On the application of storage coefficient determination by quasi-steady-state flow

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Abstract A simple analytical method has been used for estimating the storage coefficient provided that transmissivity of the aquifer is known at the quasi-steady-state condition in confined or unconfined aquifers. The application of the method has been performed for unconfined and confined aquifer test data in Chaj Doab, Pakistan with observation wells and compared with conventional methods in the groundwater flow literature dealing with pumping tests. The results from the methodology presented in this paper conform well in practice with the results obtained from the traditional methods on the basis of order of magnitude.

Keywords Aquifers; groundwater; quasi-steady state; storativity; well

Introduction

A pumping test is a controlled field experiment aimed at determining the basic aquifer parameters in the vicinity of a pumping well in groundwater studies. No prediction of drawdown produced by pumping wells is possible without information on these parameters. Generally, in pumping test measurements, constant discharge and variation of drawdown with time are observed in the main well and/or in one or more observation wells in its vicinity.

Many researchers in the literature have studied the determination of the aquifer parameters both in confined and unconfined aquifers. Among these are approaches proposed by Theis (1935), Jacob (1940), Chow (1952), Hantush (1956), Ferris (1963), Boulton (1963), Şen (1982, 1986, 1987) and others. However, since the construction of an observation well is expensive, the alternative methods for determining the aquifer characteristics using the pumping tests recorded in the abstraction well itself have been presented by Papadopulos and Cooper (1967) and Şen (1982). Although transmissivity can be estimated with a reasonable accuracy from the abstraction well data using these methods, it is not possible to calculate the storage coefficient from the pumped well data reliably. Later, Şen (1987) has proposed a unique storage coefficient determination technique for confined and unconfined aquifer tests.

In this study, the method proposed by Şen (1987) has been extended and applied to two sets of field data for the storage coefficient estimation for both confined and unconfined aquifer conditions. The results are compared with other methods already available in the literature. In this paper, the Şen method is also used for observation well time-drawdown data and the results show good agreement with the other methods in the literature. Hence, this methodology can be considered as a supplementary procedure to type curve matching where only the transmissivity value can be estimated reliably.

Theoretical background

Şen (1987) has suggested a method for estimating the storage coefficient of an aquifer using pumping well data from quasi-steady-state groundwater flow towards wells. The quasi-steady-state groundwater is related to large time groundwater flow towards the main well where the drawdown variations are practically negligible. This is the case where the Thiem (1906) method is also applicable. In practice, during a pumping test, if two successive late-time drawdown measurements are different from each other at less than 5% relative error, this situation may be considered as a quasi-steady-state groundwater flow by Şen (1995). It is this method that provides a quick and economical determination of the storage coefficient without any need for observation well in the case of time-drawdown data from large diameter pumping wells. In general, the storage coefficient, S, is defined as the ratio of water volume, $V_w(t)$, abstracted during the whole pumping test operation from the aquifer to the depression cone volume, V_D. Hence

$$S = \frac{V_w(t)}{V_D} \tag{1}$$

in which t indicates the duration of the pumping test. Based on this definition Şen (1987) has presented an explicit derivation of Eq. (1) for confined and unconfined aquifers which is not repeated here.

The formulae for the storage coefficient, S, estimation in confined and unconfined aquifers are given as

$$S = \frac{Qt - \pi r_w^2 s_w(t)}{r_w^2 Q\left\{\exp\left[\frac{4\pi T s_w(t)}{Q}\right] - 1\right\}/4T - \pi r_w^2 s_w(t)} \tag{2}$$

and

$$S = \frac{Qt - \pi r_w^2 s_w(t)}{\frac{1}{2} r_w^2 \sqrt{\frac{2\pi DQ}{T}} \exp\left\{-\frac{2\pi DT[1 - s_w(t)/D]^2}{Q}\right\}\int_a^b e^{x^2}dx - \pi r_w^2 s_w(t)} \tag{3}$$

respectively. Here, r_w is the well radius, $s_w(t)$ is the drawdown in the main well, Q is the pump discharge which is equal to the discharge from the aquifer into the well in the case of quasi-steady state flow, i.e. at large time-drawdown cases, D is the aquifer thickness, T is the transmissivity and x is a dummy variable.

