 \), to the zeroth order,

\[
\tau_{xx}' = (\lambda' + 2\mu') \frac{\partial u_x'}{\partial x} + \lambda' \left( \frac{\partial u_y'}{\partial y} + \frac{\partial u_z'}{\partial z} \right) = \left( \lambda' + 2\mu' - \frac{\lambda'^2}{\lambda' + 2\mu'} \right) \frac{\partial u_x'}{\partial x} + \left( \lambda' - \frac{\lambda'^2}{\lambda' + 2\mu'} \right) \frac{\partial u_y'}{\partial y}
\]

\[
= \frac{2\mu'}{\lambda' + 2\mu'} \left[ 2(\lambda' + \mu') \frac{\partial u_x^0}{\partial x} + \lambda' \frac{\partial u_y^0}{\partial y} \right] = \frac{\mu'}{\mu} \tau_{xx}^0 + \Gamma (\tau_{xx}^0 + \tau_{yy}^0) , \tag{2.15}
\]

where

\[
\Gamma = \frac{2\mu' (\lambda' - \lambda\mu')}{\mu (3\lambda + 2\mu)(\lambda' + 2\mu')} .
\]

Similarly

\[
\tau_{yy}' = \frac{\mu'}{\mu} \tau_{yy}^0 + \Gamma (\tau_{xx}^0 + \tau_{yy}^0) \tag{2.16}
\]

and

\[
\tau_{xy}' = \frac{\mu'}{\mu} \tau_{xy}^0 .
\]

We now have, to the same order

\[
\frac{\partial \tau_{xx}'}{\partial z} = \rho' \frac{\partial^2 u_x'}{\partial t^2} - \frac{\partial \tau_{xy}'}{\partial y} - \frac{\partial \tau_{xx}'}{\partial x} = \left( \frac{\rho'}{\rho} - \frac{\mu'}{\mu} \right) \left( \frac{\partial \tau_{xx}^0}{\partial x} + \frac{\partial \tau_{xy}^0}{\partial y} \right) + \rho' \frac{\partial \tau_{xx}^0}{\partial z} - \Gamma \left( \frac{\partial \tau_{xx}^0}{\partial x} + \frac{\partial \tau_{yy}^0}{\partial x} \right) , \tag{2.17}
\]
and so, to the first order in small quantities,

\[ [\tau_{xx}]_{z=0} = \left( g-f \right) \frac{\mu'}{\rho} - g \left[ \frac{\partial \tau_{xx}^0}{\partial z} \right]_{z=0} - \left[ (g_x-f_x) \frac{\mu'}{\mu} - g_x \right] [\tau_{xx}^0]_{z=0} \]

\[ - \left[ (g_y-f_y) \frac{\mu'}{\mu} - g_y \right] [\tau_{xy}^0]_{z=0} + (g-f) \left[ \frac{\rho'}{\rho} - \frac{\mu'}{\mu} \right] \left[ \frac{\partial \tau_{xx}^0}{\partial x} + \frac{\partial \tau_{xy}^0}{\partial y} \right] \]

\[ - \Gamma \frac{\partial}{\partial x} ((g-f)[\tau_{xx} + \tau_{yy}]_{z=0}) \]  

(2.18)

Similarly,

\[ [\tau_{yy}]_{z=0} = \left( g-f \right) \frac{\mu'}{\rho} - g \left[ \frac{\partial \tau_{yy}^0}{\partial z} \right]_{z=0} - \left[ (g_x-f_x) \frac{\mu'}{\mu} - g_x \right] [\tau_{yy}^0]_{z=0} \]

\[ - \left[ (g_y-f_y) \frac{\mu'}{\mu} - g_y \right] [\tau_{xy}^0]_{z=0} + (g-f) \left[ \frac{\rho'}{\rho} - \frac{\mu'}{\mu} \right] \left[ \frac{\partial \tau_{xx}^0}{\partial x} + \frac{\partial \tau_{yy}^0}{\partial y} \right]_{z=0} \]

\[ - \Gamma \frac{\partial}{\partial y} ((g-f)[\tau_{xx} + \tau_{yy}]_{z=0}) \]  

(2.19)

The problem is now reduced to finding elastic displacements in a uniform half-space subject to the boundary conditions (2.14), (2.18) and (2.19) on the three components of the surface stress. This can be done using standard methods.

Gilbert's and Knopoff's equations should be recovered if we put \( f = g \) and \( \partial / \partial y \equiv 0 \). Equations (2.14) and (2.18) reduce to the required form except for a difference in sign. It appears that Gilbert and Knopoff have a wrong sign in their expressions for the stress on the surface.

