Discussion: "Second Law Analysis of Laminar Viscous Flow Through a Duct Subjected to Constant Wall Temperature" (Sahin, A. Z., 1998, ASME J. Heat Transfer, 120, pp. 76–83)

M. M. Awad
Faculty of Engineering and Applied Science,
Memorial University of Newfoundland,
St. John's, Newfoundland, A1B 3X5, Canada
e-mail: awad@engr.mun.ca

In the paper Sahin, A. Z., 1998, "Second Law Analysis of Laminar Viscous Flow Through a Duct Subjected to Constant Wall Temperature" ASME J. Heat Transfer, ***120****(1), pp. 76–83, there are many errors in equations, values, etc. The errors will be summarized below.* [DOI: 10.1115/1.2745836]

Based on the definition of the dimensionless temperature difference (τ)

$$\tau = \frac{T_w - T_o}{T_w} \tag{1}$$

For $T_w = 373$ K (Table 1) and $T_o = 293$ K (Tables 2*a*, and 3*a*), $\tau = 0.214 > \tau = 0.0 - 0.2$ (Table 1). Thus, the range of τ in Table 1 must be $\tau = 0.0 - 0.25$.

Based on Eqs. (8) and (9), then Eq. (10) must be

$$\psi = \left\{ \ln\left(\frac{1 - \tau^{e^{-4St\lambda}}}{1 - \tau} \right) - \tau(1 - e^{-4St\lambda}) + \frac{1}{8}\frac{f\tau Ec}{St} \ln\left(\frac{e^{4St\lambda} - \tau}{1 - \tau} \right) \right\} \Big/ (1 - e^{-4St\lambda}) \tag{2}$$

The major error in Eq. (10) leads to other many errors in the equations. First, Eq. (14) must be

$$\psi = \left\{ \ln\left(\frac{1 - \tau^{e^{-4\Pi_1}}}{1 - \tau} \right) - \tau(1 - e^{-4\Pi_1}) + 8\tau\Pi_2 \ln\left(\frac{e^{4\Pi_1} - \tau}{1 - \tau} \right) \right\} \Big/ (1 - e^{-4\Pi_1}) \tag{3}$$

Second, Eq. (17) must be

$$\psi = \left\{ \ln\left(\frac{1 - \tau^{e^{-4\Pi_1}}}{1 - \tau} \right) - \tau(1 - e^{-4\Pi_1}) + 8\tau\Pi_2 \left[\ln\left(\frac{e^{4\Pi_1} - \tau}{1 - \tau} \right) + \left(\frac{bT_{\mathrm{ref}}}{a} \right) \ln\left(\frac{e^{4\Pi_1} - \tau}{1 - \tau} \right) - 4\left(\frac{bT_w}{a} \right)\Pi_1 \right] \right\} \Big/ (1 - e^{-4\Pi_1}) \tag{4}$$

Third, Eq. (20) must be

$$\psi = \left\{ \ln\left(\frac{1 - \tau^{e^{-4\Pi_1}}}{1 - \tau} \right) - \tau(1 - e^{-4\Pi_1}) + 32\Pi_2 \times \int_0^l \frac{\Pi_1}{l} \frac{T_w - T_o}{T_{\mathrm{ref}}^n} T^{n-1} \exp\left[B\left(\frac{1}{T} - \frac{1}{T_{\mathrm{ref}}} \right) \right] dx \right\} \Big/ (1 - e^{-4\Pi_1}) \tag{5}$$

Finally, Eq. (22) must be

$$\psi' = \psi(1 - e^{-4\Pi_1}) \tag{6}$$

These errors in the above equations lead to errors in values in both figures and tables. For example, values of dimensionless entropy generation (ψ) are incorrect and must be calculated again based on the correct form of Eqs. (14), (17), and (20), respectively. In addition, based on the correct form of Eq. (22), values of dimensionless entropy generation (ψ) must be greater than values of modified dimensionless entropy generation (ψ') at the same modified Stanton number (Π_1) (not as shown, $\psi < \psi'$ at the same Π_1, in Figs. 2 and 3, Tables 2*a* and 3*a*).

On p. 81 (second column), the total dimensionless entropy change using Eq. (14) becomes

$$\psi = 32(\tau\Pi_2)\Pi_1/(1 - e^{-4\Pi_1}) \tag{7}$$

On pp. 81 and 82 (Tables 2*b*, 3*b*), values of T_o are in K, not in °C.

In addtion, Sahin [1] made a second law comparison for optimum shape of duct subjected to constant wall temperature and laminar flow. In this paper, the following error is found.

For laminar flow the thermal entry length may be expressed as [2,3]:

$$\left(\frac{x_{fd,t}}{D_H} \right)_{\mathrm{lam}} \approx 0.05\, \mathrm{Re}_{D_H} \mathrm{Pr} \tag{8}$$

From Table 1, the hydraulic diameter (D_H) for circular duct geometry can be expressed as

$$D_H = \frac{2}{\sqrt{\pi}} \sqrt{A_c} \tag{9}$$

Combining Eqs. (8) and (9), we obtain

$$(x_{fd,t})_{\mathrm{lam}} \approx \frac{0.1}{\sqrt{\pi}} \sqrt{A_c}\, \mathrm{Re}_{D_H} \mathrm{Pr} \tag{10}$$

From Table 2, $(A_c)_{\max} = 6 \times 10^{-7}$ m^2 and Pr=7. From Figs. 1–7, $(\mathrm{Re}_{D_H})_{\max} = 3000$. Thus,

$$(x_{fd,t})_{\mathrm{lam}} \approx \frac{0.1}{\sqrt{\pi}} \sqrt{6 \times 10^{-7}}(3000)(7) \approx 0.92 \text{ m} \tag{11}$$

The above value is greater than the length of the duct: $L = 0.1$ m (Table 2). This indicated that the flow is still in thermal entrance laminar region and does not reach fully developed laminar region. This error is repeated for other duct geometries such as square, triangle, etc. As a result, the assumption of the fluid is fully developed laminar as it enters the duct in all types of geometry is not acceptable.

References

[1] Sahin, A. Z., 1998, "A Second Law Comparison for Optimum Shape of Duct Subjected to Constant Wall Temperature and Laminar Flow," Heat Mass Transfer, **33**(5-6), pp. 425–430.
[2] Kays, W. M., Crawford, M. E., and Weigand, B., 2005, *Convective Heat and Mass Transfer*, McGraw Hill, New York.
[3] Incropera, F. P., and DeWitt, D. P., 2002, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, Inc., New York.

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