Furthermore, in Eq. (3) a and b constants are given explicitly as

$$a = \sqrt{\frac{2\pi DT}{Q}} \tag{4}$$

$$b = \sqrt{\frac{2\pi DT}{Q}}\left[1 - \frac{s_w(t)}{D}\right] \tag{5}$$

Although these formulations are claimed to be valid for time-drawdown data from pumping wells, it has been shown in practice in this paper through applications to time-drawdown data from observation wells that the same formulae are equally valid. The reason for such an approach is due to the fact that Şen (1987) has not considered the well loss effects in his derivations. By taking the drawdowns in the main well he neglects the well losses completely. However, in a pumping test, the time-drawdown observations recorded in any observation well are not affected by well losses. On the other hand, for large time aquifer tests, the term Qt will be much bigger than the well storage volume contribution, namely, $\pi r_w^2 s_w(t)$ such that this contribution may be neglected. Similar arguments apply for the denominators in Eqs. (2) and (3) especially in unconfined aquifers. Hence, removal of $\pi r_w^2 s_w(t)$ from both equations leads to

$$S = \frac{4tT}{r_w^2\left\{\exp\left[\frac{4\pi T s(t)}{Q}\right] - 1\right\}} \tag{6}$$

and

$$S = \frac{2Qt}{r_w^2\sqrt{\frac{2\pi DQ}{T}}\exp\left\{-\frac{2\pi DT[1-s_w(t)/D]^2}{Q}\right\}\int_a^b e^{x^2}dx} \tag{7}$$

respectively. Let us first confine ourselves to Eq. (6) and make rearrangements by considering a dimensionless time factor, $u = r^2S/4tT$ and a well function $W(u) = (4\pi T/Q)s$ as known from any textbook on groundwater. It is then possible to write Eq. (6) as

$$u_w = \frac{1}{e^{W(u_w)} - 1} \tag{8}$$

This is a dimensionless expression which yields the same result provided that u_w and $W(u_w)$ remain the same. Fig. 1 shows that Eq. (8) approaches Thies (1935) type curves for large times. This indicates that consideration of the observation wells instead of the main well, as suggested by Şen (1987) for Eqs. (2) and (3), is possible, since the Thies solution is valid for observation well time-drawdown data. Papadopulas and Cooper curves are also shown in the same figure. Hence, it does not make any difference whether the well radius r_w or r, the distance of the observation well to the main well, are considered at large times. Similar arguments are possible for an unconfined aquifer. Consequently, the adaptation of Eqs. (2) and (3) for an observation well at a radial distance, r, from the pumping well renders them into the following forms. First, the storage coefficient estimate for a confined aquifer is

$$S = \frac{Qt - \pi r^2 s(t)}{r^2Q\left\{\exp\left[\frac{4\pi T s(t)}{Q}\right] - 1\right\}/4T - \pi r^2 s(t)} \tag{9}$$

and then for the unconfined aquifers similar to Eq. (2) one can write

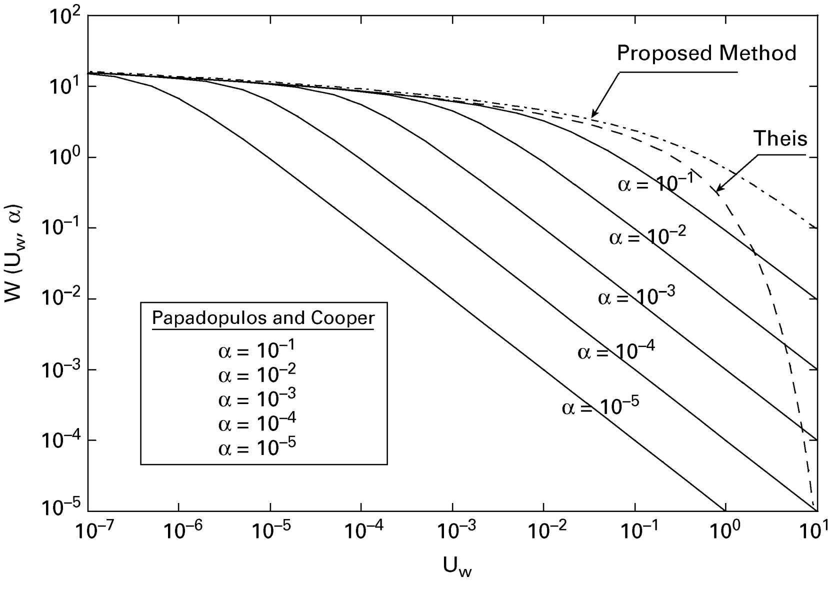

Figure 1 Comparison of methods

$$S=\frac{Qt-\pi r^2 s(t)}{\frac{1}{2}r^2\sqrt{\frac{2\pi DQ}{T}}\exp\left\{-\frac{2\pi DT[1-s(t)/D]^2}{Q}\right\}\int_a^b e^{x^2}dx-\pi r^2 s(t)} \tag{10}$$

where $s(t)$ is the drawdown in an observation well.