3. Solution to the first order equations

For simplicity we rewrite equations (2.14), (2.18) and (2.19) as

\[ \tau_{xx}^s = T_1(x, y, t) \]

\[ \tau_{xy}^s = T_2(x, y, t) \]  

\[ \tau_{xx}^s = T_3(x, y, t) \]  

(3.1)

Outside the region of the indentation \( f = 0 \) and \( g = 0 \), so \( T_1, T_2 \) and \( T_3 \) will be zero outside a closed region in the \((x, y)\) plane.

The scattered displacements may be written as

\[ u^s = \text{grad} \phi + \text{curl} \psi, \]  

(3.2)

where \( \phi \) and \( \psi \) satisfy

\[ \nabla^2 \phi = \frac{1}{\alpha^2} \frac{\partial^2 \phi}{\partial t^2}, \]  

\[ \nabla^2 \psi = \frac{1}{\beta^2} \frac{\partial^2 \psi}{\partial t^2}, \text{ div} \psi = 0, \]  

(3.3)

\( \alpha \) and \( \beta \) being the dilatational and shear wave velocities in the half-space.
We assume that the incident wave is of limited duration so we may apply a Fourier transform, e.g.

$$\tilde{\phi}(x, y, z, \omega) = \int_{-\infty}^{\infty} \phi(x, y, z, t) \exp(i\omega t) dt.$$  \hspace{1cm} (3.4)

We also apply a Fourier transform of the space variables $x$ and $y$:

$$\phi^*(\xi, \eta, z, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\phi}(x, y, z, \omega) \exp[i(\xi x + \eta y)] dx dy.$$  \hspace{1cm} (3.5)

Equations (3.3) become ordinary differential equations in $z$. The solutions (ignoring those which increase indefinitely with increasing $z$) are

$$\phi^* = A(\xi, \eta, \omega) \exp(-\sigma z) \quad \psi^* = B_i(\xi, \eta, \omega) \exp(-\sigma' z), \quad i = x, y, z,$$

where

$$\sigma^2 = \xi^2 + \eta^2 - \frac{\omega^2}{\alpha^2}, \quad \text{Re}(\sigma) \geq 0,$$

$$\sigma'^2 = \xi^2 + \eta^2 - \frac{\omega^2}{\beta^2}, \quad \text{Re}(\sigma') \geq 0.$$

The boundary conditions at $z = 0$ become

$$2i\xi A + \xi \eta B_x + \left(\frac{\omega^2}{\beta^2} - 2\xi^2 - \eta^2\right) B_y + i\eta \sigma' B_z = \frac{T_1^*}{\mu}$$

$$2i\eta A - \left(\frac{\omega^2}{\beta^2} - \xi^2 - 2\eta^2\right) B_x - \xi \eta B_y - i\xi \sigma B_z = \frac{T_2^*}{\mu}$$

$$- \left(\frac{\omega^2}{\beta^2} - 2\xi^2 - 2\eta^2\right) A - 2i\eta \sigma' B_x + 2i\xi \sigma' B_y = \frac{T_3^*}{\mu},$$

with the last of equations (3.3) giving

$$i\xi B_x + i\eta B_y + \sigma' B_z = 0.$$  \hspace{1cm} (3.8)

Solving these equations we get expressions for $\phi^*$, $\psi_x^*$, $\psi_y^*$ and $\psi_z^*$. The Fourier transforms over $x$ and $y$ may be inverted to give the time transforms of $\phi$ and $\psi$; e.g.

$$\tilde{\phi} = \frac{1}{4\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{2i\xi \sigma' T_1^* + 2i\eta \sigma' T_2^* + [2(\xi^2 + \eta^2) - (\omega/\beta^2)] T_3^*}{[2(\xi^2 + \eta^2) - (\omega/\beta^2)]^2 - 4\sigma'^2(\xi^2 + \eta^2)} \times \exp\left[-i(x\xi + y\eta) - \sigma z\right] d\xi d\eta.$$  \hspace{1cm} (3.9)

Using the convolution theorem, we obtain

$$\tilde{\phi} = \frac{1}{4\pi^2 \mu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dX dY$$

$$\times \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 2i\xi \sigma' T_1(X, Y, \omega) + 2i\eta \sigma' T_2(X, Y, \omega) + [2(\xi^2 + \eta^2) - (\omega/\beta^2)] T_3(X, Y, \omega) \frac{[2(\xi^2 + \eta^2) - (\omega/\beta^2)]^2 - 4\sigma'^2(\xi^2 + \eta^2)}{[2(\xi^2 + \eta^2) - (\omega/\beta^2)]^2 - 4\sigma'^2(\xi^2 + \eta^2)} \times \exp\left[i(x(X-x) + \eta(y-y)) - \sigma z\right] d\xi d\eta \right\}. \hspace{1cm} (3.10)$$
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The inner integral becomes, by a change of variable

\[
\int_0^\infty \int_0^{2\pi} ds \, d\chi \left\{ \frac{2i\sigma's(T_1 \cos \chi + T_2 \sin \chi) + [2s^2 - (\omega^2/\beta^2)] T_3}{[2s^2 - (\omega^2/\beta^2)]^2 - 4s^2\sigma^2} \times \exp \{i\rho [(X - x)\cos \chi + (Y - y)\sin \chi] - \sigma z\} \right\}
\]