Applications to field data

Two extensive field pump-test data have been used for the application of the methodology for both confined and unconfined aquifer conditions in order to demonstrate the ability of the proposed method described above. The study area is sandwiched between Jhelum and Chenab in the northeastern part of Pakistan and locally called Chaj Doab. Aquifer test data from this region of Pakistan are already presented and analysed by Ahmad (1998) using the classic Thies method. The same data sets are selected for storage coefficient estimation by the methodology proposed here. The first data set is taken from the CTW-29 site consisting of 7 observation wells (O1, O2, O3, O4, O5, O6 and O7) with the main well (P29) shown in Fig. 2. The second one is CTW-44 with 6 observation wells (O1, O2, O3, O4, O5 and O6) and the main well (P44) shown in Fig. 3. The pumping test was carried out by WASID (Water and Soil Investigation Division) of WAPDA (Water and Power Development Authority) in Pakistan. The data are available in the report *Basic Data Release No. 9, Volume II, Directorate General of Hydrogeology, WAPDA, Lahore, 1983*.

The first pump-test data consist of drawdowns in the main well (P29) and at the seven observation wells from the pumping well caused by pumping at a constant rate of $Q = 0.1133\,\mathrm{m}^3/\mathrm{s}$ at the CTW-29 site. The diameter of the pumping well is 25.4 cm (10 inches). The time-drawdowns from the pumping well and the observation wells for 5000 min duration are shown in Fig. 4. This aquifer is an unconfined type according to Ahmad (1998). The pumping discharge during the test at the CTW-44 site is kept constant as $Q = 0.084\,95\,\mathrm{m}^3/\mathrm{s}$. The time-drawdown data on natural scale for the pumping well and the 6 observation wells are shown in Fig. 5. This aquifer is an confined type according to Ahmad (1998). Selected late drawdown data at the observation wells that were taken together were analysed separately for each data set.

Transmissivity and storage coefficients from the aquifer data of all the observation wells in CTW-29 and CTW-44 are computed according to the Thies method (considering all drawdowns) for pumping well P29 (Ahmad 1998). Three alternative Jacob straight-line methods are applied for the treatment of field data. These are the time-drawdown (DT), distance-drawdown (DD) and composite variable (CV) approaches which are explained in detail by Şen (1995). All these three methods are valid for late time data.

In this study, the same observation well data (considering late drawdowns) are selected for storage coefficient estimation by the methodology proposed in the previous section. For this purpose, Eq. (10) is used for the determination of the storage coefficient of the

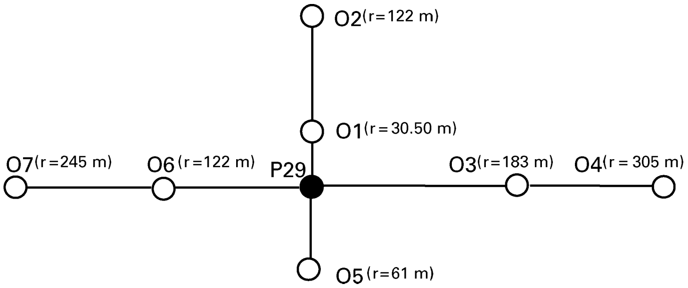

Figure 2 Plan view of PW and observation wells at CTW-29

Figure 3 Plan view of PW and observation wells at CTW-44

considered unconfined aquifer (CTW-29) and Eq. (9) is used for the confined aquifer (CTW-44) in this paper.

The results of values both for the pumping well itself and the 7 observation wells by using different methods are shown collectively in Table 1 for the unconfined aquifer of CTW-29.