\[
= 2\pi \int_0^\infty \left\{ \frac{2\sigma'(s/R) J_0(Rs)}{[2s^2 - (\omega^2/\beta^2)]^2 - 4s^2\sigma^2} \right\} \exp(-\sigma z) ds,
\]

where

\[ R^2 = [(X - x)^2 + (Y - y)^2], \quad R \geq 0, \]

and in terms of the new variable of integration,

\[ \sigma' = s^2 - \frac{\omega^2}{\alpha^2}, \quad \sigma'' = s^2 - \frac{\omega^2}{\beta^2}. \]

This integral is in fact the response due to a surface point source acting as the point \((X, Y, 0)\) and applying stresses proportional to \(T_1(X, Y, t), T_2(X, Y, t)\) and \(T_3(X, Y, t)\) in the \(x, y,\) and \(z\) directions respectively. As would be expected \(\Phi\) and \(\Psi\) are written as convolution integrals over the response due to point sources.

We now follow Lapwood's (1949) method to separate the scattered Rayleigh wave from the body waves at large distances from the source. Lapwood distorted the contour of integration in the \(s\)-plane and showed that the Rayleigh wave contribution was given by the residue at the pole \(s = \pm \omega/\gamma,\) where \(\gamma\) is the real positive root of

\[
\left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) - \frac{4}{\gamma^2} \left( \frac{1}{\alpha^2} - \frac{1}{\beta^2} \right) + \left( \frac{1}{\gamma^2} - \frac{1}{\beta^2} \right) = 0.
\]

This is, of course, the Rayleigh equation, which has been shown to have only one pair of roots on the upper Riemann surface, defined by

\[
\text{Re} \left( \frac{1}{\gamma^2} - \frac{1}{\alpha^2} \right) \geq 0, \quad \text{Re} \left( \frac{1}{\gamma^2} - \frac{1}{\beta^2} \right) \geq 0.
\]

The Rayleigh wave contribution to \(\Phi,\) asymptotically for large \(\omega R/\gamma,\) is

\[
(\Phi)_R \sim \frac{\omega^{-1} \gamma^4}{\pi^4 \mu_{16}^{1/2}} \int_{-\infty}^\infty \int_{-\infty}^\infty \left\{ \frac{2\nu'}{\gamma'} \left[ T_1 \left( \frac{X-x}{R} \right) + T_2 \left( \frac{Y-y}{R} \right) \right] - i \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) T_3 \right\}
\]

\[
\times \exp \left( \frac{i\omega R}{\gamma} - \omega v'z - \frac{i\pi}{4} \right) \frac{dX \, dY}{\Delta \sqrt{R}}
\]

for \(\text{Re}(\omega) > 0,\) where

\[ \Delta = 2 \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) - 2v'v - \frac{1}{\gamma^2} \left( \frac{v}{\nu'} + \frac{v'}{v} \right), \]

and

\[ v = \left( \frac{1}{\gamma^2} - \frac{1}{\alpha^2} \right)^{1/4}, \quad v' = \left( \frac{1}{\gamma^2} - \frac{1}{\beta^2} \right)^{1/4}. \]
Similarly we find

\[
(\psi_x)_{R} \sim -\frac{\omega^{-\frac{1}{2}}\gamma^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\mu^{\frac{3}{2}}/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ i \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) \left[ T_1 \left( \frac{X-x}{R} \right) + T_2 \left( \frac{Y-y}{R} \right) \right] + \frac{2v}{\gamma} T_3 \right\} dX dY
\]

\[
\times \left( \frac{Y-y}{R} \right) \exp \left( \frac{i\omega R}{\gamma} - \omega \nu' z - \frac{i\pi}{4} \right) \frac{dX dY}{\Delta\sqrt{R}}.
\]

\[
(\psi_y)_{R} \sim \frac{\omega^{-\frac{1}{2}}\gamma^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\mu^{\frac{3}{2}}/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ i \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) \left[ T_1 \left( \frac{X-x}{R} \right) + T_2 \left( \frac{Y-y}{R} \right) \right] + \frac{2v}{\gamma} T_3 \right\} dX dY
\]

\[
\times \left( \frac{X-x}{R} \right) \exp \left( \frac{i\omega R}{\gamma} - \omega \nu' z - \frac{i\pi}{4} \right) \frac{dX dY}{\Delta\sqrt{R}}
\]

\[
(\psi_z)_{R} = 0.
\]

(3.14)

We can now write down expressions for the Fourier transforms of the surface displacements due to the scattered Rayleigh wave:

\[
\tilde{u}_x^R = -\frac{\omega^{-\frac{1}{2}}\gamma^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\mu^{\frac{3}{2}}/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{2i\nu'}{\gamma} \left[ T_1 \left( \frac{X-x}{R} \right) + T_2 \left( \frac{Y-y}{R} \right) \right] + \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) T_3 \right\} dX dY
\]

\[
\times \left( \frac{X-x}{R} \right) \exp \left( \frac{i\omega R}{\gamma} - \frac{i\pi}{4} \right) \frac{dX dY}{\Delta\sqrt{R}}
\]

\[
\tilde{u}_y^R = -\frac{\omega^{-\frac{1}{2}}\gamma^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\mu^{\frac{3}{2}}/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{2i\nu'}{\gamma} \left[ T_1 \left( \frac{X-x}{R} \right) + T_2 \left( \frac{Y-y}{R} \right) \right] + \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) T_3 \right\} dX dY
\]

\[
\times \left( \frac{Y-y}{R} \right) \exp \left( \frac{i\omega R}{\gamma} - \frac{i\pi}{4} \right) \frac{dX dY}{\Delta\sqrt{R}}
\]

\[
\tilde{u}_z^R = \frac{\omega^{-\frac{1}{2}}\gamma^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\mu^{\frac{3}{2}}/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) \left[ T_1 \left( \frac{X-x}{R} \right) + T_2 \left( \frac{Y-y}{R} \right) \right] - \frac{2i\nu}{\gamma} T_3 \right\} dX dY
\]

\[
\times \exp \left( \frac{i\omega R}{\gamma} - \frac{i\pi}{4} \right) \frac{dX dY}{\Delta\sqrt{R}}.
\]

(3.15)

4. Scattering of a plane P pulse

Suppose a plane P pulse, given by the potential function

\[
\phi = F\left( t - \frac{r, k}{\alpha} \right), \quad (4.1)
\]

(where \( r = (x, y, z) \) and \( k \) is a unit vector in the direction of travel of the pulse) is incident on the free surface.

The Fourier transform in time of this function is

\[
\tilde{\phi} = \tilde{F}(\omega) \exp \left( \frac{i\omega}{\alpha} r, k \right), \quad (4.2)
\]

where \( \tilde{F}(\omega) \) is the transform of \( F(t) \).
The displacement due to the reflection of this pulse from the plane surface of a uniform half-space is \( u^0 \) and is given by

\[
u^0 = \text{grad}\,\phi_0 + \text{curl}\,\psi_0
\]

where

\[
\phi_0 = F(\omega) \left[ \exp \left( \frac{i\omega}{\alpha} \cdot r \cdot k \right) + R \exp \left( \frac{i\omega}{\alpha} \cdot r \cdot k' \right) \right],
\]

\[
\psi_{0i} = F(\omega) R_{Si} \exp \left( \frac{i\omega}{\alpha} \cdot r \cdot k'' \right), \quad i = x, y, z;
\]  \( (4.3) \)

and writing

\[
k = (k_x, k_y, k_z),
\]

\[
k' = (k_x, k_y, -k_z)
\]

\[
k'' = (k_x, k_y, -k_z''),
\]

where

\[
k'' = \left( \frac{\alpha^2}{\beta^2} - k_x^2 - k_y^2 \right)^rac{1}{2}.
\]

The reflection coefficients \( R_\alpha \) and \( R_{Si} \) are functions of \( k_x, k_y, k_z \) and \( k_z'' \).

The stresses and their derivatives which we need for the boundary conditions on \( u^0 \) are

\[
\left[ \begin{array}{c} \tilde{\sigma}_{ij}^0 \nabla \Phi \nabla \psi \\ \frac{\partial \tilde{\sigma}_{ij}^0}{\partial z} \end{array} \right]_{z=0} = \frac{\omega^3}{\alpha^3} \mu F(\omega) \exp \left( \frac{i\omega}{\alpha} \cdot r \cdot k_0 \right) T_{ij},
\]

\( (4.4) \)

where

\[
k_0 = (k_x, k_y, 0)
\]

and

\[
T_{xx} = 8k_x k_z'' \left[ (2 - \frac{\alpha^2}{\beta^2}) (k_x^2 + k_y^2) - k_x^2 \frac{\alpha^2}{\beta^2} \right] / \Delta_1
\]

\[
T_{xy} = -8 \frac{\alpha^2}{\beta^2} k_x k_y k_x k_z'' / \Delta_1
\]

\[
T_{yy} = 8k_x k_z'' \left[ (2 - \frac{\alpha^2}{\beta^2}) (k_x^2 + k_y^2) - k_y^2 \frac{\alpha^2}{\beta^2} \right] / \Delta_1
\]

\[
T_{xx'} = 2k_z \left[ 2k_x^2 + 2k_y^2 - \frac{\alpha^2}{\beta^2} \right] \frac{4(k_x^2 + k_y^2) \left( 1 - \frac{\alpha^2}{\beta^2} \right) + \alpha^4}{\Delta_1}
\]