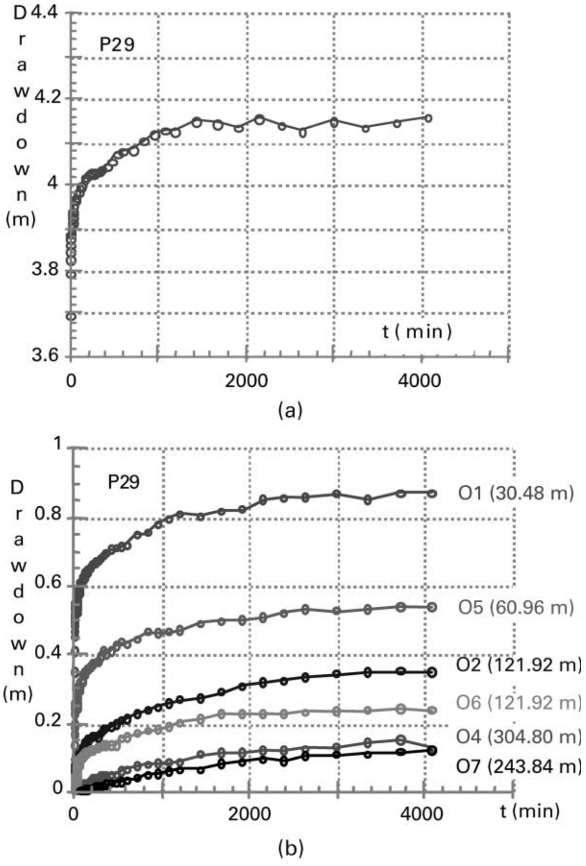

Figure 4 (a) Pumping well time-drawdown, (b) observation wells in site CTW-29

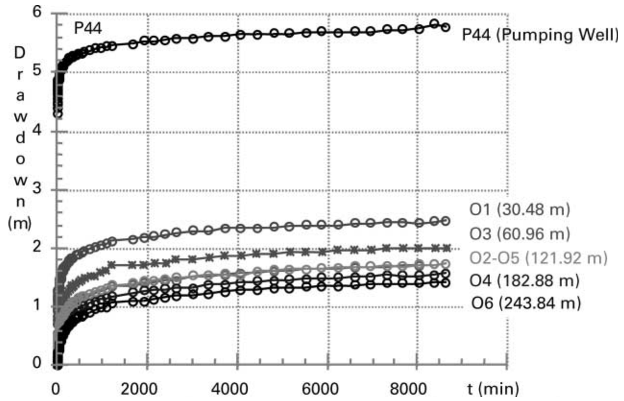

Figure 5 Pumping well and observation wells time-drawdown in site CTW-44

On the other hand, Table 2 shows the results of computed storage coefficient values in the confined aquifer of CTW-44 for both the main well and 6 observation wells along with the other methods.

Comparison of the storage coefficients indicates that the method proposed here yields results that are practically in good agreement with conventional methods. However, it is obvious that, although there is a good agreement between the methodology proposed herein and the other methods which are commonly used in the storage coefficient estimation, they both differ significantly from the results of Şen's (1987) solution. According to his approach, the storage coefficient value is extremely low ($\sim 10^{-40}$) for an unconfined aquifer at site CTW-29. Among some other factors this discrepancy is mainly due to the fact that his method does not consider well losses in the main well during the pumping. However, the modification of his method proposed here for the application of time-drawdown data in observation wells yields almost the same storage coefficient values as for the other methods mentioned above. In the application of the methodology proposed herein for the calculation of storage coefficient values, the transmissivity value is calculated from the Thies method, as stated (Ahmad 1998). The storage coefficient values of the aquifers are calculated as the average of six observations wells' storage coefficient values independently for the unconfined aquifer in the CTW-29 site and seven observation wells in the CTW-44 site.

Table 1 Comparison of storage coefficient values with the conventional methods at the CTW-29 site (unconfined aquifer)

		Jacob method			Şen (1987) analytical method
Well no.	**Thies method**	**TD**	**DD**	**CV**	S_a
P29	–	–	–	–	$\sim 10^{-40}$
O1	2.7×10^{-5}	1.3×10^{-3}	2.7×10^{-2}	4.7×10^{-5}	3.0×10^{-5}
O2	3.5×10^{-3}	9.4×10^{-3}		9.8×10^{-3}	1.9×10^{-3}
O4	2.9×10^{-2}	7.0×10^{-3}		2.0×10^{-2}	5.1×10^{-3}
O5	3.1×10^{-4}	2.0×10^{-3}		1.7×10^{-3}	1.0×10^{-4}
O6	1.5×10^{-2}	1.2×10^{-2}		9.3×10^{-3}	3.9×10^{-3}
O7	1.6×10^{-1}	4.7×10^{-2}		1.3×10^{-1}	4.5×10^{-3}
Aquifer average	3.5×10^{-2}	1.3×10^{-2}	2.7×10^{-2}	2.8×10^{-2}	2.6×10^{-3}

Şen (1987) method for P29 = $\sim 10^{-40}$.

Modified Şen method (proposed method herein) for O1, O2, O4, O5, O6 and O7.