\[
T_{yy'} = 4(k_x^2 + k_y^2) \left( 1 - \frac{\alpha^2}{\beta^2} \right) + \alpha^4 / \Delta_1
\]

\[
T_{zz'} = -4k_x k_z k_z'' \left[ 4(k_x^2 + k_y^2) \left( 1 - \frac{\alpha^2}{\beta^2} \right) + \alpha^4 \right] / \Delta_1
\]

\[
T_{xy'} = -4k_y k_z k_z'' \left[ 4(k_x^2 + k_y^2) \left( 1 - \frac{\alpha^2}{\beta^2} \right) + \alpha^4 \right] / \Delta_1
\]

with

\[
\Delta_1 = 4k_x k_z'' (k_x^2 + k_y^2) + \left( 2k_x^2 + 2k_y^2 - \frac{\alpha^2}{\beta^2} \right)^2.
\]
We can now substitute these expressions into the equations for $T_1$, $T_2$ and $T_3$ so that these in turn may be substituted into equations (3.15) for the scattered Rayleigh wave displacements. Terms involving derivatives of $f(x, y)$ and $g(x, y)$ can be integrated by parts so that only $f$ and $g$ appear in the integrand. We get

$$\tilde{u}_x^R = \frac{\omega^2 \gamma^4 F(\omega)}{\pi^2 \lambda \beta^2 \sqrt{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{2\nu'}{\gamma} \cos \zeta \exp \left( i\omega \left( \frac{R}{\gamma} + \frac{R' \cdot k_0}{\alpha} \right) - \frac{i\pi}{4} \right) dX dY$$

$$\times \left\{ \left[ (g-f) \frac{\rho'}{\rho} - g \right] \left[ T_{xy}' \cos \zeta + T_{yx}' \sin \zeta - \frac{i\gamma}{2\nu'} \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) T_{xx}' \right] 
+ (g-f) \left[ \left( T_{xx} \cos \zeta + T_{xy} \sin \zeta \right) \left( \frac{\rho'}{\rho} k_x + \frac{\mu'}{\mu} \frac{\alpha}{\gamma} \cos \zeta \right) 
+ \left( T_{xy} \cos \zeta + T_{yx} \sin \zeta \right) \left( \frac{\rho'}{\rho} k_y + \frac{\mu'}{\mu} \frac{\alpha}{\gamma} \sin \zeta \right) \right] \right\},$$

where

$$\cos \zeta = \frac{X-x}{R}, \quad \sin \zeta = \frac{Y-y}{R}$$

and

$$R' = (X, Y, 0).$$

Similar expressions can be constructed for $\tilde{u}_y^R$ and $\tilde{u}_z^R$.

The next step, in general, would be to substitute for $f$ and $g$ in the integrand, using physical intuition to form realistic estimates of these two functions. However we shall first make some simplifying deductions from the properties of the scattered signal we wish to investigate.

5. Scattering from Moffatt Water

Key noticed in his investigation (1966) that the scattered Rayleigh wave associated with Moffat Water appeared to come from exactly the same point though the incident waves generating the scattering came from different directions.

It can be seen from the expressions for $\tilde{u}_x^R$ and $\tilde{u}_y^R$ that their ratio (giving the direction of approach of the Rayleigh wave) is not constant for different values of $k$. However, variations in this ratio may be small if the angle subtended by the obstacle at the point of observation is small.

If we assume that this is so, we may choose the directions of the $x$- and $y$-axes so that $\zeta$ is everywhere small in the integrand of equation (4.6). This entails taking the $x$-axis to pass through the centre of the obstacle. We can now expand the integrand in ascending powers of $\zeta$ to get a convergent series.
The first term in the series for $\tilde{u}_y^R$ gives
\[
\tilde{u}_y^R \approx \frac{\omega^3 \gamma \beta^2 \Lambda 16/2}{\pi^3 \alpha^3 \beta^2 \Delta 16/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \left( g - f \right) \frac{\rho'}{\rho} - g \right] \left[ T_{xx}' - \frac{iy}{2\gamma'} \left( \frac{2}{\gamma'^2} - \frac{1}{\beta^2} \right) T_{zz}' + \left( k_x + \frac{\alpha}{\gamma} \right) T_{xx} + \frac{\alpha}{\gamma} \left( \frac{\mu'}{\mu} - \frac{\rho'}{\rho} \right) T_{xy} + \Gamma (T_{xx} + T_{yy}) \right] dX dY.
\]
\[
\times \zeta \exp \left\{ i\omega \left( \frac{R}{\gamma} + \frac{R' \cdot k_0}{\alpha} \right) - \frac{i\pi}{4} \right\}.
\]
(5.1)

Since, by observation, $\tilde{u}_y^R$ is zero for a variety of values of $T_{xx}, T_{xy}, T_{yy},$ etc., we may deduce that
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \zeta g(X, Y) \exp \left\{ i\omega \left( \frac{R}{\gamma} + \frac{R' \cdot k_0}{\alpha} \right) \right\} dX dY \approx 0
\]
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \zeta f(X, Y) \exp \left\{ i\omega \left( \frac{R}{\gamma} + \frac{R' \cdot k_0}{\alpha} \right) \right\} dX dY \approx 0
\]
(5.2)
for the range of $\omega$ covered by the data and for all $k_0$.