Table 2 Comparison of storage coefficient values with the conventional methods in the CTW-44 site (confined aquifer)

		Jacob method			**Şen (1987) analytical method**
Well no.	**Thies method**	**TD**	**DD**	**CV**	S_a
P44	–	–	–	–	2.4×10^{-5}
O1	2.9×10^{-4}	2.3×10^{-4}	1.4×10^{-3}	3.8×10^{-4}	1.4×10^{-3}
O2	7.0×10^{-4}	5.9×10^{-4}		6.3×10^{-4}	3.5×10^{-3}
O3	2.5×10^{-3}	7.9×10^{-4}		8.4×10^{-4}	4.0×10^{-3}
O4	4.0×10^{-4}	5.3×10^{-4}		6.6×10^{-4}	4.2×10^{-3}
O5	6.9×10^{-4}	5.9×10^{-4}		6.3×10^{-4}	3.5×10^{-3}
O6	8.3×10^{-4}	6.4×10^{-4}		6.7×10^{-4}	1.2×10^{-2}
Aquifer average	9.0×10^{-4}	5.6×10^{-4}	1.4×10^{-3}	6.4×10^{-4}	4.8×10^{-3}

Şen (1987) method for P44 = $\sim 2.4 \times 10^{-5}$.

Modified Şen method (proposed method herein) for O1, O2, O3, O4, O5 and O6.

Discussion and conclusion

Şen (1987) has proposed for determination a unique storage coefficient value for confined and unconfined aquifers tapped by a large diameter well for the steady or quasi-steady-state flow conditions. The application of the method does not require any complicated mathematical procedure of classical type curve matching procedures. However, two critical points underlying the derivation of Eqs. (1) and (2) are the neglect of well losses and the finite radius of influence. None of these equations are valid for infinitesimally small diameter wells. The application of the Şen (1987) method becomes very effective when it is coupled with the Theim formula.

In this study, the method of Şen (1987) has been modified, extended and then applied to two sets of selected field data (confined and unconfined) with small diameter wells in steady or quasi-steady-state flow conditions. The storage coefficient estimates from the methodology presented in this paper conform practically very well with the results obtained from the traditional methods on the basis of order of magnitude. The application of the practical method proposed here is especially useful in the case of transmissivity values estimated from the very well known Thies method.

Application of this method to the selected data sets shows that the storage coefficient can be estimated more accurately using the present method even when only a few late drawdowns are used. The estimated storage coefficient values from late drawdowns are as good as those obtained using Thies or Jacob straight line methods for both confined and unconfined aquifers under quasi-steady-state flow conditions.

References

Ahmad, N. (1998). Evaluation of groundwater resources in the upper middle part of Chaj Doab area, Pakistan. PhD Dissertation. Istanbul Technical University, Institute of Science and Technology, Istanbul, Turkey.

Boulton, N.S. (1963). Analysis of data from nonequilibrium pumping test allowing for delayed yield from storage. *Proc. Inst. Civil Engrs., London*, **26**, 469–482.

Chow, V.T. (1952). On the determination of transmissivity and storage coefficients from pumping test data. *Am. Geophys. Union Trans.*, **33**, 397–404.

Ferris, J.G. (1963). Correction of drawdowns caused by a pumped well tapping less than the full thickness of an aquifer. In R. Pentall (Ed.), *Methods of Determining Permeability Transmissivity and Drawdown*. USGS Water Supply Paper, 1536, I.

Hantush, M.S. (1956). Analysis of data from pumping test in leaky aquifers. *Am. Geophys. Union Trans.*, **37**, 702–714.

Jacob, C.E. (1940). On the flow of water in an elastic aquifer. *Am. Geophys. Union Trans.*, **72**, Part II, 574–586.

Papadopulos, I.S. and Cooper, H.H. (1967). Drawdown in a well of large diameter. *Wat. Res. Res.*, **3**, 241–244.

Şen, Z. (1982). Type curves for large diameter wells near barriers. *Ground Water*, **20**(3), 274–277.

Şen, Z. (1986). Determination of aquifer parameters by the slope matching method. *Ground Water*, **24**(2), 217–223.

Şen, Z. (1987). Storage coefficient determination from quasi-steady state flow. *Nord. Hydrol.*, **18**, 101–110.

Şen, Z. (1995). *Applied Hydrogeology for Scientists and Engineers*, CRC Lewis, New York, 468.

Theim, G. (1906). *Hydrologische Methoden*, J. M. Gebhart, Leipzig, 56.

Thies, C.V. (1935). The relation between the lowering of piezometric surface and the rate and duration of discharge of a well using groundwater storage. *Am. Geophys. Union Trans.*, **16**, 519–524.