We may now expand $\tilde{u}_x^R$ and $\tilde{u}_z^R$ in a similar way to get approximations to the horizontal and vertical displacements due to the scattered Rayleigh wave. Using equations (5.2) we find that the first order terms are negligible; so, to the first order,
\[
\tilde{u}_x^R = \frac{F(\omega) \omega^3 \gamma \beta^2 \Lambda 16/2}{\pi^3 \gamma^3 \beta^2 \Delta 16/2} \left( T_{xx}' - \frac{iy}{2\gamma'} \left( \frac{2}{\gamma'^2} - \frac{1}{\beta^2} \right) T_{zz}' + \left( k_x + \frac{\alpha}{\gamma} \right) T_{xx} \right.
\]
\[
+ \left( k_y T_{xy} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ (g - f) \frac{\rho'}{\rho} - g \right] \exp \left\{ i\omega \left( \frac{R}{\gamma} + \frac{R' \cdot k_0}{\alpha} \right) \right\} dX dY
\]
\[
+ \left( \frac{\mu'}{\mu} - \frac{\rho'}{\rho} \right) T_{xx} + \Gamma (T_{xx} + T_{yy}) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (g - f) \exp \left\{ i\omega \left( \frac{R}{\gamma} + \frac{R' \cdot k_0}{\alpha} \right) \right\} dX dY \right).
\]
(5.3)

One further approximation may be made. If $\omega k_y (X - x)/\alpha$ is of the order of, or less than, unity (i.e. the apparent surface wavelength of the incoming $P$ pulse is around the same size, or greater than, the distance from the scatterer to the observer) then we have $\omega k_y (Y - y)/\alpha$ to be small. Similarly, we may expect $\omega (Y - y)^2/\gamma (X - x)$ to be small. The second integral in the first of equations (5.3) becomes
\[
\exp \left\{ i\omega \left( \frac{k_x}{\alpha} y - \frac{x}{\gamma} \right) \right\} \int_{-\infty}^{\infty} \left[ g_1(X) - f_1(X) \right] \exp \left\{ i\omega X \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right\} dX,
\]
(5.4)
where
\[
g_1(X) = \int_{-\infty}^{\infty} g(X, Y) dY
\]
\[
f_1(X) = \int_{-\infty}^{\infty} f(X, Y) dY.
\]
Now
\[
\int_{-\infty}^{\infty} g_1(X) \exp \left[ i \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right] dX = \tilde{g}_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right],
\]
(5.5)

the bar denoting, as usual, the Fourier transform.

Therefore the transform of the vertical component of the scattered displacement (the vertical component is that measured by the Eskdalemuir array) is
\[
\tilde{u}_z = \frac{i \omega^3 \gamma^4 (2 / \gamma^2 - 1 / \beta^2) \exp \left( -i \pi / 4 \right) \tilde{F}(\omega)}{\pi^4 \alpha^3 \beta^2 \Delta x \Delta y \Delta z} \left\{ \left[ T_{xx} - \frac{i \gamma}{2 \nu} \left( \frac{2}{\gamma^2} - \frac{1}{\beta^2} \right) T_{zz} \right] + \left( k_x + \frac{\alpha}{\gamma} \right) T_{xx} + k_y T_{yy} \right\} \left[ (\tilde{g}_1 - f_1) \frac{\rho'}{\rho} - \tilde{g}_1 \right]
+ \frac{\alpha}{\gamma} \left[ \left( \frac{\mu'}{\mu} - \frac{\rho'}{\rho} \right) T_{xx} + \Gamma (T_{xx} + T_{yy}) \right] (\tilde{g}_1 - f_1),
\]
(5.6)

where
\[
\tilde{g}_1 = \tilde{g}_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right],
\]
\[
f_1 = f_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right].
\]

Equation (5.6) is still rather complicated for comparison with the experimental data. Three simple cases may be identified. First, if the upper surface of the solid medium is everywhere plane, i.e. \( f = 0 \). In this case, the Fourier spectrum of the scattered wave is proportional to
\[
\tilde{F}(\omega) \omega^3 \tilde{g}_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right].
\]
(5.7)
The Fourier spectrum of the displacement of the incident wave is proportional to \( \omega F(\omega) \), and so the ratio of these two is proportional to
\[
\omega^4 \tilde{g}_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right]
\]
(5.8)
for all direction of incidence.

A second case is when there is no intrusion of a second type of solid; i.e. \( g \equiv f \).
In this case the ratio of the spectra is again proportional to
\[
\omega^4 \tilde{g}_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right].
\]

A third case is when isostasy holds; i.e. \( (g-f) \rho' = gp \). The first term in equation (5.6) disappears and the ratio of the spectra is again proportional to expression (5.8).

6. Comparison with data

The data used were the Eskdalemuir array recordings of four earthquakes; these were at Kamchatka, 1963 July 30, of magnitude 5.3, another at Kamchatka, 1963 September 7, of magnitude 5.4, at Kuriles, 1964 October 23, of magnitude 5.6, and the Longshot explosion of 1965 October 29.
The recordings of the separate seismometers were summed, using time delays which gave the best correlated output, for both the incident P-arrival and the scattered
Scattered surface waves from a surface obstacle

Table 1

Data used in the analysis

<table>
<thead>
<tr>
<th>Date</th>
<th>Area</th>
<th>Magnitude</th>
<th>Azimuth (P)</th>
<th>Surface velocity (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963 July 30</td>
<td>Kamchatka</td>
<td>5.3</td>
<td>12°</td>
<td>19.2 km/s</td>
</tr>
<tr>
<td>1963 September 7</td>
<td>Kamchatka</td>
<td>5.4</td>
<td>10°</td>
<td>18.2 km/s</td>
</tr>
<tr>
<td>1964 October 23</td>
<td>Kuriles</td>
<td>5.6</td>
<td>15°</td>
<td>20.2 km/s</td>
</tr>
<tr>
<td>1965 October 29</td>
<td>Rat Isles (Longshot)</td>
<td>6.1</td>
<td>358°</td>
<td>24.0 km/s</td>
</tr>
</tbody>
</table>

Rayleigh wave: azimuth 315°, surface velocity 2.5 km/s.

Rayleigh wave. This provides a measurement of the azimuth and surface velocity for each arrival. The results are listed in Table 1.

It was found that the azimuth of the scattered wave was the same in each case. The sampling interval for the azimuth was 5°, or 0.09 radians, we may take the azimuth of the Rayleigh arrival to be constant with an error of less than one in ten. Thus the approximations leading to the expressions (5.3) for the displacement of the scattered wave are in error by approximately one in a hundred.

Rayleigh waves, of course, show dispersion when propagating through layered media, and so there is no single velocity for the Rayleigh arrival. However, the frequencies involved are fairly high and the available band width narrow (see Fig. 2), so we shall ignore the effects of dispersion.

The distance of the source of scattering from the observation point is given by the surface velocities and arrival times of the direct $P$ and scattered Rayleigh waves.

![Amplitude spectra of the $P$ and Rayleigh ($R$) waves from the Kamchatka (July) event.](https://academic.oup.com/gji/article-abstract/13/4/441/709439/13.441.004-038)
It was found to be 13 km. This means that the factor $\omega R/y$ which was assumed to be large to justify the use of the asymptotic expansion of the expressions (3.13) for the Rayleigh wave, is approximately 10.

One further approximation which may be checked is that the surface wavelength of the direct $P$-wave should be of the order of, or greater than, the distance from source to observer. It can be seen from Table 1 that at frequencies of about 2 c/s this condition is satisfied.

The summed signal for each arrival was Fourier analysed so that the ratios of the Rayleigh wave amplitude to the $P$-wave amplitude could be measured at different frequencies. The recorded pulses were quite short in duration (4–4½ s) so that the intervals between points on the Fourier spectrum are large. In addition, the band width imposed by the recording apparatus is narrow (see Fig. 2), so rather few points were obtained.

The amplitude of the noise preceding each event was measured and this was taken to be an estimate of the possible error.

![Diagram](https://academic.oup.com/gji/article-abstract/13/4/441/709439)

**Fig. 3(a).** The function $g_1(X)$

**Fig. 3(b).** The function $f_1(k)$

The curves with the broken lines describe equations (6.1), and those with continuous lines, equations (6.2).
The amount of data remaining when all the points with unreasonably large errors had been eliminated were small; five points from the Kamchatka (July) event, three points from the Kamchatka (September) event, two points from the Kuriles event and three points from Longshot.

Five points do not justify curve fitting and Fourier inversion to find the function \( g_1 \), giving the shape of the obstacle.

It was thought, however, that a simple form for \( g_1 \) might be used in order to calculate \( \tilde{g}_1 \left[ \frac{1}{\gamma} + \frac{k_x}{\alpha} \right] \) and the points for each event fitted by least squares. The form of \( g_1 \) would contain two or three parameters, related to the height and width of the scattering obstacle, and estimates of these quantities could be obtained by this method.

If
\[
g_1(X) = h, \quad |X| < d
\]
\[
= \left[ \frac{b-|X|}{b-d} \right] h, \quad d < |X| < b
\]
\[
= 0, \quad b < |X|,
\]
then
\[
\tilde{g}_1(k) = \frac{4h}{k^2(b-d)} \sin k \left( \frac{b+d}{2} \right) \sin k \left( \frac{b-d}{2} \right).
\]

If
\[
g_1(X) = h \exp \left( -\frac{X^2}{a^2} \right),
\]
then
\[
\tilde{g}_1(k) = ha \sqrt{\pi} \exp \left( -\frac{k^2 a^2}{4} \right).
\]

(see Fig. 3).

Either of the two forms for \( g_1 \) could be used (a simpler form of (6.1) is obtained by putting \( d = 0 \)). We used the second example here to fit the data.

The ratio of the spectral amplitudes is equal to

\[
K \omega^\phi \tilde{g}_1 \left[ \omega \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right) \right]
\]

by equations (5.6) and (5.8) where \( K \) is independent of frequency but depends on the azimuth of the P-wave. We fitted the data points by least squares to the expression

\[
K' \omega^\phi \exp \left[ -\frac{a^2 \omega^2}{4} \left( \frac{1}{\gamma} + \frac{k_x}{\alpha} \right)^2 \right],
\]

obtaining at the same time estimates of \( K' \) and \( a \) from each event.

The results are listed on Table 2. Understandably, there are large variations in the value of \( a \), which is an approximate measure of the length of the obstacle along a

<table>
<thead>
<tr>
<th>Event</th>
<th>( a )</th>
<th>( K' )</th>
<th>( K' ) (using ( a = 5.2 ) km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kamchatka (July)</td>
<td>5.5 km</td>
<td>1.85</td>
<td>1.55</td>
</tr>
<tr>
<td>Kamchatka (September)</td>
<td>5.0 km</td>
<td>1.45</td>
<td>1.41</td>
</tr>
<tr>
<td>Kuriles</td>
<td>2.9 km</td>
<td>0.54</td>
<td>0.80</td>
</tr>
<tr>
<td>Longshot</td>
<td>6.1 km</td>
<td>2.53</td>
<td>1.61</td>
</tr>
</tbody>
</table>
FIG. 4. The ratio of the Rayleigh wave to the \( P \) wave spectral amplitudes for the Kamchatka (July) event.

FIG. 5. Comparison of all data points for the ratio of Rayleigh to \( P \) wave spectral amplitudes with the mean theoretical curve (using \( a = 5.2 \text{ km} \)) (\( \times \) Kamchatka (July), +Kamchatka (September), • Kuriles, ○ Longshot).
straight line from the obstacle to the observer. The best set of data came from the Kamchatka (July) event, and the accuracy of the fit of the theoretical curve is shown in Fig. 4.

We can obtain a mean estimate of $a$ from the four values obtained above, weighting each value according to the number of data points used in its calculation. By this method

$$a = 5.2 \text{ km.}$$  \hfill (6.4)

With this value of $a$, new estimates of $K'$ were made for each event (see Table 2) and theoretical points calculated for each data point. It was found that the difference between the observed and the theoretical points always lay within the possible error. Fig. 5 shows all the data, scaled to be able to be presented on a single graph, and the mean theoretical curve.

Fig. 6 is a contour map of the neighbourhood of Moffatt water. The continuous line running SE–NW is the direction of the scattered signal observed at the Eskdalemuir array. The two broken lines on either side are inclined at $5^\circ$ to this direction ($5^\circ$ is the resolution of the array).

The two broken lines enclosing the river valley are 5.2 km apart, the theoretical width of the scatterer. It can be seen that this distance is a little greater than the distance from peak to peak on either side of the valley. Thus, assuming that scattering is due to topographic features, our theoretical estimate appears to be a reasonably good one.

Fig. 6. Relief map (1 inch to 1 mile) of the neighbourhood of Moffatt Water. (Neighbouring contours show changes of 250 ft in height.)
A study of a relief map of the area surrounding the seismograph array shows that the Moffatt river valley is the steepest piece of terrain for some miles in either direction. This corresponds with the fact that the scattered signal from Moffatt Water is by far the most prominent recorded at the array.

We may conclude, therefore, that the scattering is due to the steep nature of the sides of the valley, and may be expected wherever seismographs are established in a region of sharply uneven terrain.

7. Conclusions

It appears that first order scattering theory can usefully be compared with the data available from arrays.

The data used here were not very good, but if the results from several events with the clarity of the Kamchatka (July) event could be found, deductions from the data could be made with greater confidence. Results could also be taken into greater detail.

It is interesting to compare estimates of the scattering obstacle with surface features. It would, moreover, be very useful to be able to predict the shape of the scattered arrival in a recording where it has been obscured by noise, so that it can be removed to facilitate the analysis of other, nearby, arrivals. The present estimate of \( a \) together with the variation of \( K' \) with azimuth given by equation (5.6) could be used in this way at Eskdalemuir.

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